



Roll No. _____

**PRESIDENCY UNIVERSITY
BENGALURU**

SCHOOL OF ENGINEERING

TEST 1

Sem: Odd Sem 2019-20

Date: 12.10.2019

Course Code: ECE 401

Time: 1.30 PM to 2.30 PM

Course Name: ARTIFICIAL NEURAL NETWORKS

Max Marks: 40

Program & Sem: B.Tech (ECE) & VII (OE)

Weightage: 20%

Instructions:

- (i) Read the question properly and answer accordingly.
- (ii) Question paper consists of 3 parts.
- (iii) Scientific and Non-programmable calculators are permitted.

Part A [Memory Recall Questions]

Answer all the Questions. Each Question carries six marks. (3Qx6M=18M)

1. Discuss the applications of Neural Networks in detail. (C.O.NO.1)[Knowledge]
2. Explain the Neuron Model with Suitable Diagrams consider bias and activation function. (C.O.NO.1)[Knowledge]
3. What is Signal flow graph? Explain the rules associated with signal flow graph representation of a model. (C.O.NO.1)[Knowledge]

Part B [Thought Provoking Questions]

Answer both the Questions. Each Question carries seven marks. (2Qx7M=14M)

4. a) In a neuron model what are the two functions which can acts as activation function? Name it and explain the same with suitable mathematical and graphical representation. (C.O.NO.1)[Comprehension]
- b) Name the property which can exhibits a graceful balance between linear and nonlinear model and decide which activation function having this property.

(C.O.NO.1)[Comprehension]

5. a) Identify the networks which have at least one feedback loop and can capable to distinguish feed forward neural networks, then draw the same network.

Part C [Problem Solving Questions]

Answer the Question. The Question carries eight marks.

(1Qx8M=8M)

6. What is the importance of feedback in neural network? Consider 'w' is the initial weight and feedback is 'z⁻¹', determine $Y_k(n)$ for the input $X_k(n)$ where 'k' is neuron. Realize the system and also discuss on its stability (C.O.NO.1)[Application]



SCHOOL OF ENGINEERING

Semester: 2019-2020 (ODD Semester)

Course Code: ECE-401

Course Name: Artificial Neural Networks

Program & Sem: B.Tech(ECE) & 7th Sem

Date: 27-09-2019 (Friday)

Time: 1.00PM to 2.00PM

Max Marks: 40

Weightage: 20%

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO	Unit/Module Number/Unit /Module Title	Memory recall type		Thought provoking type		Problem Solving type [Marks allotted]	Total Marks
			[Marks allotted]	Bloom's Levels	[Marks allotted]	Bloom's Levels		
1	CO1	MODULE-1	K	6			A	6
2	CO1	MODULE-1	K	6				6
3	CO1	MODULE-1	K	6				6
4	CO1	MODULE-1			C	7		7
5	CO1	MODULE-1			C	7		7
6	CO1	MODULE-1					A	8
	Total			18		14	8	40

K = Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that on above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

[I hereby certify that All the questions are set as per the above guide lines.]

Reviewers' Comments

① Q & A Timing ② Q & A → Q is something else, Ans is defin
③ More Control Sys Oriented Less AN N
④ - Check Remark in scheme.

Annexure- II: Format of Answer Scheme



SCHOOL OF ENGINEERING

SOLUTION

Semester: 2019-2020 (ODD Semester)

Course Code: ECE-401

Course Name: Artificial Neural Networks

Program & Sem: B.Tech(ECE) & 7th Sem

Date: 27-09-2019 (Friday)

