Roll No.													

PRESIDENCY UNIVERSITY BENGALURU

SCHOOL OF ENGINEERING

TEST – 2

Sem & AY: Odd Sem 2019-20 **Date:** 16.11.2019

Course Code: ECE 314 **Time:** 9.30 AM to 10.30 AM

Course Name: LINEAR ALGEBRA FOR COMMUNICATION ENGINEERING **Max Marks:** 40

Program & Sem: B.Tech & VII **Weightage:** 20%

Instructions:

(i) Read the questions carefully and answer them all

(ii) Only non-programmable scientific calculators are allowed

(iii) Part A requires no explanation. Part-B & Part-C require explanation wherever necessary.

Part A [Memory Recall Questions]

Answer all the Questions. Each Question carries one mark. (10Qx1M=10M)

1.

(a) The symmetric matrices in M form a subspace. True or False?

(b) One particular solution x_p has all free variables equal to zero. True or False?

(c) If A is square and invertible then $Ax = b$ has infinite solutions. True or False?

(d) The pivot columns are one basis for the row space. True or False?

(e) If the row space equals the column space then $A^T = A$. True or False?

(f) The __________ is a subspace of R^n. It contains all solutions to $Ax = 0$.

(g) The rank r is the number of __________

(h) The complete solution to $Ax = 0$ is a combination of the __________ solutions.

(i) The special solutions are a basis for ______________ subspace.

(j) _____________ is the orthogonal complement of the column space.

(Q.No.1(a to j)(C.O.NO.2)[Knowledge]

Part B [Thought Provoking Questions]

Answer both the Questions. Each Question carries five marks. (2Qx5M=10M)

2. (a) Find the projection matrix P_C onto the column space of A.

$$A = \begin{bmatrix} 3 & 6 & 6 \\ 4 & 8 & 8 \end{bmatrix}$$

(b) Find the 3 by 3 projection matrix P_R onto the row space of A.

(C.O.NO.2) [Application]

3. For which right sides (find a condition on b_1, b_2, b_3) are these systems solvable?

(a) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ (C.O.NO.2) [Application]

Part C [Problem Solving Questions]

Answer both the Questions. Each Question carries ten marks. (2Qx10M=20M)

4. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$ find $\hat{x}, p$ *and* P. (C.O.NO.2) [Comprehension]

5. Find the complete solution in the form $x_p + x_n$ to these full rank systems

(a) $x + y + z = 4$ (b) $x + y + z = 4;\ \ x - y + z = 4$ (C.O.NO.2) [Comprehension]

SCHOOL OF ENGINEERING

Semester: 7

Course Code: ECE 314

Course Name: LACE

Date: 16/11/2019

Time: 9 30 am to 10.30 am

Max Marks: 40

Weightage: 20%

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels			Thought provoking type [Marks allotted] Bloom's Levels			Problem Solving type [Marks allotted]			Total Marks
			K			C			A			
1	2	2	10									
2	2	2							5			
3	2	2							5			
4	2	2				10						
5	2	2				10						
	Total Marks		10			20			10			40

K =Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

Annexure- II: Format of Answer Scheme

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

SOLUTION

Date: 16/11/2019

Semester: 7

Time: 9.30 am to 10.30 am

Course Code: ECE 314

Max Marks: 40

Course Name: LACE

Weightage: 20%

Part A (10Q x 1M = 10Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	True	1	0.5min
	True	1	0.5min
	False	1	0.5min
	False	1	0.5min
	False	1	0.5min
	nullspace	1	0.5min
	pivots	1	0.5min
	special	1	0.5min
	nullspace	1	0.5min
	Left nullspace	1	0.5min

Part B (2Q x 5M = 10 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question

