Roll No.

PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING

SET A

TEST 1

Sem & AY: Odd Sem. 2019-20
Course Code: MAT 105
Course Name: CALCULUS & LINEAR ALGEBRA
Program & Sem: B.Tech (All Program) & I

Date: 27.09.2019
Time: 1.00PM to 2.00PM
Max Marks: 30
Weightage: 15%

Instructions:

(i) *Read the question properly and answer accordingly.*
(ii) *Question paper consists of 3 parts.*
(iii) *Scientific and Non-programmable calculators are permitted.*

Part A [Memory Recall Questions]

Answer all the Questions. Each Question carries two marks. (5Qx2M=8M)

1. Define Cauchy's mean Value Theorem. (C.O.NO.1) [Knowledge]

2. State Rolle's theorem. (C.O.NO.1) [Knowledge]

3. Evaluate $\lim_{x \to 0} \frac{\sin^2 x}{x}$ (C.O.NO.1) [Comprehension]

Part B [Thought Provoking Questions]

Answer all the Questions. Each Question carries four marks. (3Qx4M=12M)

4. Expand $\sqrt{1+\sin 2x}$ by Maclaurin's series up to the term containing x^2

(C.O.NO.1) [Comprehension]

5. Verify Lagrange's Mean value theorem for $f(x) = e^x$ in $[0, 1]$

(C.O.NO.1) [Comprehension]

6. Evaluate $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$ (C.O.NO.1) [Comprehension]

Part C [Problem Solving Questions]

Answer both Questions. Each Question carries six marks. (2Qx6M=12M)

7. If f(x) and g(x) are respectively e^{x} and e^{-x}, prove that 'c' of Cauchy's mean value theorem is the arithmetic mean between a and b.

(C.O.NO. 1) [Comprehension]

8. Obtain Taylor's series expansion of $\log(\cos x)$ about the point $x=\frac{\pi}{3}$ up to third degree term.

(C.O. NO.1) [Comprehension]

SCHOOL OF ENGINEERING

Semester: 1st Sem

Course Code: MAT105

Course Name: Calculus and Linear Algebra

Date: 27/09/2019

Time: 1:00 PM to 2:00 PM

Max Marks: 30

Weightage: 15%

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels			Thought provoking type [Marks allotted] Bloom's Levels			Problem Solving type [Marks allotted] Bloom's Levels			Total Marks
			K	C	A	K	C	A	K	C	A	
1	CO 1	Module 1	2M									2M
2	CO 1	Module 1		2M								2M
3	CO 1	Module 1		2M								2M
4	CO 1	Module 1					4 M					4M
5	CO 1	Module 1					4 M					4M
6	CO 1	Module 1					4 M					4M
7	CO 1	Module 1								6 M		6M
8	CO 1	Module 1								6 M		6M
9	CO 1	Module 1								6 M		6M
	Total Marks						12			12		30 M

K =Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60% of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

[I hereby certify that All the questions are set as per the above guide lines. Dr V Ramalatha]

Reviewers' Comments

Annexure- II: Format of Answer Scheme

SCHOOL OF ENGINEERING

SOLUTION

Semester: 1st Sem

Course Code: MAT105

Course Name: Calculus and Linear Algebra

Date: 27/09/2019

Time 1:00 PM to 2:00 PM

Max Marks: 30

Weightage: 15%

Part A (3Q x 2M = 6 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	Def	2 M	3 M
2	Statement	2M	3 M
3	$Lt_{x \to 0} \frac{\sin^2 x}{x} = \frac{0}{0}$ Ans = 0	2 M	3 M

Part B (3Q x 4M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
4	Maclaurin's series $f(0)=1,\ f'(0)=1,\ f''(0)=-1$ Substitution	1 M 2 M 1 M	7 M
5	Continuous , Differentiable	1 M 1 M	7 M

	$f'(c) = \frac{f(b)-f(a)}{b-a}$ $c = 0.531$	1 M	
6	$\infty - \infty$ and $\lim_{x \to 0} \frac{x - \sin x}{x \sin x}$ $\lim_{x \to 0} \frac{x - \sin x}{x^2} \rightarrow \frac{0}{0}$ $\lim_{x \to 0} \frac{1 - \cos x}{2x} \rightarrow \frac{0}{0}$ Ans = 0	1 M 1 M 1 M 1M	7 M

Part C (2Q x 6M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
7	Continuous , Differentiable and $g'(x) \neq 0$ $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$ $c = \frac{a+b}{2} \in (a,b)$	2 M 1 M 3 M	10 M
8	Taylor's series $f(\pi/3) = \log(1/2)$, $f'(\pi/3) = -\sqrt{3}$, $f''(\pi/3) = -4$, $f'''(\pi/3) = -8\sqrt{3}$ Substitution	1 M 1M+1M 1M+1M 1 M	10 M
9	1^{∞} and Apply log on both sides $\lim_{x \to 0} \frac{\frac{1}{\sin x / x} \quad \frac{x\cos x - \sin x}{x^2}}{1}$ $\lim_{x \to 0} \frac{x\cos x - \sin x}{x^2}$ $\lim_{x \to 0} \frac{-x\sin x}{2x} = 0$ Ans) 1	1M 1 M 1 M 1 M 1 M 1M	10 M

Roll No.													

PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING — SET A

TEST - 2

Sem & AY: Odd Sem. 2019-20 | **Date**: 16.11.2019

Course Code: MAT 105 | **Time**: 1:00 PM to 2:00 PM

Course Name: CALCULUS & LINEAR ALGEBRA | **Max Marks**: 30

Program & Sem: B.Tech (All Programs) & I | **Weightage**: 15%

Instructions:

I. Read the question properly and answer accordingly.
II. Question paper consists of 3 parts.
III. Scientific and Non-programmable calculators are permitted.

Part A [Memory Recall Questions]

Answer both the Questions. Each Question carries three marks. (2Qx3M=6M)

1. Evaluate $\int_0^{\pi} x\cos^6 x\ dx$ (C.O.NO.3) [Knowledge]

2. If $u = \log\left[\frac{x^4+y^4}{x+y}\right]$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$ (C.O.NO.2) [Knowledge]

Part B [Thought Provoking Questions]

Answer both the Questions. Each Question carries six marks. (2Qx6M=12M)

3. Find the extreme values of the following function
$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ (C.O.NO.2) [Comprehension]

4. Evaluate $\int_0^{\infty} \frac{x^4}{\left(1+x^2\right)^4}\ dx$ (C.O.NO.3) [Comprehension]

Part C [Problem Solving Questions]

Answer the Question. The Question carries twelve marks. (1Qx12M=12M)

5. Obtain the reduction formula for $\int \sin^n x\ dx$ and $\int_0^{\pi/2} \sin^n x\ dx$ (C.O.NO.3) [Comprehension]

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

Semester: 1st Sem

Course Code: MAT105

Course Name: Calculus and Linear Algebra

Date: 16/11/2019

Time: 1:00 PM to 2:00 PM

Max Marks: 30

Weightage: 15%

Extract of question distribution [outcome wise & level wise]

Q.NO	C.O.NO	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels			Thought provoking type [Marks allotted] Bloom's Levels			Problem Solving type [Marks allotted]			Total Marks
			K	C	A	K	C	A	K	C	A	
1	CO 3	Module 3	3M									3M
2	CO 2	Module 2	3M									3M
3	CO 2	Module 2					6M					6M
4	CO 3	Module 3					6M					6M
5	CO 3	Module 3								12M		12M
	Total Marks		6M				12M			12M		30 M

K =Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

Annexure- II: Format of Answer Scheme

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

SOLUTION

Semester: 1st Sem
Course Code: MAT105
Course Name: Calculus and Linear Algebra

Date: 16 /11/2019
Time: 1:00 PM to 2:00 PM
Max Marks: 30
Weightage: 15%

Part A (2Q x 3M = 6 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	$I = \int_0^{\pi} (\pi - x)\cos^6(\pi - x)xdx$ $I = \pi \int_0^{\pi/2} \cos^6 x dx$ $5\pi/32$	1 Mark 1 Mark 1 Marks	5 Minutes
2	e^u is homogeneous function of degree n = 1 formula $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$	1 Marks 1 Mark 1 Mark	5 Minutes

Part B (2Q x 6M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
3	$f_x = 3x^2 + 3y^2 - 30x + 72 \quad f_y = 6xy - 30y$ Stationary points are (4, 0), (6, 0), (5, 1), (5, -1) A = 6x-30, B =6y, C = 6x-30 At (4,0) , A= -6<0 & AC – B^2 =36 > 0 Maximum value is f (4,0) = 112 At (6, 0) , A= 6 > 0 & AC – B^2 =-36 > 0 Minimum value is f (6, 0) = 108	1 Mark + 1 Mark 2 Mark 1 Mark 1 Mark	8 Minutes

4	$Put\ x=\tan\theta,\ dx=\sec^2\theta,\ \theta=0\ to\ \frac{\pi}{2}$ Using Substitution I = $\int_0^{\pi/2}\sin^2\theta\cos^3\theta\,d\theta$ Formula Using reduction formula I = 2/15	2 Marks 2 Mark 1 Mrak 1 Mark	8 Minutes

Part C (1Q x 12M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
5	$I_n=\int\sin^n x\ dx=\int\sin^{n-1}x\ \sin x\ dx$ Integration by parts $I_n=\frac{-\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}I_n$ $\int_0^{\pi/2}\sin^n x=\frac{n-1}{n}I_n$ I_0, I_1 $\int_0^{\pi/2}\sin^n x=\frac{n-1}{n}\frac{n-3}{2}....k$ Getting Where k=$\pi/2$ if n is even k=1 if n is *odd*	1 Mark 5 Mark 1 Marks 1 Mark 4 Mark	20 Minutes

Roll No.													

PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING — SET B

TEST - 2

Sem & AY: Odd Sem. 2019-20
Course Code: MAT 105
Course Name: CALCULUS & LINEAR ALGEBRA
Program & Sem: B.Tech (All Programs) & I

Date: 16.11.2019
Time: 1:00 PM to 2:00 PM
Max Marks: 30
Weightage: 15%

Instructions:

I. Read the question properly and answer accordingly.
II. Question paper consists of 3 parts.
III. Scientific and Non-programmable calculators are permitted.

Part A [Memory Recall Questions]

Answer both the Questions. Each Question carries three marks. (2Qx3M=6M)

1. Evaluate $\int_0^{\pi} x \sin^8 x \, dx$ (C.O.NO.2) [Knowledge]

2. If $u = \tan^{-1}\left[\frac{x^2+y^2}{x-y}\right]$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$

(C.O.NO.2) [Knowledge]

Part B [Thought Provoking Questions]

Answer both the Questions. Each Question carries six marks. (2Qx6M=12M)

3. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

(C.O.NO.2) [Comprehension]

4. Evaluate $\int_0^{\infty} \frac{x^2}{\left(1+x^2\right)^{7/2}} dx$ (C.O.NO.3) [Comprehension]

Part C [Problem Solving Questions]

Answer the Question. The Question carries twelve marks. (1Qx12M=12M)

5. Obtain the reduction formula for $\int \cos^n x \ dx$ and $\int_0^{\pi/2} \cos^n x \ dx$

(C.O.NO.3) [Comprehension]

SCHOOL OF ENGINEERING

Semester: 1st Sem
Course Code: MAT105
Course Name: Calculus and Linear Algebra

Date: 16/11/2019
Time: 1:00 PM to 2:00 PM
Max Marks: 30
Weightage: 15%

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels			Thought provoking type [Marks allotted] Bloom's Levels			Problem Solving type [Marks allotted]			Total Marks
			K	C	A	K	C	A	K	C	A	
1	CO 3	Module 3	3M									3M
2	CO 2	Module 2	3M									3M
3	CO 2	Module 2					6M					6M
4	CO 3	Module 3					6M					6M
5	CO 3	Module 3								12M		12M
	Total Marks		6M				12M			12M		30 M

K =Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

Annexure- II: Format of Answer Scheme

SCHOOL OF ENGINEERING

SOLUTION

Semester: 1st Sem | **Date:** 16 /11/2019

Course Code: MAT105 | **Time:** 1:00 PM to 2:00 PM

Course Name: Calculus and Linear Algebra | **Max Marks:** 30 | **Weightage:** 15%

Part A (2Q x 3M = 6 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	Using property $I = \int_0^{\pi} (\pi - x) \sin^8(\pi - x)\ dx$	1 Mark	6 Minutes
	Reducing to $I = \pi \int_0^{\pi/2} \sin^8 x\ dx$	1 Mark	
	Using Reduction formula $I = \frac{35\pi^2}{256}$	1 Mark	
2	$\tan u$ is homogeneous function of degree n = 1	1 Marks	6 Minutes
	$x\sec^2 u \frac{\partial u}{\partial x} + y\sec^2 u \frac{\partial u}{\partial y} = \tan u$	1 Mark	
	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\tan u}{\sec^2 u} = \sin u \cos u$	1 Mark	

Part B (2Q x 6M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
3	$f_x = 3x^2 - 3,\ f_y = 3y^2 - 12$	1 Mark	10 Minutes
	Stationary points are (1, 2), (1, -2), (-1, 2), (-1, -2)	1 Mark	
	A = 6x, B =0, C = 6y	1 Mark	
	At (-1, -2) , A= -6<0 & AC – B^2 =72 > 0	1 Mark	

	Maximum value is f (-1, -2) = 38 At (1, 2) , A= 6 > 0 & AC – B^2 =72 > 0 Minimum value is f (1, 2) = 2	1 Mark 1 Mark	
4	$Put\ x=\tan\theta,\ dx=\sec^2\theta,\ \theta=0\ to\ \frac{\pi}{2}$ Using Substitution I = $\int_0^{\pi/2}\sin^2\theta\cos^3\theta\,d\theta$ Using reduction formula I = 2/15	2 Marks 3 Marks 1 Mark	10 Minutes