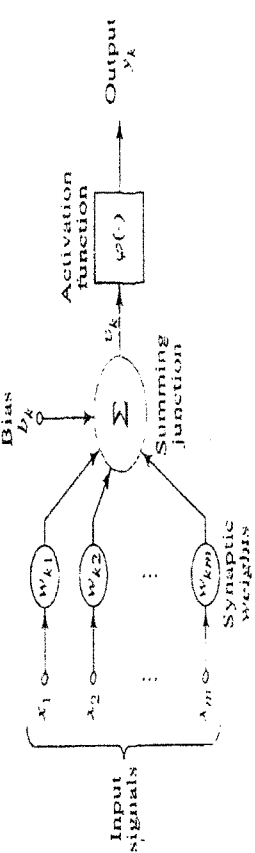
Time: 1.00PM to 2.00PM

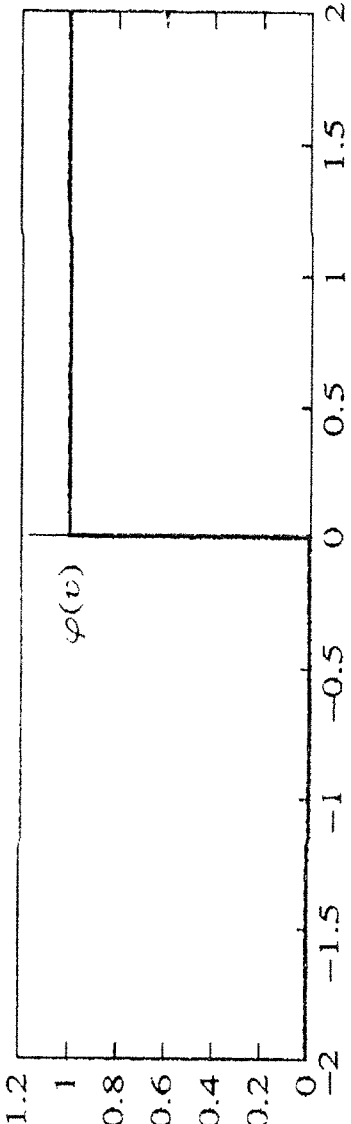
Max Marks: 40

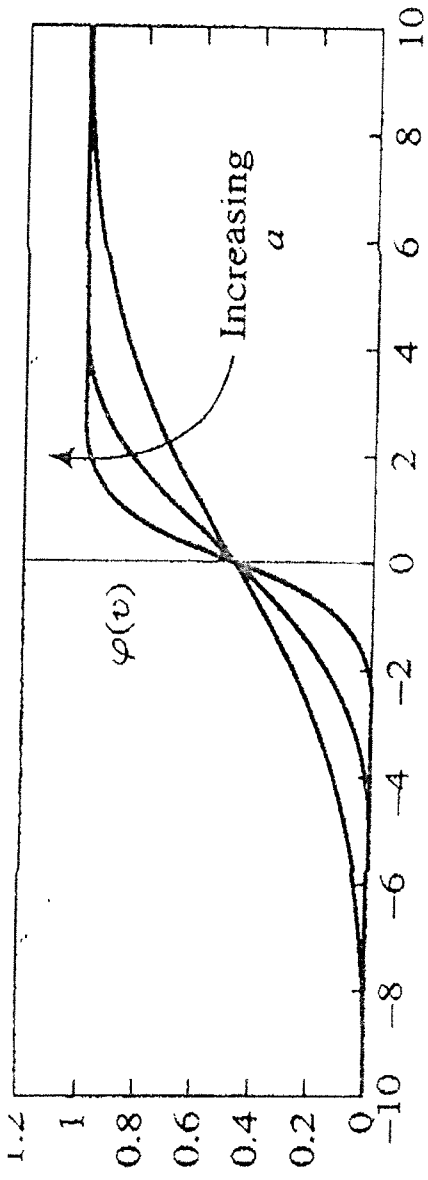
Weightage: 20%

Part A (3Q x 6 M = 18 Marks)

Q No	Solution	Scheme of Marking	Max. Time required

	Question		
1	<p>A neural network is a massively parallel distributed processor made up of simple processing units, which has a natural propensity for storing experiential knowledge and making it available for use. It resembles the brain in two respects.</p> <ol style="list-style-type: none"> 1. Knowledge is acquired by the network from its environment through a learning process. 2. Interneuron connection strengths, known as synaptic weights, are used to store the acquired knowledge. 	<p>3+3</p>	5min
2		<p>3+3</p>	10min
3	<p>A signal-flow graph is a network of directed links (branches) that are interconnected at certain points called nodes. A typical node j has an associated node signal x_j. A typical directed link originates at node j and terminates on node k; it has an associated</p> <p>Rule 1. A signal flows along a link only in the direction defined by the arrow on the link.</p> <p>Two different types of links may be distinguished:</p> <p>Rule 2. A node signal equals the algebraic sum of all signals entering the pertinent node via the incoming links.</p> <p>Rule 3. The signal at a node is transmitted to each outgoing link originating from that node, with the transmission being entirely independent of the transfer functions of the outgoing links.</p>	<p>2+4</p>	5min

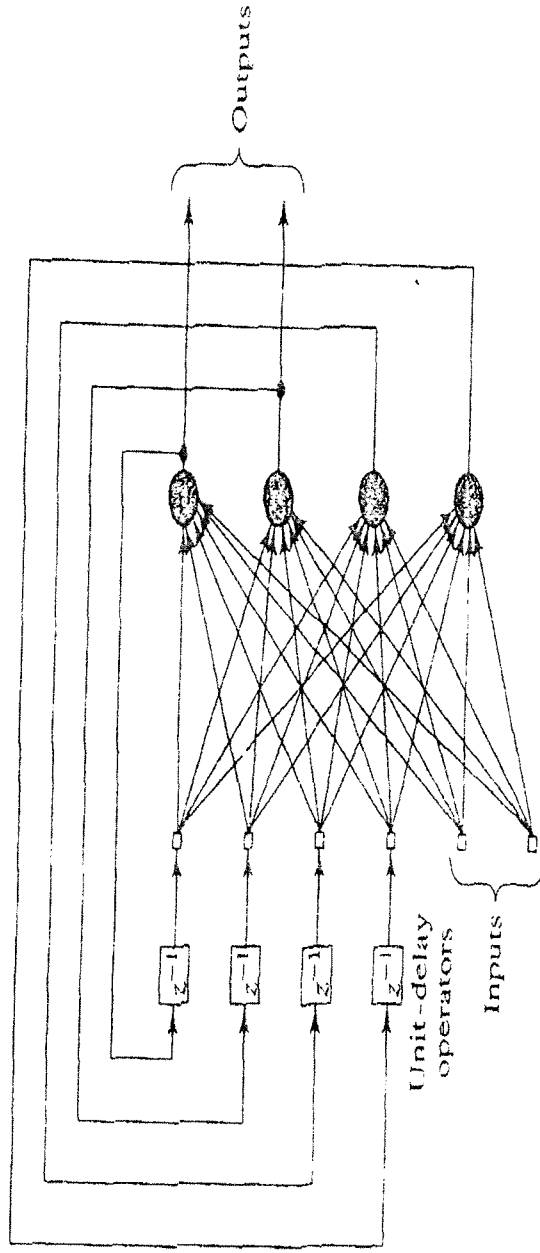
	Solution	Marking	Time required for each Question
4	<p>1. <i>Threshold Function.</i> For this type of activation function, described in Fig. 1.8a, we have</p> $\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases} \quad (1.8)$  <p>3. <i>Sigmoid Function.</i> The sigmoid function, whose graph is s-shaped, is by far the most common form of activation function used in the construction of artificial neural networks. It is defined as a strictly increasing function that exhibits a graceful balance between linear and nonlinear behavior.³ An example of the sigmoid function is the <i>logistic function</i>,⁴ defined by</p> $\varphi(v) = \frac{1}{1 + \exp(-av)} \quad (1.12)$	3+4	10min



Sigmoidal function have the differentiability property where as threshold function not

Other Fine is also allowed

Recurrent Networks With Hidden layers




5

2+5

69

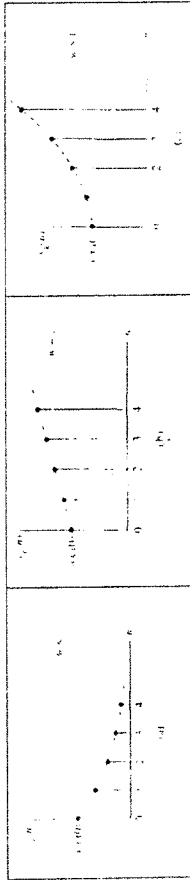
Part C

(1Q x 8 M = 8Mark

Q No	Solution	Scheme of Marking	Max. Time required for each Question
6	<p>For a single loop feedback system shown in fig. below</p>  <p>a) Find the close loop operator of the system. b) Express the output signal $Y_k(n)$ as an infinite weighted summation of present and past samples of input signal $X_1(n)$ c) How the selection of the values of W will lead to divergence .</p> $1 - AB = \sum_{k=0}^{\infty} (AB)^k$ <p>Using the binomial expansion for $1 - a^{-1}$ we can rewrite the closed loop operator of the system as</p> $\frac{A}{1 - AB} = \sum_{k=0}^{\infty} (AB)^k \tag{1.19}$ <p>Hence, substituting Eq. (1.19) in (1.18), we get</p> $Y_1(n) = W \sum_{k=0}^{\infty} (AB)^k X_1(n) \tag{1.20}$ <p>where again we have included square brackets to emphasize the fact that ϵ^{-1} is an operator. In particular, from the definition of ϵ^{-1} we have</p> $\epsilon^{-1} (A(n)) = A(n-1) \tag{1.21}$	2+4+2	15min

may express the output signal $y_k(n)$ as an infinite weighted summation of present and past samples of the input signal $x_j(n)$, as shown by

$$y_k(n) = \sum_{l=0}^{\infty} w^{l(n-k)} x_j(n-l) \quad (1.22)$$



5. What is Linear Model for regression, extend this Model and find pseudo inverse by using a function which is having the property of "linear combination of fixed non-linear function."
(C.O.NO.2)[Comprehension]

Part C [Problem Solving Questions]

Answer the Question. The question carries eight marks.