2	a) $A = \begin{bmatrix} 3 & 6 & 6 \\ 4 & 8 & 8 \end{bmatrix}$ Column space is space spanned by $\begin{pmatrix}3\\4\end{pmatrix} \rightarrow$ $P_c = \dfrac{aa^T}{a^Ta}$ $aa^T = \begin{pmatrix}3\\4\end{pmatrix}(3\ 4) = \begin{pmatrix}9 & 12\\12 & 16\end{pmatrix}$. $a^Ta = (3\ 4)\begin{pmatrix}3\\4\end{pmatrix} = 25$ $\therefore P_c = \dfrac{1}{25}\begin{pmatrix}9 & 12\\12 & 16\end{pmatrix}$. b) Row space is spanned by $b = \begin{pmatrix}3\\6\\6\end{pmatrix} \rightarrow \mathbb{R}^1$. or $\begin{pmatrix}1\\2\\2\end{pmatrix}$. $\therefore P_R = \dfrac{bb^T}{b^Tb}$ $bb^T = \begin{pmatrix}3\\6\\6\end{pmatrix}(3\ 6\ 6) = \begin{pmatrix}9 & 18 & 18\\18 & 36 & 36\\18 & 36 & 36\end{pmatrix}$ $b^Tb = (3\ 6\ 6)\begin{pmatrix}3\\6\\6\end{pmatrix} = 9+36+36 = 81$ $P_R = \dfrac{1}{81}\begin{pmatrix}9 & 18 & 18\\18 & 36 & 36\\18 & 36 & 36\end{pmatrix}$	2.5+2.5	8mins
3	3) a) $[A \mid b]$ augmented matrix $\rightarrow \left(\begin{array}{ccc\|c} 1 & 4 & 2 & b_1 \\ 2 & 8 & 4 & b_2 \\ -1 & -4 & -2 & b_3 \end{array}\right) \rightarrow$ $\begin{pmatrix}1 & 4 & 2 & b_1\\0 & 0 & 0 & b_2-2b_1\\0 & 0 & 0 & b_3+b_1\end{pmatrix}$ $b_3+b_1 = 0 \Rightarrow \boxed{b_1 = -b_3}$. $b_2 - 2b_1 = 0 \rightarrow \boxed{b_2 = 2b_1}$ b) $\begin{pmatrix}1 & 4\\2 & 9\\-1 & -4\end{pmatrix}\begin{matrix}b_1\\b_2\\b_3\end{matrix} \longrightarrow \begin{pmatrix}1 & 4 & b_1\\0 & 1 & b_2-2b_1\\0 & 0 & b_3+b_1\end{pmatrix}$ $\boxed{b_1 = -b_3}$	2.5+2.5	8mins