Part C (1Q x 12M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
5	$I_n=\int\cos^n x\ dx=\int\cos^{n-1}x\ \cos x\ dx$ Integration by parts Simplification and getting $I_n=\int\cos^n x\ dx=\frac{\cos^{n-1}x\sin x}{n}+\frac{n-1}{n}I_{n-2}$ Applying limits $I_n=\frac{n-1}{n}I_{n-2}$ Using recurrence relation and obtaining $I_n=\frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\ldots\ldots\ldots\frac{2}{3}I_1\ if\ n\ is\ odd$ $I_n=\frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\ldots\ldots\ldots\frac{2}{3}I_0\ if\ n\ is\ even$ $I_1=1\ and\ I_0=\frac{\pi}{2}$ and the result	1 Mark 1 Mark 4 Marks 2 Marks 2 Marks 2 Marks	18 Minutes

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PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING — SET A

TEST - 2

Sem & AY: Odd Sem. 2019-20 | **Date**: 16.11.2019

Course Code: MAT 105 | **Time**: 1:00 PM to 2:00 PM

Course Name: CALCULUS & LINEAR ALGEBRA | **Max Marks**: 30

Program & Sem: B.Tech (All Programs) & I | **Weightage**: 15%

Instructions:

I. Read the question properly and answer accordingly.
II. Question paper consists of 3 parts.
III. Scientific and Non-programmable calculators are permitted.

Part A [Memory Recall Questions]

Answer both the Questions. Each Question carries three marks. (2Qx3M=6M)

1. Evaluate $\int_0^{\pi} x\cos^6 x\ dx$ (C.O.NO.3) [Knowledge]

2. If $u = \log\left[\frac{x^4+y^4}{x+y}\right]$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$ (C.O.NO.2) [Knowledge]

Part B [Thought Provoking Questions]

Answer both the Questions. Each Question carries six marks. (2Qx6M=12M)

3. Find the extreme values of the following function
$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ (C.O.NO.2) [Comprehension]

4. Evaluate $\int_0^{\infty} \frac{x^4}{\left(1+x^2\right)^4}\ dx$ (C.O.NO.3) [Comprehension]

Part C [Problem Solving Questions]

Answer the Question. The Question carries twelve marks. (1Qx12M=12M)

5. Obtain the reduction formula for $\int \sin^n x\ dx$ and $\int_0^{\pi/2} \sin^n x\ dx$ (C.O.NO.3) [Comprehension]

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

Semester: 1st Sem

Course Code: MAT105

Course Name: Calculus and Linear Algebra

Date: 16/11/2019

Time: 1:00 PM to 2:00 PM

Max Marks: 30

Weightage: 15%

Extract of question distribution [outcome wise & level wise]

Q.NO	C.O.NO	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels			Thought provoking type [Marks allotted] Bloom's Levels			Problem Solving type [Marks allotted]			Total Marks
			K	C	A	K	C	A	K	C	A	
1	CO 3	Module 3	3M									3M
2	CO 2	Module 2	3M									3M
3	CO 2	Module 2					6M					6M
4	CO 3	Module 3					6M					6M
5	CO 3	Module 3								12M		12M
	Total Marks		6M				12M			12M		30 M

K =Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

Annexure- II: Format of Answer Scheme

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

SOLUTION

Semester: 1st Sem
Course Code: MAT105
Course Name: Calculus and Linear Algebra

Date: 16 /11/2019
Time: 1:00 PM to 2:00 PM
Max Marks: 30
Weightage: 15%

Part A (2Q x 3M = 6 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	$I = \int_0^{\pi} (\pi - x)\cos^6(\pi - x)xdx$ $I = \pi \int_0^{\pi/2} \cos^6 x dx$ $5\pi/32$	1 Mark 1 Mark 1 Marks	5 Minutes
2	e^u is homogeneous function of degree n = 1 formula $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$	1 Marks 1 Mark 1 Mark	5 Minutes

Part B (2Q x 6M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
3	$f_x = 3x^2 + 3y^2 - 30x + 72 \quad f_y = 6xy - 30y$ Stationary points are (4, 0), (6, 0), (5, 1), (5, -1) A = 6x-30, B =6y, C = 6x-30 At (4,0) , A= -6<0 & AC – B^2 =36 > 0 Maximum value is f (4,0) = 112 At (6, 0) , A= 6 > 0 & AC – B^2 =-36 > 0 Minimum value is f (6, 0) = 108	1 Mark + 1 Mark 2 Mark 1 Mark 1 Mark	8 Minutes

4	$Put\ x=\tan\theta,\ dx=\sec^2\theta,\ \theta=0\ to\ \frac{\pi}{2}$ Using Substitution I = $\int_0^{\pi/2}\sin^2\theta\cos^3\theta\,d\theta$ Formula Using reduction formula I = 2/15	2 Marks 2 Mark 1 Mrak 1 Mark	8 Minutes

Part C (1Q x 12M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
5	$I_n=\int\sin^n x\ dx=\int\sin^{n-1}x\ \sin x\ dx$ Integration by parts $I_n=\frac{-\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}I_n$ $\int_0^{\pi/2}\sin^n x=\frac{n-1}{n}I_n$ I_0, I_1 $\int_0^{\pi/2}\sin^n x=\frac{n-1}{n}\frac{n-3}{2}....k$ Getting Where k=$\pi/2$ if n is even k=1 if n is *odd*	1 Mark 5 Mark 1 Marks 1 Mark 4 Mark	20 Minutes

Roll No.													

PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING — SET B

TEST - 2

Sem & AY: Odd Sem. 2019-20
Course Code: MAT 105
Course Name: CALCULUS & LINEAR ALGEBRA
Program & Sem: B.Tech (All Programs) & I

Date: 16.11.2019
Time: 1:00 PM to 2:00 PM
Max Marks: 30
Weightage: 15%

Instructions:

I. Read the question properly and answer accordingly.
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III. Scientific and Non-programmable calculators are permitted.

Part A [Memory Recall Questions]

Answer both the Questions. Each Question carries three marks. (2Qx3M=6M)

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2. If $u = \tan^{-1}\left[\frac{x^2+y^2}{x-y}\right]$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$

(C.O.NO.2) [Knowledge]

Part B [Thought Provoking Questions]

Answer both the Questions. Each Question carries six marks. (2Qx6M=12M)

3. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

(C.O.NO.2) [Comprehension]

4. Evaluate $\int_0^{\infty} \frac{x^2}{\left(1+x^2\right)^{7/2}} dx$ (C.O.NO.3) [Comprehension]

Part C [Problem Solving Questions]

Answer the Question. The Question carries twelve marks. (1Qx12M=12M)

5. Obtain the reduction formula for $\int \cos^n x \ dx$ and $\int_0^{\pi/2} \cos^n x \ dx$

(C.O.NO.3) [Comprehension]

SCHOOL OF ENGINEERING

Semester: 1st Sem
Course Code: MAT105
Course Name: Calculus and Linear Algebra

Date: 16/11/2019
Time: 1:00 PM to 2:00 PM
Max Marks: 30
Weightage: 15%

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels			Thought provoking type [Marks allotted] Bloom's Levels			Problem Solving type [Marks allotted]			Total Marks
			K	C	A	K	C	A	K	C	A	
1	CO 3	Module 3	3M									3M
2	CO 2	Module 2	3M									3M
3	CO 2	Module 2					6M					6M
4	CO 3	Module 3					6M					6M
5	CO 3	Module 3								12M		12M
	Total Marks		6M				12M			12M		30 M

K =Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

Annexure- II: Format of Answer Scheme

SCHOOL OF ENGINEERING

SOLUTION

Semester: 1st Sem | **Date:** 16 /11/2019

Course Code: MAT105 | **Time:** 1:00 PM to 2:00 PM

Course Name: Calculus and Linear Algebra | **Max Marks:** 30 | **Weightage:** 15%

Part A (2Q x 3M = 6 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	Using property $I = \int_0^{\pi} (\pi - x) \sin^8(\pi - x)\ dx$	1 Mark	6 Minutes
	Reducing to $I = \pi \int_0^{\pi/2} \sin^8 x\ dx$	1 Mark	
	Using Reduction formula $I = \frac{35\pi^2}{256}$	1 Mark	
2	$\tan u$ is homogeneous function of degree n = 1	1 Marks	6 Minutes
	$x\sec^2 u \frac{\partial u}{\partial x} + y\sec^2 u \frac{\partial u}{\partial y} = \tan u$	1 Mark	
	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\tan u}{\sec^2 u} = \sin u \cos u$	1 Mark	

Part B (2Q x 6M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
3	$f_x = 3x^2 - 3,\ f_y = 3y^2 - 12$	1 Mark	10 Minutes
	Stationary points are (1, 2), (1, -2), (-1, 2), (-1, -2)	1 Mark	
	A = 6x, B =0, C = 6y	1 Mark	
	At (-1, -2) , A= -6<0 & AC – B^2 =72 > 0	1 Mark	

	Maximum value is f (-1, -2) = 38 At (1, 2) , A= 6 > 0 & AC – B^2 =72 > 0 Minimum value is f (1, 2) = 2	1 Mark 1 Mark	
4	$Put\ x=\tan\theta,\ dx=\sec^2\theta,\ \theta=0\ to\ \frac{\pi}{2}$ Using Substitution I = $\int_0^{\pi/2}\sin^2\theta\cos^3\theta\,d\theta$ Using reduction formula I = 2/15	2 Marks 3 Marks 1 Mark	10 Minutes