(1Qx8M=8M)

6. Consider a Linear model for regression with design matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and target

vector $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ then identify the number of weights in the model and find maximum likelihood weight vector.
(C.O.NO.2)[Application]

Annexure- II: Format of Answer Scheme



SCHOOL OF ENGINEERING

SOLUTION

Semester: 2019-2020 (ODD Semester)

Course Code: ECE-401

Course Name: Artificial Neural Networks

Program & Sem: B.Tech(ECE) & 7th Sem

Date: 27-09-2019 (Friday)

Time: 1.00PM to 2.00PM

Max Marks: 40

Weightage: 20%

Part A (3Q x6 M =18 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	Learning with teacher is referred to supervised learning . Explanation	2+2+2	5min

2	<p>The diagram illustrates a learning system architecture. It consists of the following components and connections:</p> <ul style="list-style-type: none"> Environment: Provides the input vector to both the Teacher and the Learning system. Teacher: Receives the environment vector and outputs a Desired response to the summation node. Learning system: Receives the environment vector and outputs an Actual response to the summation node. Summation Node (Σ): Combines the desired response (positive sign) and the actual response (negative sign) to produce an Error signal. Error signal: Is fed back into the Learning system to adjust its output. 	
		<p>4(steps)+2(diagram)</p>
		<p>10min</p>

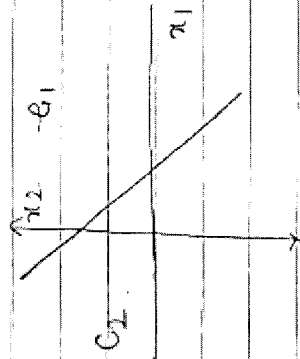
3	If two mutually independent variables are there then perceptron will give 100% guarantee to classify.	2(statement)+1+2+1	5min

The simplest form of perceptron.

The two decision regions separated by hyperplane, and is defined by

$$\sum_{i=1}^m w_i x_i + b = 0$$

The fig explains the case of two input variable x_1 and x_2



(A) x_1, x_2 lies above the boundary lies in class C_1

(B) x_1, x_2 lies below the boundary is assigned to C_2 .

(C) The role of bias is to shift decision boundary away from origin.

The weights w_1, w_2, \dots, w_m of the perceptron are adapted on an iteration by iteration basis.

For this a error-correction rule known as perceptron convergence algorithm is used.

	<p>If the hyperplane that separates the inputs into 2 different class by drawing a hyperplane clearly then this is called as linearly separable class of data . if the perceptron receive this data it will converge or else it will fail to classify this data.</p> <p>Perceptron has limitation that it can take only 2 class problems.</p>	
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Part B (2Q x 7M = 14 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
5	Name of the function is Basis Function	2 for the definition +4 for the derivation +1 for the comment	10min

$$X = \{x_1, x_2, x_3, \dots, x_N\}$$

$$x_1 \rightarrow b_1$$

$$x_2 \rightarrow b_2$$

$$\vdots$$

$$x_N \rightarrow b_N$$

$$t = Y(x, w)$$

$$Y = f(x, w) = w_0 x_0 + w_1 x_1 + w_2 x_2 + \dots + w_N x_N$$

$$Y(x, w) = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_N x_N$$

The above eqⁿ is called Linear model. We can extend this class of model by considering "Basis Function".

* Basis function is linear combination of fixed non-linear function.

$$Y(x, w) = w_0 + \sum_{j=1}^M w_j \phi_j(x)$$

\downarrow Fixed offset or Basis parameter \uparrow Basis func

$$Y(x, w) = \sum_{j=0}^{M-1} w_j \phi_j(x)$$

$$\phi_0(x) = 1 \rightarrow \text{dummy basis func}$$

$$t = \sum_{j=0}^{M-1} w_j \phi_j(x)$$

In matrix form,

$$\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}_{N \times 1} = \begin{bmatrix} \phi_0(x_1) & \phi_1(x_1) & \phi_2(x_1) & \dots & \phi_{m-1}(x_1) \\ \phi_0(x_2) & \phi_1(x_2) & \phi_2(x_2) & \dots & \phi_{m-1}(x_2) \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \phi_0(x_N) & \phi_1(x_N) & \phi_2(x_N) & \dots & \phi_{m-1}(x_N) \end{bmatrix}_{N \times m} \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{m-1} \end{bmatrix}_{m \times 1}$$

$$\phi^T w = t$$

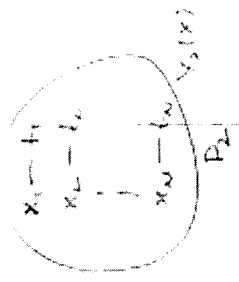
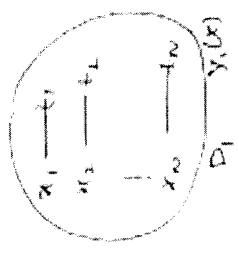
$$\phi^T \phi w = \phi^T t$$

$$w = \underbrace{(\phi^T \phi)^{-1} \phi^T t}_{\text{pseudo inverse}}$$

eq for Least Square

4

Basis . Variance decomposition:



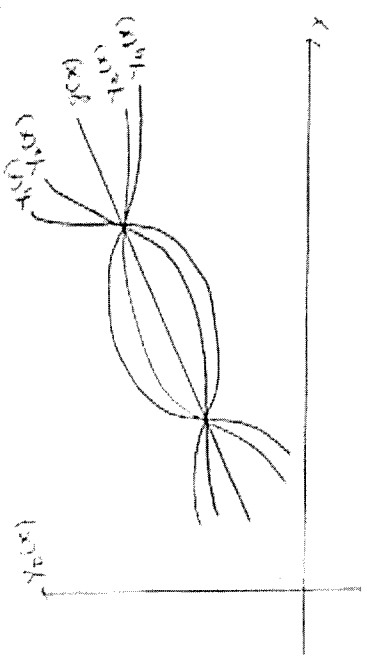
similarly D_3, D_4, \dots, D_q

$Y_1(x), Y_2(x), Y_3(x), Y_4(x)$ are considered as q datasets $\beta_1, \beta_2, \beta_3, \beta_4$

estimating t as

$$\hat{T} = g(x)$$

$g(x)$: reference only



2 (graphical) + 5 (mathematical)

10 min

while estimating the target value.

$$E[(y(x) - g(x))^2]$$

$$E_D[(y(x, D) - g(x))^2]$$

By adding & subtracting with $E_D(y(x, D))$

$$E_D \left[\underbrace{(y(x, D) - E_D(y(x, D)))}_a + \underbrace{(E_D(y(x, D)) - g(x))}_b \right]^2 \rightarrow \text{Variance}$$

3rd term $E_D[(y(x, D) - E_D(y(x, D)))^2] \rightarrow \text{Variance}$

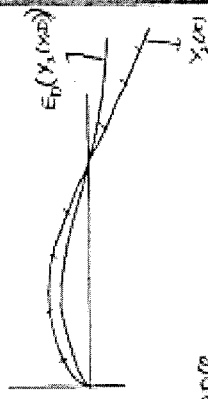
2nd term $E_D[(E_D(y(x, D)) - g(x))^2] \rightarrow \text{Bias}^2$

~~1st term~~ $E_D[(y(x, D) - E_D(y(x, D))) (E_D(y(x, D)) - g(x))]$

4th term $E_D[(y(x, D) - E_D(y(x, D))) (E_D(y(x, D)) - g(x))] \rightarrow \text{Error}$

Actually, while estimating the target value

$$\boxed{\text{Expected loss} = \text{Variance} + (\text{Bias})^2 + \text{Noise}}$$





Roll No																			
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**PRESIDENCY UNIVERSITY
BENGALURU**

SCHOOL OF ENGINEERING

END TERM FINAL EXAMINATION

Semester: Odd Sem. 2019 - 20

Date: 23 December 2019

Course Code: ECE 401

Time: 9:30 AM to 12:30 PM

Course Name: ARTIFICIAL NEURAL NETWORKS

Max Marks: 80

Program & Sem: B.Tech (All Programs) & VII (OE-I)

Weightage: 40%

Instructions:

- (i) Read the question properly and answer accordingly.
- (ii) Question paper consists of 3 parts.
- (iii) Scientific and Non-programmable calculators are permitted.

Part A [Memory Recall Questions]

Answer all the Questions. Each Question carries 4 marks.

(5Qx4M=20M)

1. What is the use of activation function in neuron model? Explain any two activation functions. (C.O.No.1) [Knowledge]
2. What are the methods used to estimate target variables and weight parameters. (C.O.No.2) [Knowledge]
3. Explain the steepest descent algorithm with diagrams. (C.O.No.3) [Comprehension]
4. What is Design matrix? Explain with an Example. (C.O.No.1) [Knowledge]
5. Explain Perceptron convergence theorem. (C.O.No.3) [Knowledge]

Part B [Thought Provoking Questions]

Answer all the Questions. Each Question carries 8 marks.

(3Qx8M=24M)

6. How support vector machine treated as a 'Minimization problem'? Consider two classes and Explain with suitable diagrams. (C.O.No.3) [Application]
7. Which algorithm is used to reduce data from higher dimension to lower dimension? Will the same algorithm is used for classification also? Identify that algorithm and write the step by step procedure involved in it. (C.O.No.3) [Comprehension]
8. For three class problem (Say C1, C2, C3), Consider two scatter matrix S_B (between class Scatter matrix) and S_W (Within Class Scatter Matrix). (C.O.No.3) [Comprehension]
Statement: trace of S_B will be maximized and trace of S_W will be minimized.
 - i) The statement is correct or wrong, if it is correct, why? If it is wrong, why?
 - ii) Formulate those scatter matrix S_B and S_W
 - iii) Formulate the Objective function.

Part C [Problem Solving Questions]

Answer all the Questions. Each Question carries 12 marks.

(3Qx12M=36M)

9. Implement Back Propagation Model and find $P_{ij}(t+1)$, $q_{jk}(t+1)$, by Minimizing the objective function "J" (where $i=1,2,3,4$; $j=1,2$; $k=1,2$, t = target variable) from the following specifications (C.O.No.3) [Application]

i) Number of inputs are 4(say i_1, i_2, i_3, i_4) Hidden Layer nodes are 2(say h_1, h_2) and outputs are 2(say O_1, O_2)

ii) Hidden layer biasing elements are U_1, U_2 and output layer biasing elements are V_1, V_2

10. Using Discriminant Approach and prove that the minimum value for objective function will be zero if the target value is equals to the actual output. For minimization of objective function you may use steepest descent algorithm. (C.O.No.2) [Comprehension]

11. Consider three data sets $C_1 = [X_{11}, X_{12}, X_{13}, \dots, X_{1n1}]$, $C_2 = [X_{21}, X_{22}, X_{23}, \dots, X_{2n2}]$, $C_3 = [X_{31}, X_{32}, X_{33}, \dots, X_{3n3}]$. Expand the three datasets (C.O.No.3) [Comprehension]

i) as a Gram Matrix

ii) as a Gram Matrix with basis function $\Phi(x)$

iii) as a Gram Matrix with Kernel



SCHOOL OF ENGINEERING
END TERM FINAL EXAMINATION

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Course Code: ECE401
Course Name: ARTIFICIAL NEURAL NETWORKS
Program & Sem: B.Tech (ECE, EEE, CSE, MEC) & 7th Sem

Extract of question distribution [outcome wise & level wise]

Q.NO.	C.O.NO (% age of CO)	Unit/Module Number/Unit /Module Title	Memory recall type	Thought provoking type	Problem Solving type [Marks allotted]	Total Marks
			[Marks allotted] Bloom's Levels	[Marks allotted] Bloom's Levels		
1	1	Module 1	K 4	C 4	A 4	4
2	2	Module 2	4			4
3	3	Module 3	4			4
4	1	Module 1	4			4

5	3	Module 3	4			4
6	3	Module 3			8	8
7	3	Module 3		8		8
8	3	Module 3		8		8
9	3	Module 3			12	12
10	2	Module 2		12		12
11	3	Module 3		12		12
Total Marks			20	40	20	80

K = Knowledge Level C = Comprehension Level, A = Application Level

Note: While setting all types of questions the general guideline is that about 60%

Of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

I hereby certify that all the questions are set as per the above guidelines.