Part C (1Q x 10M = 20Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
4	$A^TA = \begin{pmatrix}1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}1&0\\1&1\\1&2\end{pmatrix} = \begin{pmatrix}3&3\\3&5\end{pmatrix}$. $A^Tb = \begin{pmatrix}1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}6\\0\\0\end{pmatrix} = \begin{pmatrix}6\\0\end{pmatrix}$. $\hat{x} = (A^TA)^{-1}A^Tb \rightarrow A^TA\hat{x} = A^Tb$. $\begin{pmatrix}3&3\\3&5\end{pmatrix}\begin{pmatrix}\hat{x}_1\\\hat{x}_2\end{pmatrix} = \begin{pmatrix}6\\0\end{pmatrix}$. $\begin{pmatrix}3&3&6\\3&5&0\end{pmatrix} \rightarrow \begin{pmatrix}3&3&6\\0&2&-6\end{pmatrix} \rightarrow \begin{pmatrix}1&1&2\\0&1&-3\end{pmatrix}$ $\hat{x}_2 = -3,\quad \hat{x}_1 + \hat{x}_2 = 2 \quad \hat{x}_1 = 5$. $\therefore \hat{x} = \begin{pmatrix}5\\-3\end{pmatrix}$ $p = A\hat{x}$ $= \begin{pmatrix}1&0\\1&1\\1&2\end{pmatrix}\begin{pmatrix}5\\-3\end{pmatrix} = \begin{pmatrix}5\\2\\-1\end{pmatrix}$ $P = A(A^TA)^{-1}A^T$ $(A^TA)^{-1}$: $\left(\begin{array}{cc\|cc}3&3&1&0\\3&5&0&1\end{array}\right) \rightarrow \begin{array}{cccc}3&3&1&0\\0&2&-1&1\end{array} \rightarrow \left(\begin{array}{cccc}1&1&1/3&0\\0&1&-1/2&1/2\end{array}\right)$ $\left(\begin{array}{cccc}1&0&5/6&-1/2\\0&1&-1/2&1/2\end{array}\right)$ $(A^TA)^{-1} = \begin{pmatrix}\frac{5}{6}&\frac{-1}{2}\\\frac{-1}{2}&\frac{1}{2}\end{pmatrix}$. $A(A^TA)^{-1} = \begin{pmatrix}1&0\\1&1\\1&2\end{pmatrix}\begin{pmatrix}\frac{5}{6}&-\frac{1}{2}\\\frac{-1}{2}&\frac{1}{2}\end{pmatrix} = \begin{pmatrix}\frac{5}{6}&\frac{-1}{2}\\\frac{2}{6}&0\\\frac{-1}{6}&\frac{1}{2}\end{pmatrix}$ $A(A^TA^{-1})A^T = \begin{pmatrix}\frac{5}{6}&\frac{-3}{6}\\\frac{2}{6}&0\\\frac{-1}{6}&\frac{3}{6}\end{pmatrix}\begin{pmatrix}1&1&1\\0&1&2\end{pmatrix} = \begin{pmatrix}\frac{5}{6}&\frac{2}{6}&\frac{-1}{6}\\2/6&2/6&2/6\\-1/6&2/6&5/6\end{pmatrix}$ 3x2 2x3	5+5	10 min

5	a) $x + y + z = 4$ $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$ (1×3, 3×1) y & z are free variables. $x_p \rightarrow$ when $y=0$, $z=0 \Rightarrow x=4$. $\therefore x_p = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$ $x_n \rightarrow$ when $y=1$ & $z=0 \rightarrow Ax=0$ solution for. $x+1+0=0 \Rightarrow x=-1$. $\therefore y\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ when $z=1$ & $y=0$, $x+1=0$ $x=-1$ $x_{n_2} = z\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\therefore$ complete solution $= \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. b) $x+y+z=4$ $x-y+z=4$ $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$ (2×3, 3×1) ~~when y=0, z=0~~ $\left(\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 1 & -1 & 1 & 4 \end{array}\right) = \begin{array}{cccc} 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 0 \end{array} \quad \begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 0 \end{array}$ $= \left(\begin{array}{ccc\|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 0 \end{array}\right)$ z is the free variable	5+5	10min

When $z = 0$

$x + y = 4$
$x - y = 4$

$$\begin{pmatrix} 1 & 1 & 4 \\ 1 & -1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \\ 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \end{pmatrix}.$$

$y = 0 \quad x = 4.$

$\therefore x_p = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}.$

When $z = 1$ solve $Ax = 0$.

$x + y + 1 = 0 \to x + y = -1$
$x - y = -1$

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \downarrow$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \leftarrow \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \end{pmatrix} \leftarrow \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \end{pmatrix}$$

$y = 0 \; ; \; x = -1.$

$\therefore x_n = z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$\therefore$ complete solution $= \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$

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GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

PRESIDENCY UNIVERSITY
BENGALURU

SCHOOL OF ENGINEERING

END TERM FINAL EXAMINATION

Semester: Odd Semester: 2019 - 20

Course Code: ECE 314

Course Name: LINEAR ALGEBRA FOR COMMUNICATION ENGINEERING

Program & Sem: B.Tech. (ECE) & VII (DE-III)

Date: 20 December 2019

Time: 9.30 AM to 12.30 PM

Max Marks: 80

Weightage: 40%

Instructions:

(i) Read all the questions carefully and answer accordingly.

(ii) Scientific and non-programmable calculators may be used.