Part C (1Q x 12M = 12 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
5	$I_n=\int\cos^n x\ dx=\int\cos^{n-1}x\ \cos x\ dx$ Integration by parts Simplification and getting $I_n=\int\cos^n x\ dx=\frac{\cos^{n-1}x\sin x}{n}+\frac{n-1}{n}I_{n-2}$ Applying limits $I_n=\frac{n-1}{n}I_{n-2}$ Using recurrence relation and obtaining $I_n=\frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\ldots\ldots\ldots\frac{2}{3}I_1\ if\ n\ is\ odd$ $I_n=\frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\ldots\ldots\ldots\frac{2}{3}I_0\ if\ n\ is\ even$ $I_1=1\ and\ I_0=\frac{\pi}{2}$ and the result	1 Mark 1 Mark 4 Marks 2 Marks 2 Marks 2 Marks	18 Minutes

Roll No												

PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING — SET A

END TERM FINAL EXAMINATION

Semester: Odd semester- 2019-20 | **Date:** 03 January 2020

Course Code: MAT 105 | **Time:** 1.00 PM to 4.00 PM

Course Name: CALCULUS AND LINEAR ALGEBRA | **Max Marks:**100

Program &Sem : B.Tech (Physics & Chemistry cycle) & I | **Weightage:**50%

Instructions:

(i) Read the all questionscarefully and answer accordingly.

(ii) Scientific and Non-programmable calculators are permitted.

Part A[Memory Recall Questions]

Answer all the Questions. Each Question carries 02 marks. **(10Qx2M=20M)**

1. Lagrange's mean value theorem states that if $f(x)$ is continuous in _____ (ii)differentiable in _____ then there exists at least one value '*c*' in (*a, b*) such that___ (C.O.No.5) [Knowledge]

2. $\lim_{x \to 0} \frac{a^x - b^x}{x}$ =_____ (C.O.No.1) [Comprehension]

3. If $z = y\sin x + e^{xy}$ then $\frac{\partial z}{\partial y}$ = _____ (C.O.No.2) [Knowledge]

4. If $u = \frac{x^3 + y^3}{\sqrt{x} - \sqrt{y}}$, then by Euler's theorem $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ _____ (C.O.No.2) [Knowledge]

5. $\int_0^\infty x^{3/2} e^{-x} dx =$_____ (C.O.No.3) [Knowledge]

6. $\beta\left(\frac{5}{2}, \frac{5}{2}\right)$ =_____ (C.O.No.3) [Comprehension]

7. Is $\sum \frac{1}{n^{1/2}}$ convergent or divergent? (C.O.No.4) [Knowledge]

8. The half range cosine series of f(x) in (0, π) is _____where a_n = _____ (C.O.No.4) [Knowledge]

9. Rank of $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ = _____ (C.O.No.5) [Knowledge]

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

SCHOOL OFENGINEERING

END TERM FINAL EXAMINATION

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO (% age of CO)	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels			Thought provoking type [Marks allotted] Bloom's Levels			Problem Solving type [Marks allotted]			Total Marks
			K	C	A	K	C	A	K	C	A	
1	1	1	2M									2M
2	1	1		2M								2M
3	2	2	2M									2M
4	2	2	2M									2M
5	3	3		2M								2M
6	3	3	2M									2M
7	4	4	2M									2M
8	4	4	2M									2M
9	5	5	2M									2M
10	5	5	2M									2M
11	1	1				10M						10M
12	2	2				10M						10M
13	3	3				10M						10M
14	4	4					10M					10M
15	5	5					10M					10M
16	3	3								15M		15M
17	5	5							15M			15M
	Total Marks		16M	4M		30M	20M		15M	15M		100M

K =Knowledge Level C = Comprehension Level, A = Application Level

8	$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$	2M	4 Minutes
9	2	2M	4 Minutes
10	1,6	2M	4 Minutes