Faculty Signature:

Reviewer Comment: Dr. M. Levy.

1. Faculty not signed
2. Thought provoking Question — ?
3. Answer scheme Stop marks — ?

Format of Answer Scheme



SCHOOL OF ENGINEERING SOLUTION

Semester: Odd Sem. 2019-20

Course Code: ECE401

Course Name: ARTIFICIAL NEURAL NETWORKS

Program & Sem: B.Tech (ECE, EEE, CSE, MEC) & 7th Sem

Date: 23rd December 2019

Time: 9.30 AM to 12.30 PM

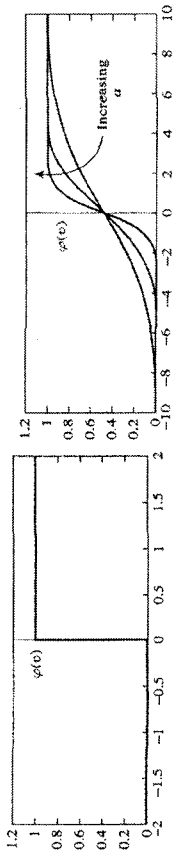
Max Marks: 80

Weightage: 40%

Part A

(5Q x4M =20Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
1	Activation function defines o/p of neuron in terms of the induced local field. 1. Threshold Function. For this type of activation function, described in Fig. 1.8a, we have $\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases} \quad (1.8)$	2M+2M=4M	8min



3. **Sigmoid Function.** The sigmoid function, whose graph is s-shaped, is by far the most common form of activation function used in the construction of artificial neural networks. It is defined as a strictly increasing function that exhibits a graceful balance between linear and nonlinear behavior.³ An example of the sigmoid function is the *logistic function*,⁴ defined by

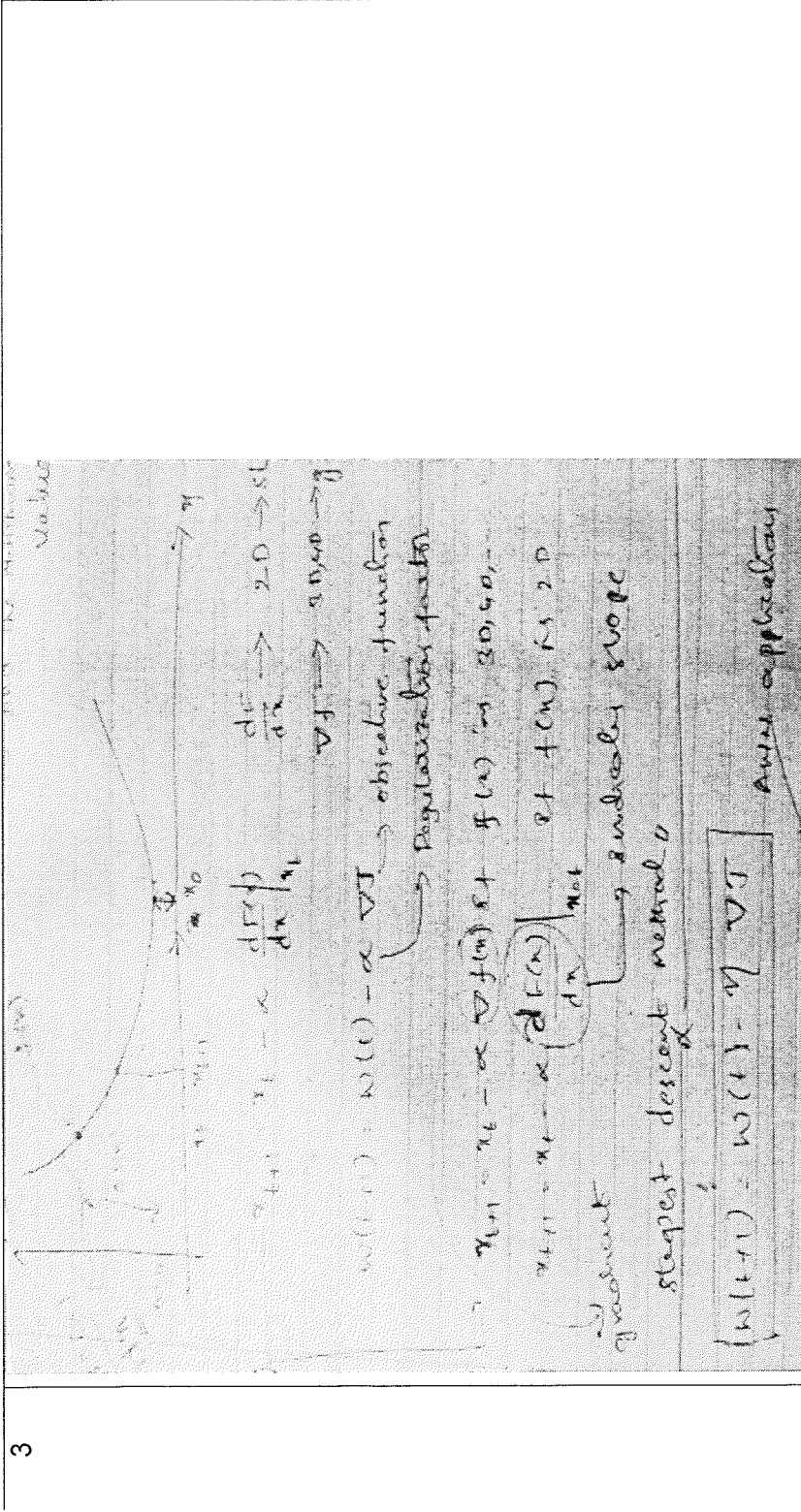
$$\phi(v) = \frac{1}{1 + \exp(-av)} \quad (1.12)$$

2

$f_1(w) \rightarrow$ Prior density
 $f_2(w/x) \rightarrow$ Aposterior density
 $f_3(x/t) \rightarrow$ Likelihood
 , for estimating weight w parameter
 $f_1(w) \rightarrow$ Prior estimation
 $f_2(w/t, X) \rightarrow$ Aposterior estimation
 $f_3(t/w, X) \rightarrow$ Likelihood estimation