Part A [Memory Recall Questions]

Answer all the Questions. **(2Q=15M)**

(5Qx2M=10M)

1. For a matrix $A_{3\times4}$, write down the following:

(a) The elimination matrix E which subtracts twice of row 1 from row 2.
(b) The permutation matrix P which exchanges row 2 and row 3.
(c) If its rank r is 2, how many solutions can it take?
(d) If it has full row rank, how many solutions can it take?
(e) What matrix should you multiply to make it square and symmetric?

(Q.No. a to e) (C.O.No.1) [Knowledge]

2. Fill in the blanks **(5Qx1M=5M)**

(a) If we try to fit *m* points by a combination of *n*<*m* functions, the *m* equations $Ax = b$ are generally unsolvable. The *n* equations __________ give the least squares solution with the smallest mean square error.
(b) Diagonalization makes *A*=_________ with an orthogonal matrix *Q*.
(c) Gram-Schmidt produces _______ vectors from independent vectors.
(d) The orthonormal columns of *U* and *V* in Singular Value Decomposition are eigenvectors of the ______ and _______ respectively.
(e) The eigenvalues of a symmetric matrix are ______.

(Q.No. a to e) (C.O.No.3) [Knowledge]

Part B [Thought Provoking Questions]

Answer all the Questions. Each Question carries 7 marks. **(5Qx7M=35M)**

3. The matrix form of Gaussian elimination is introduced as *LU* decomposition by the Polish mathematician Tadeusz Banachiewicz in 1938. Factorize the square matrix $\begin{bmatrix} 3 & 1 & 6 \\ -6 & 0 & -16 \\ 0 & 8 & -17 \end{bmatrix}$ into lower and upper triangular matrices through elimination. (C.O.No.1) [Comprehension]

4. Elimination and back substitution helps in solving a system of linear equations. If so, using the same, find the solution of the system $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$.

(C.O.No.1) [Comprehension]

5. For the square matrix $\begin{bmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{bmatrix}$, the sum of dimensions of its eigenspaces is equal to its own dimension. The matrix is then said to be diagonalizable or nondefective. Find its diagonal matrix and corresponding eigenvector matrix.

(C.O.No.3) [Comprehension]

6. If you are in a space of R^4, find the projection matrix onto one of its subspaces (column space) defined by $W = span \left(\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right)$.

(C.O.No.3) [Comprehension]

7. What line $y = mx + c$ will best fit through the data points $(0,0), (1,1), (2,3)$?

(C.O.No.2) [Comprehension]

Part C [Problem Solving Questions]

Answer all the Questions. Each Question carries 10 marks. (3Qx10M=30M)

8. Let Π be the space in R^3 spanned by the vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Find an orthonormal basis set for Π, instead of these.

(C.O.No.3) [Application]

9. If $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$, produce orthonormal bases $\boldsymbol{v}$'s and $\boldsymbol{u}$'s for its four fundamental subspaces by finding out its singular values.

(C.O.No.3) [Application]

10. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$.

(C.O.No.2) [Application]

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

END TERM FINAL EXAMINATION

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO (% age of CO)	Unit/Module Number/Unit /Module Title	Memory recall type [Marks allotted] Bloom's Levels	Thought provoking type [Marks allotted] Bloom's Levels	Problem Solving type [Marks allotted]	Total Marks
			K	C	A	
1	1	1	10			10
2	3	3	5			5
3	1	1		7		7
4	1	1		7		7
5	3	3		7		7
6	3	3		7		7
7	2	2		7		7
8	3	3			10	10
9	3	3			10	10
10	2	2			10	10
	Total Marks		15	35	30	80

K =Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

I hereby certify that all the questions are set as per the above guidelines.

