Roll No.												

PRESIDENCY UNIVERSITY
BENGALURU

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

SCHOOL OF ENGINEERING

TEST - 1

Sem & AY: Odd Sem, 2019-20 | Date: 30.09.2019

Course Code: MEC 325 | Time: 9.30 AM to 10:30 AM

Course Name: Engineering Dynamics | Max Marks: 40

Program & Sem: B. Tech. (MEC) & VII (DE) | Weightage: 20%

Instructions:

(i) Answer all the questions!

Part A [Memory Recall Questions]

Answer all the Questions. (2Q=11M)

1. State whether the following statements are true or false. Justify your answer.
   (a) In this Course we say a particle is special kind of rigid body whose dimensions are assumed to change with the application of loads. (2 Marks) (C. O. No. 1) [Knowledge]
   (b) The magnitude of the acceleration of a particle will be different in different coordinate systems. (2 Marks) (C. O. No. 1) [Knowledge]
   (c) Suppose O is a point whose position does not change with respect to time. Consider a velocity vector with its tail at O. This vector represents absolute velocity. (2 Marks) (C. O. No. 1) [Knowledge]
   (d) The total work done by all the forces acting on a particle as it moves from a position 1 to a position 2 equals the sum of the corresponding changes in potential energy. (2 Marks) (C. O. No. 1) [Knowledge]
2. Use the definitions of velocity and acceleration in rectilinear motion to express acceleration as a function of the velocity and position of the particle. The relation being asked for is $vdv = a\,ds$ where v, a and s are the velocity, acceleration and position of the particle, respectively, at the instant. (3 Marks)(C. O. No. 1) [Knowledge]

Part B [Thought Provoking Questions]

Answer all the Questions. (2Q=15M)

3. Distinguish between the rectangular coordinate system and the normal-tangential coordinate system in the following aspects.
   (a) the unit vectors in the two coordinate systems, (2 Marks)
   (b) the position vectors in terms of unit vectors, (1 Marks)
   (c) the velocity vectors in terms of unit vectors, and, (3 Marks)
   (d) the acceleration vectors in terms of unit vectors. (3 Marks)
   You answer must have figures and mathematical expressions for the position, velocity and acceleration vectors in the two coordinate systems. (C. O. No. 1) [Comprehension]

4. Answer the following questions based on the information in Figure 1. The figure shows the spacecraft Goggapest located at a distance r_1 from the centre of Kepler-452b.

(C. O. No. 1) [Comprehension]

(a) Sketch the free body diagram of Goggapest at any arbitrary position r. (1 Mark)

(b) Determine the work down by the force in your free body diagram in moving Goggapest from r_1 to r_2 (3 Marks)

(c) Determine the potential energy of Goggapest from your answer to (b). (2 Marks)

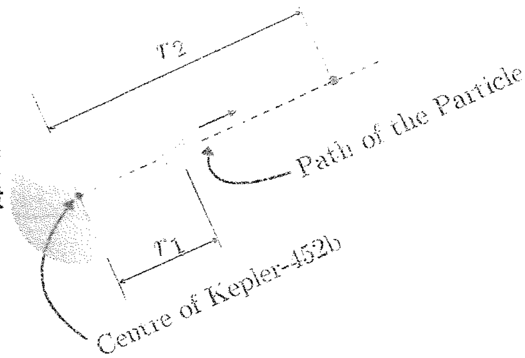


Figure 1: Goggapest near Kepler-452b

Part C [Problem Solving Questions]

Answer all the Questions. (2Q=14M)

5. In solving impact problems concerning systems of particles it is easier to use one of the impulse-momentum equations. The problem you must solve in this question concerns the development of the linear impulse-momentum equation from Newton's second law of motion. Answer the following questions to solve the problem.

(C. O. No. 1) [Application]

(a) Define linear momentum for a particle in three-dimensional curvilinear motion. Your definition must have a figure showing the terms used in the definition. (3 Marks)

(b) Define linear impulse. (1 Mark)

(c) Derive the linear impulse-momentum principle for a particle. (3 Marks)

6. Figure 2 shows the sketch of a Volvo F12 6x2 Tungbärgare tow truck loaded with a Mercedes-Benz GLA 250 Compact SUV. The GLA is being towed to a service station for repairs. Unfortunately, the GLA is not fully secure on the truck and may slide off if the truck accelerates "quickly."