2M+2M=4M

8min

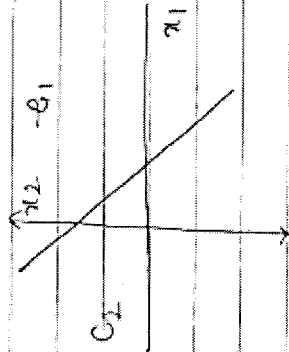
3	 <p> $x_{t+1} = x_t - \alpha \left. \frac{df}{dx} \right _{x_t}$ $w(t+1) = w(t) - \alpha \nabla f$ → objective function Population fitness $x_{t+1} = x_t - \alpha \left. \frac{df}{dx} \right _{x_t}$ for 2D gradient → steepest descent method $w(t+1) = w(t) - \eta \nabla J$ → matrix application </p>	4M	8min
4	<p> In matrix form, $\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{bmatrix}_{N \times 1} = \begin{bmatrix} \phi_1(x_0) & \phi_1(x_1) & \phi_1(x_2) & \dots & \phi_1(x_N) \\ \phi_2(x_0) & \phi_2(x_1) & \phi_2(x_2) & \dots & \phi_2(x_N) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_N(x_0) & \phi_N(x_1) & \phi_N(x_2) & \dots & \phi_N(x_N) \end{bmatrix}_{N \times N} \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{N-1} \end{bmatrix}_{N \times 1}$ </p>	4M	8min

The Simplest form of perceptron.

The two decision regions separated by hyperplane, and is defined by

$$\sum_{i=1}^m w_i x_i + b = 0$$

The fig explains the case of two input variable x_1 and x_2 .



(A) $x_{1,2}$ lies above the boundary lies in class C_1

(B) $x_{1,2}$ lies below the boundary is assigned to C_2 .

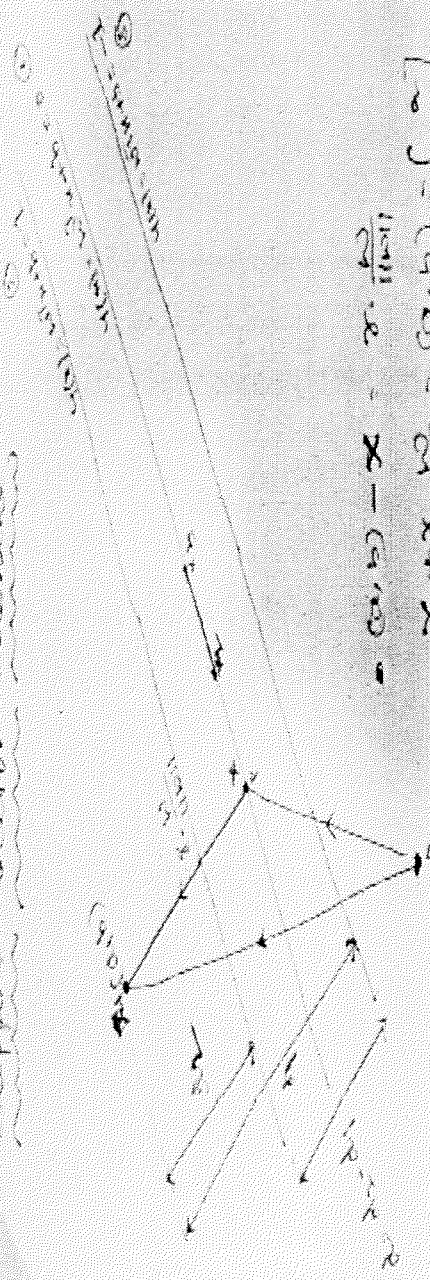
(C) The role of bias is to shift decision boundary away from origine.

The weights w_1, w_2, \dots, w_n of the perceptron are adapted on an iteration by Hebbian basis.

For this a error-correction rule known as perceptron convergence algorithm is used.

Part B

(3Q x 8M = 24 Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
6	<p><u>Support Vector Machine</u></p>  <p> $d_1 = \frac{a^T x + b + 1}{\ w\ }$ $d_2 = \frac{a^T x + b - 1}{\ w\ }$ $d_3 = \frac{d_1 - d_2}{2} \rightarrow \text{maximized}$ so $\ w\ \rightarrow \text{minimized}$ </p> <p> $\ w\ = \ a + x\$ $\ w\ ^2 = \ a + x\ ^2 = \ a\ ^2 + \ x\ ^2 + 2a^T x$ $\frac{d}{dx} \ w\ ^2 = 2x + 2a = 0 \Rightarrow x = -a$ $\ w\ = \ a - a\ = 0$ (This part of the handwritten solution is partially obscured and seems to be a different derivation or correction.) </p>	8M	19min
7	<p>Algorithm is Principle Component Analysis, Yes, PCA we can use as a dimensionality reduction technique.</p>	1M+1M+6M	19min

Principal component analysis (PCA) is one of the most popular techniques for dimensionality reduction. Starting from an original set of l samples (features), which form the elements of a vector $x \in \mathbb{R}^l$, the goal is to apply a linear transformation to obtain a new set of samples: $y = A^T x$

so that the components of y are uncorrelated. In a second stage, one chooses the most significant of these components. The steps are summarized below:

- Estimate the autocorrelation matrix for N feature vectors
- Perform the eigen decomposition of S and compute the l e $R \approx \frac{1}{N} \sum_{i=1}^N x_i x_i^T$ n vectors, $\lambda_i, a_i \in \mathbb{R}^l$, $i = 0, 2, \dots, l-1$.
- Arrange the eigenvalues in descending order, $\lambda_0 \geq \lambda_1 \geq \dots \geq \lambda_{l-1}$.
- Choose the m largest eigenvalues. Usually m is chosen so that the gap between λ_{m-1} and λ_m is large. Eigenvalues $\lambda_0, \lambda_1, \dots, \lambda_{m-1}$ are known as the m principal components.
- Use the respective (column) eigen vectors $a_i, i = 0, 1, 2, \dots, m-1$ to form the transformation matrix $A = [a_0, a_1, a_2, \dots, a_{m-1}]$
- Transform each l -dimensional vector x in the original space to an m -dimensional vector y via the transformation $y = A^T x$. In other words, the i th element $y(i)$ of y is the projection of x on a_i , $y(i) = a_i^T x$

i) Yes, The given statement is correct because if we maximizing the trace of S_w we can get good classification of data sets. S_B will be minimized because all the data point will come closer to centroid of the clusters.