Faculty Signature:

Reviewer Commend:

Format of Answer Scheme

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

SOLUTION

Semester: Odd Sem. 2019-20
Course Code: ECE 314
Course Name: Linear Algebra for Communication Engineering
Program & Sem: BTech/7

Date: 20.12.2019
Time: 3 HRS
Max Marks: 80
Weightage: 40%

Part A (1Q x 15M = 15Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	a) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ c) No or infinity d) Infinite e) Transpose	**2+2+2+2+2=10**	**10 min**
2	a) $A^TAx=A^Tb$ b) $Q\Lambda Q^T$ c) Orthonormal d) Column space and row space e) real	**1+1+1+1+1=5**	**5 min**

Part B (7Q x 5M = 35 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question

3	$\begin{pmatrix}3&1&6\\-6&0&-16\\0&8&-17\end{pmatrix} \xrightarrow[\begin{pmatrix}1&0&0\\2&1&0\\0&0&1\end{pmatrix}]{E_1} \begin{pmatrix}3&1&6\\0&2&-4\\0&8&-17\end{pmatrix}$ $\xrightarrow{E_2 \begin{pmatrix}1&0&0\\0&1&0\\0&-4&1\end{pmatrix}} \begin{pmatrix}3&1&6\\0&2&-4\\0&0&-1\end{pmatrix}$ (2.5 marks) $E = E_2 \times E_1 = \begin{pmatrix}1&0&0\\0&1&0\\0&-4&1\end{pmatrix}\begin{pmatrix}1&0&0\\2&1&0\\0&0&1\end{pmatrix} = \begin{pmatrix}1&0&0\\2&1&0\\-8&-4&1\end{pmatrix}$ $(E_2E_1)^{-1} = \left(\begin{array}{ccc\|ccc}1&0&0&1&0&0\\2&1&0&0&1&0\\-8&-4&1&0&0&1\end{array}\right) \rightarrow \left(\begin{array}{ccc\|ccc}1&0&0&1&0&0\\0&1&0&-2&1&0\\0&-4&1&8&0&1\end{array}\right)$ $\downarrow$ $\left(\begin{array}{ccc\|ccc}1&0&0&1&0&0\\0&1&0&-2&1&0\\0&0&1&0&4&1\end{array}\right)$ (2.5 marks) $\therefore \begin{pmatrix}3&1&6\\-6&0&-16\\0&8&-17\end{pmatrix} = \underbrace{\begin{pmatrix}1&0&0\\-2&1&0\\0&4&1\end{pmatrix}}_{L}\underbrace{\begin{pmatrix}3&1&6\\0&2&-4\\0&0&-1\end{pmatrix}}_{U}$ (2 marks)	7	15 min
4	$\left(\begin{array}{ccc\|c}1&2&4&3\\3&8&14&13\\2&6&13&4\end{array}\right) \longrightarrow \left(\begin{array}{cccc}1&2&4&3\\0&2&2&4\\0&2&5&-2\end{array}\right)$ (2 marks) $\downarrow$ $\left(\begin{array}{cccc}1&2&4&3\\0&2&2&4\\0&0&3&-6\end{array}\right)$ (1 mark) $\therefore 3z=-6 \Rightarrow z=-2.$ (1 mark) $2y+2z=4 \Rightarrow 2y=8 \Rightarrow y=4.$ (1 mark) $x+2y+4z=3 \Rightarrow x=3-8+8=3.$ (1 mark) $\therefore \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\4\\-2\end{pmatrix}$ (1 mark)	7	15 min
5	$\det(A-\lambda I)=0.$ $\begin{vmatrix}3-\lambda&0&0\\-3&4-\lambda&9\\0&0&3-\lambda\end{vmatrix} = (3-\lambda)(4-\lambda)(3-\lambda)=0.$ $\therefore \lambda=4$ & $\lambda=3$ (repeated) (2 marks) for $\lambda=4$. $(A-\lambda I)=0$ gives eigenvectors. $\left(\begin{array}{ccc\|c}-1&0&0&0\\-3&0&9&0\\0&0&-1&0\end{array}\right) \rightarrow \begin{pmatrix}-1&0&0\\0&0&9\\0&0&-1\end{pmatrix} \rightarrow \begin{pmatrix}-1&0&0\\0&0&9\\0&0&9\end{pmatrix} \rightarrow \begin{pmatrix}1&0&0\\0&0&1\\0&0&0\end{pmatrix}$ $\therefore y$ is free variable. when $y=1$, $z=0$, $x=0$. $\therefore$ eigenvector $= \begin{pmatrix}0\\1\\0\end{pmatrix}$ (1 mark)	7	20 min