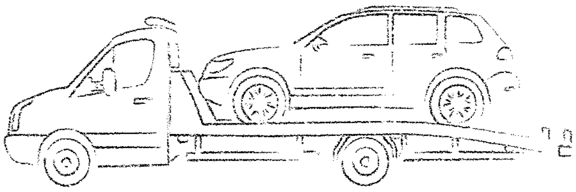


Figure 2: A Volvo Tungbärgare Loaded with a Mercedes-Benz GLA

The driver of the truck, Baby, wants to determine a safe acceleration to prevent the GLA from sliding off. He knows that the truck and car need to be treated as rigid bodies but he doesn't fully understand rigid body dynamics yet. So he decides to frame two related problems assuming they are particles. Solve the following problems Baby has framed.

(5M+2M=7 Marks)(C. O. No. 1) [Application]

(a) A flatbed truck carrying a 90-kg crate starts from rest and attains a velocity of 65 kilometres per hour over a distance of 80 meters on level road with constant acceleration. The coefficients of static and kinetic friction between the crate and flatbed are 0.32 and 0.25, respectively. Help Baby by telling him if the crate "slides" on the flatbed under these conditions.

(b) Baby is also interested in the theoretical power used by the truck to transport the crate. Help Baby by determining the work done by the friction force on the crate. ☐

Annexure I: Summary of Question Distribution [C. O. Wise and Bloom's Level Wise]

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

Date: September 27, 2019

Semester: Odd Semester

Time: 2:30 PM to 3:30 PM

Course Code: MEC 325

Max Marks: 40

Course Name: Engineering Dynamics

Weightage: 20%

Extract of Question Distribution [Outcome Wise and Level Wise]

Q. No.	C. O. No.	Module No. and Title	Memory Recall Type [Marks Allotted] Bloom's Level	Thought Provoking Type [Marks Allotted] Bloom's Level	Problem Solving Type [Marks Allotted] Bloom's Level	Total Marks
			K	C	A	
1	1	Dynamics of Particles and Systems of Particles	8	-	-	8
2	1		3	-	-	3
3	1		-	9	-	9
4	1		-	6	-	6
5	1		-	-	7	7
6	1		-	-	7	7
		Total Marks	11	15	14	40

K: Knowledge Level, C: Comprehension Level, A: Application Level

Note: While setting all types of questions the general guideline is that about 60% of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

SOLUTION

Date: September 27, 2019

Semester: Odd Semester

Time: 2:30 PM to 3:30 PM

Course Code: MEC 325

Max Marks: 40

Course Name: Engineering Dynamics

Weightage: 20%

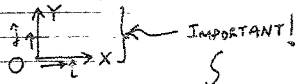
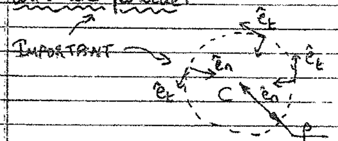
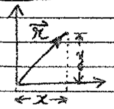
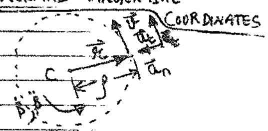
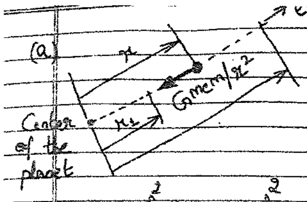
Part A

(2Q=11M)

Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
1	(a) False! A particle is not even a rigid body.	✦ 1 M for each truth value. ✦ 1 M for justifying each truth value.	5 Minutes
	(b) False! Coordinate systems are mathematical means of representing vectors, like accelerations. Such representation do not change the properties, like magnitudes and directions, of the vectors being represented. So the magnitude of a vector is the same in all coordinate systems. (c) True! This follows from the definition of absolute velocity. (d) False! The statement should have said kinetic energy instead of potential energy to correctly state the work-energy principle.		
2	We define velocity of a particle as $v \triangleq \frac{ds}{dt}$ and its acceleration as $a \triangleq \frac{dv}{dt}$. Then $dt = \frac{ds}{v} = \frac{dv}{a}$ $\Longrightarrow ads = vdv.$ $\square$	✦ 1 M for the definition of velocity. ✦ 1 M for the definition of acceleration. ✦ 1 M for manipulating the two definitions to get the final answer.	5 Minutes