Part B (5Q x 10M = 50 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
11 (i)	(i). Taylor's series about the point $x=\frac{\pi}{4}$ $f(x)=f\left(\frac{\pi}{4}\right)+\left(x-\frac{\pi}{4}\right)f'\left(\frac{\pi}{4}\right)+\frac{\left(x-\frac{\pi}{4}\right)^2}{2!}f''\left(\frac{\pi}{4}\right)+.....$ $f\left(\frac{\pi}{4}\right)=1, f'\left(\frac{\pi}{4}\right)=2, f''\left(\frac{\pi}{4}\right)=8, f'''\left(\frac{\pi}{4}\right)=16$ $f(x)=1+2\left(x-\frac{\pi}{4}\right)+2\left(x-\frac{\pi}{4}\right)^2+\frac{8}{3}\left(x-\frac{\pi}{4}\right)^3$	2 Marks 3 Marks 1 Marks	6Minutes
11 (ii)	b. Maclaurin's series $f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+.....$ $f(0)=0, f'(0)=1, f''(0)=-1,...$ $f(x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$	1 Mark 2 Marks 1 Mark	4 Minutes
12	$f_x=3x^2-3$ $f_y=3y^2-12$ Stationary points are (1, 2), (1, -2), (-1, 2), (-1, -2) A = 6x, B = 0, C = 6y At (-1, -2) , A= -6<0 & AC – B^2 =72> 0 Maximum value is f (-1, -2) = 38 At (1, 2) , A= 6 > 0 & AC – B^2 =72> 0 Minimum value is f (1, 2) = 2	2 Marks 2 Marks 2 Marks 2 Marks 2 Marks	15 Minutes
13	Applying the property $\int_0^{\pi}f(x)dx=\int_0^{\pi}f(\pi-x)dx$ Upto Applying even function property Changing the limit and integrating	2 marks 4 marks 4marks	15 Minutes

	$I_n = \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\ldots\ldots\ldots\frac{2}{3}I_1$ *if n is odd* $I_n = \frac{n-1}{n}\frac{n-3}{n-2}\frac{n-5}{n-4}\ldots\ldots\ldots\ldots\frac{2}{3}I_0$ *if n is even* $I_1 = 1$ *and* $I_0 = \frac{\pi}{2}$ and the result	3 Marks	
17	Characteristic equation $\begin{vmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{vmatrix} = 0$	1 Mark	25 Minutes
	Eigen values are $\lambda = 3,6,9$	4 Marks	
	$(7-\lambda)x - 2y = 0$ System of equations are $-2x + (6-\lambda)y - 2z = 0$ $-2y + (5-\lambda)z = 0$	1 Mark	
	$\lambda_1 = 3 \quad X_1 = [1, 1, 1]^T$	3 Marks	
	$\lambda_1 = 6 \quad X_2 = [1, 2, 1]^T$	3 Marks	
	$\lambda_1 = 9 \quad X_3 = [1, -2, 1]^T$	3 Marks	

Roll No												

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING — SET B

END TERM FINAL EXAMINATION

Semester: Odd Semester: 2019 - 20
Course Code: MAT 105
Course Name: CALCULUS AND LINEAR ALGEBRA
Program & Sem: B.Tech (Physics & Chemistry Cycle) & I

Date: 03 January 2020
Time: 1:00 PM to 4:00 PM
Max Marks: 100
Weightage: 50%

Instructions:
(i) Read the all questions carefully and answer accordingly.
(ii) Scientific and Non-programmable calculators are permitted.

Part A [Memory Recall Questions]

Answer all the Questions. Each Question carries 2 marks. **(10Qx2M=20M)**

1. If a function $f(x)$ is (i)_____ in [a, b], (ii)_____ in (a, b) then by Cauchy's mean value theorem there exist at least one point c in (a, b) such that $f'(c) =$ _____ (C.O.No.1) [Knowledge]

2. $\lim_{x\to 0} \frac{\tan^2 x}{x} =$ _____ (C.O.No.1) [Comprehension]

3. If $u = x^y$ then $\frac{\partial u}{\partial x} =$ _____ and $\frac{\partial u}{\partial y} =$ _____ (C.O.No.2) [Comprehension]

4. If $u = \frac{2x^5 - 4y^5}{x^2 - 2y^2}$ then by Euler's theorem $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$_____ (C.O.No.2) [Comprehension]

5. The value of $\int_0^\infty x^{1/2} e^{-x} dx =$ is _____ (C.O.No.3) [Comprehension]

6. $\beta\left(\frac{1}{2}, \frac{1}{2}\right) =$ _____ (C.O.No.3) [Comprehension]

7. The half range sine series of f(x) in $(0, \pi)$ is _____ where $b_n =$ _____ (C.O.No.4) [Comprehension]

8. Is $\sum \frac{1}{n^{5/2}}$ convergent or divergent? _____ (C.O.No.1) [Knowledge]

9. Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 3 & 4 \end{bmatrix}$ is _____ (C.O.No.5) [Comprehension]

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

SCHOOL OF ENGINEERING

END TERM FINAL EXAMINATION

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO (% age of CO)	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels		Thought provoking type [Marks allotted] Bloom's Levels		Problem Solving type [Marks allotted]		Total Marks
			K	C	K	C	K	C	
1	CO 1	Module 1	2M						2M
2	CO 1	Module 1		2M					2M
3	CO 2	Module 2		2M					2M
4	CO 2	Module 2		2M					2M
5	CO 3	Module 3		2M					2M
6	CO 3	Module 3		2M					2M
7	CO 4	Module 4		2M					2M
8	CO 4	Module 4	2M						2M
9	CO 5	Module 5	2M						2M
10	CO 5	Module 5		2M					2M
11	CO 1	Module 1				10M			10M
12	CO 2	Module 2				10M			10M
13	CO 4	Module 4				10M			10M
14	CO 4	Module 4				10M			10M
15	CO 5	Module 5				10M			10M
16	CO 3	Module 3						15M	15M
17	CO 5	Module 5						15M	15M
	Total Marks		20M		50M		30M		100M