	for $\lambda = 3$ $\begin{pmatrix} 0 & 0 & 0 \\ -3 & 1 & 9 \\ 0 & 0 & 0 \end{pmatrix}$ y & z are free variables. y=3 & z=0. $-3x + 3 = 0$, $x = 1$ $v_1 = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ (1.5 marks) y=0 & z=1 $-3x + 9 = 0$, $x = 3$. $v_2 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ (1.5 marks) $\therefore \begin{pmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{pmatrix} = S \Lambda S^{-1}$ $= \begin{pmatrix} 0 & 1 & 3 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 1 & 3 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1}$ (1 mark)		
6	$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$ $P = A(A^TA)^{-1}A^T$ — ~~(1 mark)~~ (1.5 marks) $A^TA = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ — (1 mark) $(A^TA)^{-1} = \left(\begin{array}{cc\|cc} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \to \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & -3 & 1 & -2 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & -1/3 & 2/3 \end{pmatrix} \to \left(\begin{array}{cc\|cc} 1 & 0 & 2/3 & -1/3 \\ 0 & 1 & -1/3 & 2/3 \end{array}\right)$ → ~~(marks)~~ (2.5 marks) $A(A^TA)^{-1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}$ $= \begin{pmatrix} 0 & 0 \\ 2/3 & -1/3 \\ 1/3 & 1/3 \\ -1/3 & 2/3 \end{pmatrix}$ — ~~(1 mark)~~ (1.[illegible] marks) $A(A^TA)^{-1}A^T = \begin{pmatrix} 0 & 0 \\ 2/3 & -1/3 \\ 1/3 & 1/3 \\ -1/3 & 2/3 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ $= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2/3 & 1/3 & -1/3 \\ 0 & 1/3 & 2/3 & 1/3 \\ 0 & -1/3 & 1/3 & 2/3 \end{pmatrix}$ (1 marks)	7	20 min

7	for (0,0) $0 = 0m + c$, $1 = 1m + c$, $3 = 2m + c$ — 1 mark $\therefore \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} m \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ — 1 mark $Ax = b$. $A^TA\hat{x} = A^Tb$ — 1 mark. $A^TA = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$ $= \begin{pmatrix} 5 & 3 \\ 3 & 3 \end{pmatrix}$ — 1 mark. $A^Tb = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$ — 1 mark $\left(\begin{array}{cc\|c} 5 & 3 & 7 \\ 3 & 3 & 4 \end{array}\right) \rightarrow \begin{pmatrix} 5 & 3 & 7 \\ 15 & 15 & 20 \end{pmatrix} \rightarrow \begin{pmatrix} 5 & 3 & 7 \\ 0 & 6 & -15 \end{pmatrix}$. $6\hat{x}_2 = -1$ $\hat{x}_2 = -1/6$. — (1 mark) $5\hat{x}_1 + 3\hat{x}_2 = 7 \Rightarrow 5\hat{x}_1 = 7 + \frac{3}{6} = \frac{15}{2}$ $\hat{x}_1 = 3/2$ — (1 mark) $\therefore$ new $m, c = (3/2, -1/6)$	7	20 min

Part C

(3Q x 10M = 10Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
8	let $y_1 = x_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ — 1 mark $q_1 = \frac{y_1}{\\|y_1\\|} = \begin{pmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{pmatrix}$ — 1 mark $y_2 = x_2 - P_{y_1} x_2$ — 1 mark $P_{y_1} = \frac{y_1 y_1^T}{y_1^T y_1} = \frac{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}(1\ 2\ 0)}{(1\ 2\ 0)\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}} = \frac{\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}}{5}$ — 1 mark $y_2 = \begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} - \frac{1}{5}\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} - \frac{1}{5}\begin{pmatrix} 10 \\ 20 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 8 \\ 1 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ -6 \end{pmatrix} = +3\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ — 1 mark $q_2 = \frac{y_2}{\\|y_2\\|} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ — 1 mark	10	25 min