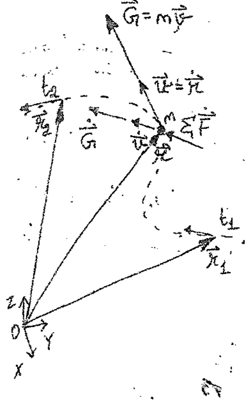
Part B

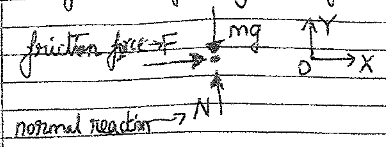
(2Q=15M)

Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
3	(a) Let O be the origin of the rectangular coordinate system. Then it is represented as shown below  The unit vectors $\hat{i}$ and $\hat{j}$ are fixed and are used to express position, velocity and acceleration in the rectangular coordinate system. Consider a particle in circular motion with C being the center of the circle. The figure below shows the unit vectors $\hat{e}_n$ and $\hat{e}_t$ which move with the particle.  (b), (c) and (d) CARTESIAN COORDINATES | NORMAL-TANGENTIAL COORDINATES   Position: $\vec{r} = x\hat{i} + y\hat{j}$ | $\vec{r} = -\rho\hat{e}_n$ Velocity: $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$ | $\vec{v} = \rho\dot{\beta}\hat{e}_t$ Acceleration: $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$ | $\vec{a} = \rho\ddot{\beta}\hat{e}_t + \rho\dot{\beta}^2\hat{e}_n$	(a) 1 M for the two figures. 1 M for distinguishing the between the fixed and moving unit vectors. (b) 1 M for the two position vectors. (c) 3 M for the two velocity vectors. (d) 3 M for the two acceleration vectors.	10 Minutes
4	(a)  (b) $U_{1-2} = \int_1^2 \vec{F}\cdot d\vec{r} = \int_1^2 \left(-\frac{Gm_em}{r^2}\hat{e}_r\right)\cdot(dr\,\hat{e}_r)$ $= \int_{r_1}^{r_2} -\frac{Gm_em}{r^2}dr = Gm_em\int_{r_1}^{r_2}\frac{-dr}{r^2} = Gm_em\left(\frac{1}{r}\right)\Big\|_{r_1}^{r_2}$ $\Rightarrow U_{1-2} = Gm_em\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ (c) $V_g = -U_{1-2} = Gm_em\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.	(a) 1 M for the free body diagram. (b) 3 M to determine the work done. (c) 1 M to determine the potential energy.	5 Minutes

Part C

(2Q=14M)

Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
5	LINEAR IMPULSE AND LINEAR MOMENTUM  A particle of mass m moves from the position $\vec{r}_1$ at time t_1 to $\vec{r}_2$ at t_2. At an arbitrary instant t the particle is located by $\vec{r}$, has a velocity $\vec{v}$ and acceleration $\dot{\vec{v}}$. The equation of motion of the particle is: $\Sigma\vec{F} = m\dot{\vec{v}}$. Since we are considering the mass of the particle to be fixed, we have $\Sigma\vec{F} = \frac{d}{dt}(m\vec{v})$. Define $\vec{G} = m\vec{v}$ to be the linear momentum of the particle. Then $\Sigma\vec{F} = \dot{\vec{G}}$. Integrating this from t_1 to t_2 gives us $\int_{t_1}^{t_2}\Sigma\vec{F}dt = \int_{t_1}^{t_2}\dot{\vec{G}}\,dt = \int_{t_1}^{t_2}\frac{d}{dt}(\vec{G})\,dt = (\vec{G})\Big\|_{t_1}^{t_2}$ $\Rightarrow \int_{t_1}^{t_2}\Sigma\vec{F}\,dt = \vec{G}(t_2) - \vec{G}(t_1)$. — (A) The integral $\int_{t_1}^{t_2}\Sigma\vec{F}\,dt$ is defined as the linear impulse. [The equation (A) says that the total linear impulse equals the change in linear momentum.] THE LINEAR IMPULSE-MOMENTUM PRINCIPLE	(a) 2 M for the figure with all the terms. 1 M for the definition of linear impulse. (b) 1 M to define linear impulse. (c) 3 M to derive the principle of linear impulse-momentum.	10 Minutes
6	The truck starts at rest $\Rightarrow v_1 = 0$. It attains a speed of 65 kmph $= \frac{65\times1000}{(60)(60)}$ m/s ≈ 18 m/s $= v_2$ over the distance $s = 80$ m. The mass of the crate $m = 90$ kg and its acceleration is given by: $v_2^2 - v_1^2 = 2as$ $\Rightarrow a = \frac{v_2^2 - v_1^2}{2s} = \frac{(18)^2-(0)^2}{2(80)} = 2.025\ m/s^2$. The inertial force on the crate is $ma = (90)(2.025) = 182.25$ N.	◆ 2 M to determine the acceleration of the crate. ◆ 1 M for the free body diagram of the crate.	5 Minutes for This Part of the Question