Linear Discriminant Analysis (LDA)

$S_B = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T$ → S_B between class scatter matrix
 $S_w = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_c)(x_i - \mu_c)^T$ → S_w within class scatter matrix

Separation b/w classes $S_B = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T$

Eg: $(x_1 - \mu)(x_1 - \mu)^T, (x_2 - \mu)(x_2 - \mu)^T, (x_3 - \mu)(x_3 - \mu)^T$
 separation within class $S_w = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_c)(x_i - \mu_c)^T$

$S_B = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T$
 $S_w = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_c)(x_i - \mu_c)^T$

NOTE: S_B and S_w are not calculated

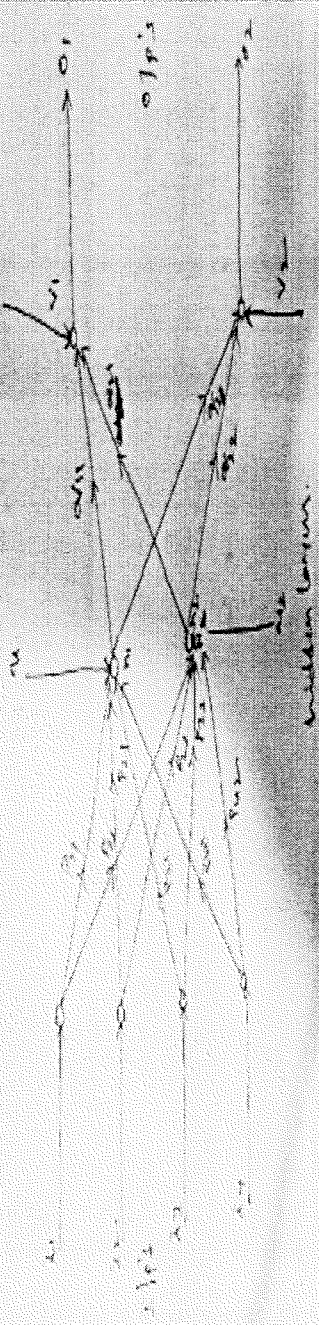
	<p>Similarly $S_2 = \text{col}(A - \lambda_2 I)$ we can write $S_2 = \text{col}(A - \lambda_2 I)$ $S_2 = \text{col}(A - \lambda_2 I)$ then the orthogonal matrix $S = [S_1 \ S_2]$ similar matrix $J = \text{diag}(\lambda_1, \lambda_2)$ $A = S^{-1} J S$ the orthogonal matrix A and take single element λ_1 $A \text{col}(S_1) = \lambda_1 \text{col}(S_1)$ $A S_1 = \lambda_1 S_1$ Eigenvalue $(S_1)^T A S_1 = \lambda_1$ Eigen Vector</p>		
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Part C

(3Q x 12M = 36Marks)

Q No	Solution	Scheme of Marking	Max. Time required for each Question
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Block propagation model



$$w_1 = (w_{11} + w_{12} + w_{13} + w_{14}) + w_1$$

$$w_2 = (w_{21} + w_{22} + w_{23} + w_{24}) + w_2$$

$$o_1 = A(w_{11} + w_{12} + w_{13} + w_{14}) + v_1$$

$$o_2 = A(w_{21} + w_{22} + w_{23} + w_{24}) + v_2$$

24min

6M+6M

$$J = (t_1 - t) - f(v_1, v_{11} + h_1 v_{11} + v_1) + (t_2 - t) + (h_2 v_{12} + h_3 v_{12} + v_2)$$

This is a minimization problem so, $J \rightarrow \downarrow$ (minimized)

Using Stokpit's direct algorithm.

$$v_{11}(t+1) = v_{11}(t) - \eta \gamma_2 \frac{dJ}{dv_{11}} \Big|_t$$

$$v_{11}(t+1) = v_{11}(t) + \eta e_1 h_1$$

$$v_{12}(t+1) = v_{12}(t) + \eta e_2 h_2$$

$$v_{21}(t+1) = v_{21}(t) + \eta e_3 h_3$$

$$v_{22}(t+1) = v_{22}(t) + \eta e_4 h_4$$

$$p_{11}(t+1) = p_{11}(t) - \eta \frac{dJ}{dp_{11}}$$

$$p_{12}(t+1) = p_{12}(t) - \eta \gamma_2 \frac{dJ}{dp_{12}}$$

$$\text{||} \gamma \text{||} \quad p_{21}, p_{22}, p_{31}, p_{32}, p_{41}, p_{42}$$

$$J = (t_1 - t) - f \left(+ (\lambda_1 p_{11} + \lambda_2 p_{21} + \lambda_3 p_{31} + \lambda_4 p_{41}) v_{11} + h_1 v_{11} + v_1 \right)$$

$$+ (t_2 - t) - f \left(p_{12} v_{12} + \lambda_2 p_{22} + \lambda_3 p_{32} + \lambda_4 p_{42} \right) v_{12} + h_2 v_{12} + v_2$$

$$\frac{dJ}{dp_{11}} = e_1 e_2 + (-\lambda_2 v_{11}) + e_2 (-e_1 v_{12})$$

$$= -e_1 \lambda_2 v_{11} - e_2 e_1 v_{12}$$

$$= -2 \lambda_1 (e_1 v_{11} + e_2 v_{12})$$

$$p_{11}(t+1) = p_{11}(t) + \eta \lambda_1 (e_1 v_{11} + e_2 v_{12})$$

||} \gamma \text{||} \quad e_3, e_4

In order to minimize a function with two class problems
 using gradient descent, we have

$$f(x) = \frac{1}{2} x^T A x + b^T x + c$$

$$\nabla f(x) = Ax + b = 0$$

$$Ax = -b$$

$$x = -A^{-1}b$$

Using the gradient method, we have

$$x_{k+1} = x_k - \eta \nabla f(x_k)$$

$$x_{k+1} = x_k - \eta (Ax_k + b)$$

$$x_{k+1} = (I - \eta A)x_k - \eta b$$

Let $P = I - \eta A$

$$x_{k+1} = P x_k - \eta b$$

The error $e_k = x_k - x^*$

$$e_{k+1} = P e_k$$

The convergence rate is determined by the spectral radius of P

$$\rho(P) = \max_i |\lambda_i(P)|$$

To minimize the error, we want to choose η such that $\rho(P)$ is minimized.
 For a symmetric matrix A , the eigenvalues of P are $1 - \eta \lambda_i(A)$.
 The spectral radius is $\max_i |1 - \eta \lambda_i(A)|$.
 To minimize this, we choose η such that $1 - \eta \lambda_{\min}(A) = 1 - \eta \lambda_{\max}(A)$.
 This gives $\eta = \frac{2}{\lambda_{\min}(A) + \lambda_{\max}(A)}$.
 The convergence rate is $\frac{\lambda_{\max}(A) - \lambda_{\min}(A)}{\lambda_{\max}(A) + \lambda_{\min}(A)}$.