	$y_3 = x_3 - P_{y_1} x_3 - P_{y_2} x_3$ [1 mark] $P_{y_2} = \frac{y_2 y_2^T}{y_2^T y_2} = \frac{\begin{pmatrix}2\\-1\\-2\end{pmatrix}(2\ \ -1\ \ -2)}{(2\ \ -1\ \ -2)\begin{pmatrix}2\\-1\\-2\end{pmatrix}}$ $= \frac{1}{9}\begin{pmatrix}4 & -2 & -4\\-2 & 1 & 2\\-4 & 2 & 4\end{pmatrix}$ — 1 mark $y_3 = \begin{pmatrix}0\\0\\1\end{pmatrix} - \frac{1}{5}\begin{pmatrix}1 & 2 & 0\\2 & 4 & 0\\0 & 0 & 0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} - \frac{1}{9}\begin{pmatrix}4 & -2 & -4\\-2 & 1 & 2\\-4 & 2 & 4\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$. $= \begin{pmatrix}0\\0\\1\end{pmatrix} - \frac{1}{5}\begin{pmatrix}0\\0\\0\end{pmatrix} - \frac{1}{9}\begin{pmatrix}-4\\2\\4\end{pmatrix}$. $= \begin{pmatrix}4/9\\-2/9\\5/9\end{pmatrix} = \frac{1}{9}\begin{pmatrix}4\\-2\\5\end{pmatrix}$. — 1 mark $q_3 = \frac{y_3}{\lVert y_3 \rVert} = \frac{1}{3}\begin{pmatrix}4/\sqrt{5}\\-2/\sqrt{5}\\5/\sqrt{5}\end{pmatrix}$ — 1 mark $q_1 = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\\0\end{pmatrix}$; $q_2 = \frac{1}{3}\begin{pmatrix}2\\-1\\-2\end{pmatrix}$ $q_3 = \frac{1}{3\sqrt{5}}\begin{pmatrix}4\\-2\\5\end{pmatrix}$.		
9	$A^T_{2\times 3} = \begin{bmatrix}3 & 2 & 2\\2 & 3 & -2\end{bmatrix}$ $A^TA = 3\times 3$. X. $AA^T = \begin{pmatrix}3 & 2 & 2\\2 & 3 & -2\end{pmatrix}\begin{pmatrix}3 & 2\\2 & 3\\2 & -2\end{pmatrix}$ $= \begin{pmatrix}17 & 8\\8 & 17\end{pmatrix}$ — 1 mark $(17-\lambda)^2 - 64 = 0$. $289 - 34\lambda + \lambda^2 - 64 = 0$. $\lambda^2 - 34\lambda + 225 = 0$. $\lambda = 25,\ \lambda = 9$. — 1 mark when $\lambda = 25$. $\begin{matrix}-8 & 8 & \vert & 0\\8 & -8 & \vert & 0\end{matrix} \rightarrow \begin{matrix}+1 & -1 & \vert & 0\\1 & -1 & \vert & 0\end{matrix} \rightarrow \begin{matrix}1 & -1 & 0\\0 & 0 & 0\end{matrix}$. $x - 1 = 0 \Rightarrow x = 1$. $\therefore$ eigen vector $\begin{pmatrix}1\\1\end{pmatrix}$. orthonormal vector. $u_1 = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$. — 1 mark	10	25 min