K =Knowledge Level C = Comprehension Level, A = Application Level

SCHOOL OF ENGINEERING

SOLUTION

Semester: Odd Sem. 2019-20
Course Code: MAT105
Course Name: CALCULUS AND LINEAR ALGEBRA
Program & Sem: B.TECH (All Programs) & I

Date: 03.01.2020
Time: 1:00PM to 4:00PM
Max Marks: 100
Weightage: 50%

Part A (10Q x 2M = 20Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	Continuous, Differentiable $f'(c) = \frac{f(b)-f(a)}{b-a}$	1 Mark 1 Mark	3 Minutes
2	$\lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} \frac{\sin x}{x} \lim_{x\to 0} \sin x$ = 1 x 0 = 0	2 Marks	3 Minutes
3	$\frac{\partial u}{\partial x} = y^2$ $\frac{\partial^2 u}{\partial x^2} = 0$	1 Mark 1 Mark	3 Minutes
4	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$	2 Marks	3 Minutes
5	$\int_0^\infty e^{-x}\, x^{5/2}\, dx = \Gamma\left(\frac{3}{2}\right)$ $= \frac{\sqrt{\pi}}{2}$	2 Marks	3 Minutes
6	$\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{\Gamma(1/2)\Gamma(1/2)}{\Gamma(1/2+1/2)}$ $= \pi$	2 Marks	4 Minutes
7	$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$	1 Mark 1 Mark	3 Minutes

	At (6, 0) , A= 6 > 0 & AC – B^2 =36 > 0 Minimum value is f (6, 0) = 108	2 Marks	
13	a. Here $u_n = \dfrac{n^2}{(3n+1)(3n+4)(3n+7)}$ Take $v_n = \dfrac{1}{n}$ $\lim\limits_{n\to\infty} \dfrac{u_n}{v_n} = \dfrac{1}{27} \neq 0$ By Comparison test both series converge or diverge together By p-series $\sum v_n$ is divergent, Hence $\sum u_n$ is divergent	1 Mark 1 Mark 2 Marks 1 Mark	9 Minutes
	b. Here $u_n = \dfrac{n^p}{n!}, u_{n+1} = \dfrac{(n+1)^p}{(n+1)!}$ $\lim\limits_{n\to\infty} \dfrac{u_n}{u_{n+1}} = \dfrac{n}{\left(1+\frac{1}{n}\right)^{p-1}} = \infty > 1$ By D'Alembert's test given series is convergent	2 Marks 2 Marks 1 Mark	8 Minutes
14	half range cosine series of the function $f(x)$ in $(0, 2)$ is $f(x) = \dfrac{a_0}{2} + \sum\limits_{n=1}^{\infty} a_n \, Cos\left(\dfrac{n\pi x}{2}\right)$ $a_0 = \dfrac{2}{l}\int\limits_0^l f(x)\,dx = \dfrac{2}{2}\int\limits_0^2 x\,dx = 2$ $a_n = \dfrac{2}{2}\int\limits_0^2 x\cos\left(\dfrac{n\pi x}{2}\right)dx = \dfrac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$ $f(x) = 1 + \dfrac{4}{\pi^2}\sum\limits_{n=1}^{\infty} \dfrac{\left[(-1)^n - 1\right]}{n^2}\cos\left(\dfrac{n\pi x}{2}\right)$	2 Marks 2 Marks 4 Marks 2 Marks	17 Minutes
15	Augmented matrix interchanging row1 and row 3 $[A:B] = \begin{bmatrix} 1 & -1 & 5 & : 7 \\ 2 & 10 & 1 & : 13 \\ 10 & 1 & 1 & : 12 \end{bmatrix}$ Using Row transformation Step 1: $[A:B] = \begin{bmatrix} 1 & 1 & 5 & : 7 \\ 0 & 8 & -9 & : -1 \\ 0 & -9 & -49 & : -58 \end{bmatrix}$	2 Marks 2 Marks	17 Minutes

	System of equations are $(1-\lambda)x+y+3z=0$ $x+(5-\lambda)y+z=0$ $3x+y+(1-\lambda)z=0$ $\lambda_1=-2 \quad X_1=[1, 0, -1]$ $\lambda_1=3 \quad X_2=[1, -1, 1]$ $\lambda_1=6 \quad X_3=[1, 2, 1]$	3 Marks 3 Marks 3 Marks	