$$J = \sum_{k=1}^N P(c_1 | x_k) \cdot P(c_2 | x_k)$$

$$J = \sum_{k=1}^N \frac{N}{N} \cdot (1 - y_k)^{c_1 - t_k} \cdot (1 - y_k)^{c_2 - t_k}$$

note taking 'ln' and '...' will leads to minimize the objective function

$$J = - \sum_{k=1}^N t_k \ln y_k - (c_1 - t_k) \ln(1 - y_k)$$

$$\nabla J = - \sum_{k=1}^N \frac{t_k}{y_k} \cdot \left(\frac{dy_k}{dx_k} \right) + \frac{(c_1 - t_k)}{(1 - y_k)} \cdot \left(\frac{dy_k}{dx_k} \right)$$

$$y_k = \frac{e^{-s_1(x_k)} + e^{-s_2(x_k)}}{e^{-s_1(x_k)} + e^{-s_2(x_k)} + e^{-s_3(x_k)}}$$

$$\left[\frac{e^{-s_1(x_k)} \cdot (-s_1(x_k))}{e^{-s_1(x_k)} + e^{-s_2(x_k)} + e^{-s_3(x_k)}} - \frac{e^{-s_2(x_k)} \cdot (-s_2(x_k))}{e^{-s_1(x_k)} + e^{-s_2(x_k)} + e^{-s_3(x_k)}} \right] \cdot \left(\frac{dy_k}{dx_k} \right)$$

$$\frac{dy_k}{dx_k} = y_k - y_k^2 - y_k(1 - y_k)$$

$$\frac{dy_k}{dx_k} = \frac{d}{dx_k} [\ln P(x_k)] = \ln P(x_k)$$

$$= \sum_{k=1}^n \frac{y_k}{y_k} (1-y_k) + \sum_{k=1}^n \frac{(1-y_k)}{(1-y_k)} y_k (1-y_k) \phi(x_k)$$

$$= \sum_{k=1}^n -t_k \phi(x_k) + t_k y_k \phi(x_k) + y_k \phi(x_k) - t_k y_k \phi(x_k)$$

$$= \sum_{k=1}^n (y_k - t_k) \phi(x_k)$$

$$WJ = \Phi^T(x) (y-t)$$

$x_1^T x_1$	$x_1^T x_2$	$x_1^T x_3$	$x_1^T x_4$	$x_1^T x_5$	$x_1^T x_6$	$x_1^T x_7$	$x_1^T x_8$	$x_1^T x_9$	$x_1^T x_{10}$
$x_2^T x_1$	$x_2^T x_2$	$x_2^T x_3$	$x_2^T x_4$	$x_2^T x_5$	$x_2^T x_6$	$x_2^T x_7$	$x_2^T x_8$	$x_2^T x_9$	$x_2^T x_{10}$
$x_3^T x_1$	$x_3^T x_2$	$x_3^T x_3$	$x_3^T x_4$	$x_3^T x_5$	$x_3^T x_6$	$x_3^T x_7$	$x_3^T x_8$	$x_3^T x_9$	$x_3^T x_{10}$
$x_4^T x_1$	$x_4^T x_2$	$x_4^T x_3$	$x_4^T x_4$	$x_4^T x_5$	$x_4^T x_6$	$x_4^T x_7$	$x_4^T x_8$	$x_4^T x_9$	$x_4^T x_{10}$
$x_5^T x_1$	$x_5^T x_2$	$x_5^T x_3$	$x_5^T x_4$	$x_5^T x_5$	$x_5^T x_6$	$x_5^T x_7$	$x_5^T x_8$	$x_5^T x_9$	$x_5^T x_{10}$
$x_6^T x_1$	$x_6^T x_2$	$x_6^T x_3$	$x_6^T x_4$	$x_6^T x_5$	$x_6^T x_6$	$x_6^T x_7$	$x_6^T x_8$	$x_6^T x_9$	$x_6^T x_{10}$
$x_7^T x_1$	$x_7^T x_2$	$x_7^T x_3$	$x_7^T x_4$	$x_7^T x_5$	$x_7^T x_6$	$x_7^T x_7$	$x_7^T x_8$	$x_7^T x_9$	$x_7^T x_{10}$
$x_8^T x_1$	$x_8^T x_2$	$x_8^T x_3$	$x_8^T x_4$	$x_8^T x_5$	$x_8^T x_6$	$x_8^T x_7$	$x_8^T x_8$	$x_8^T x_9$	$x_8^T x_{10}$
$x_9^T x_1$	$x_9^T x_2$	$x_9^T x_3$	$x_9^T x_4$	$x_9^T x_5$	$x_9^T x_6$	$x_9^T x_7$	$x_9^T x_8$	$x_9^T x_9$	$x_9^T x_{10}$
$x_{10}^T x_1$	$x_{10}^T x_2$	$x_{10}^T x_3$	$x_{10}^T x_4$	$x_{10}^T x_5$	$x_{10}^T x_6$	$x_{10}^T x_7$	$x_{10}^T x_8$	$x_{10}^T x_9$	$x_{10}^T x_{10}$

Given matrix is

$$\begin{bmatrix} \phi(x_{11}) & \phi(x_{21}) \\ \phi(x_{12}) & \phi(x_{22}) \\ \phi(x_{13}) & \phi(x_{23}) \end{bmatrix}$$

(not to mix (x_{12}, x_{21}))
 instead to calculate $\phi(x_{11})$, instead ~~use~~ replaced

with kernel matrix

$$\begin{bmatrix} k(x_{11}, x_{11}) & k(x_{11}, x_{21}) \\ k(x_{12}, x_{11}) & k(x_{12}, x_{21}) \\ k(x_{21}, x_{11}) & k(x_{21}, x_{21}) \end{bmatrix}$$

inner product
 matrix

$$(x_{11}, x_{21}) \times (x_{11}, x_{21})$$

GO X GO.