	$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$. $A_{2\times3} = U_{2\times2}\ \Sigma_{2\times3}\ V^T_{3\times3}$ $\Sigma = \begin{pmatrix} \sqrt{25} & 0 & 0 \\ 0 & \sqrt{9} & 0 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{pmatrix}$ — 1 mark $\frac{Au_1}{\sigma_1} = v_1 \quad \frac{Au_2}{\sigma_2} = v_2 \quad v_3 = ?$ $v_1 = \frac{\begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \end{pmatrix}}{5} = \frac{1}{5}\begin{pmatrix} \frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$ — 1 mark $v_2 = \frac{Au_2}{\sigma_2} = \frac{\begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}}{3} = \frac{1}{3}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} \end{pmatrix}$ $= \begin{pmatrix} \frac{1}{3\sqrt{2}} \\ \frac{-1}{3\sqrt{2}} \\ \frac{4}{3\sqrt{2}} \end{pmatrix}$ — 1 mark V_3 is ⊥r to both V_1 & V_2 $\begin{array}{ccc\|c} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{3\sqrt{2}} & \frac{-1}{3\sqrt{2}} & \frac{4}{3\sqrt{2}} & 0 \end{array} \rightarrow \begin{array}{ccc\|c} 1 & 1 & 0 & 0 \\ 1 & -1 & 4 & 0 \end{array} \rightarrow \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 \end{array}$ $\begin{array}{ccc\|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 \end{array} \rightarrow$		
10	$A = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$. $\det(A - \lambda I) = 0$. $\begin{vmatrix} (3-\lambda) & 6 & -8 \\ 0 & -\lambda & 6 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (3-\lambda)(-\lambda)(2-\lambda)$ $= \lambda(2-\lambda)(\lambda-3) = 0$. $\lambda = 0,\ \lambda = 2,\ \lambda = 3$. — 3 marks When $\lambda = 3$, $\begin{array}{ccc\|c} 0 & 6 & -8 & 0 \\ 0 & -3 & 6 & 0 \\ 0 & 0 & -1 & 0 \end{array} \rightarrow \begin{array}{ccc\|c} 0 & 6 & -8 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & -1 & 0 \end{array}$	10	15 min

$x \Rightarrow$ free variable.

eigen vectors $= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. [2 marks]

$\lambda = 2$.

$$\left(\begin{array}{ccc|c} 1 & 6 & -8 & 0 \\ 0 & -2 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{array}{ccc} 1 & 6 & 8 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}$$

z is free variable

$y - 3z = 0 \quad y = 3.$

$x + 6y - 8z = 0. \quad v_2 = \begin{pmatrix} -10 \\ 3 \\ 1 \end{pmatrix}$ [3 marks]

$x + 18 - 8 = 0.$

$x = -10.$

when $\lambda = 0$.

$$\left(\begin{array}{ccc|c} 3 & 6 & -8 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|c} 3 & 6 & -8 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

y is a free variable.

$3x + 6y = 0.$

$3x = -6$

$x = -2.$

$\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$. [2 marks]

4. Elimination and back substitution helps in solving a system of linear equations. If so, using the same, find the solution of the system $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$.

(C.O.No.1) [Comprehension]

5. For the square matrix $\begin{bmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{bmatrix}$, the sum of dimensions of its eigenspaces is equal to its own dimension. The matrix is then said to be diagonalizable or nondefective. Find its diagonal matrix and corresponding eigenvector matrix. (C.O.No.3) [Comprehension]

6. If you are in a space of R^4, find the projection matrix onto one of its subspaces (column space) defined by $W = span\left(\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}\right)$. (C.O.No.3) [Comprehension]

7. What line $y = mx + c$ will best fit through the data points $(0,0), (1,1), (2,3)$?

(C.O.No.2) [Comprehension]

Part C [Problem Solving Questions]

Answer all the Questions. Each Question carries 10 marks. **(3Qx10M=30M)**

8. Let Π be the space in R^3 spanned by the vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Find an orthonormal basis set for Π, instead of these. (C.O.No.3) [Application]

9. If $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$, produce orthonormal bases v's and u's for its four fundamental subspaces by finding out its singular values. (C.O.No.3) [Application]

10. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$.

(C.O.No.2) [Application]