6	The free body diagram of the crate is:  From Newton's second law of motion: $\Sigma F_Y = ma_Y$ $\Rightarrow N - mg = 0 \Rightarrow N = mg = (90)(9.81) \approx 883$ N. The maximum available force from static friction is $\mu_s N = (0.32)(883) \approx 282.6$ N. This is ~~less~~ more than the inertial force $\Rightarrow$ the crate does not "slip". ~~Therefore~~ We use the work-energy principle to determine the work done by the force of friction on the crate to be: $U_{1-2} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} m (v_2^2 - v_1^2)$ $= \frac{1}{2}(90)((18)^2 - (0)^2) = 14{,}580$ N·m.	◆ 1 M to determine the maximum force from static friction. ◆ 1 M to conclude the crate is fixed to the truck. ◆ 1 M for the right formula for work done. ◆ 1 M to determine the work done.	10 Minutes for This Part of the Question

The End

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SCHOOL OF ENGINEERING

TEST - 2

Sem & AY: Odd Sem, 2019-20
Date: 18.11.2019
Course Code: MEC 325
Time: 9:30 AM to 10:30 AM
Course Name: ENGINEERING DYNAMICS
Max Marks: 40
Program & Sem: B. Tech. (MEC) & VII (DE)
Weightage: 20%

Instructions:
(i) Answer all the questions.

Part A [Memory Recall Questions]

(2Q=11M)

Answer **all** the Questions.

1. State whether the following statements are true or false. Justify your answer.
   (a) All points on a rigid body in general plane motion have the same velocity.
   (2 M) (C. O. No. 2) [Knowledge]
   (b) Different points on a rigid body have different angular accelerations.
   (2 M) (C. O. No. 2) [Knowledge]
   (c) The acceleration term $\vec{\omega} \times \vec{\omega} \times \vec{r}$ is in the direction normal to the motion of the point in consideration.
   (2 M) (C. O. No. 2) [Knowledge]
   (d) The principle of angular impulse and angular momentum is not applicable to particles and systems of particles.
   (2 M) (C. O. No. 2) [Knowledge]

2. Prove that any rigid body has an instantaneous centre of zero velocity.
   (3 M)(C. O. No. 2) [Knowledge]

Part B [Thought Provoking Questions]

(2Q=14M)

Answer **all** the Questions.

3. Answer the following questions on rotational kinematics of rigid bodies.
   (a) Is it correct to say a particle has an angular velocity of 0.75 rad/s? Justify your answer. (2 M)(C. O. No. 2) [Comprehension]
   (b) Prove or disprove the following claim: All lines in a rigid body have the same angular velocity and angular acceleration. (4 M)(C. O. No. 2) [Comprehension]

4. Figure 1 shows a rigid body that is translating and rotating. The rigid body has a particle A sliding on it. Answer the following questions based on the information in Figure 1.

(C. O. No. 2) [Comprehension]

(a) Sketch the position vector of A relative to B and give it a suitable name. (2 M)

(b) Suppose the position of A from B is x units along the $\hat{i}$ direction and y units along the $\hat{j}$ direction. Write down the expressions for the position and velocity of A relative to B in vector notation. (2 M)

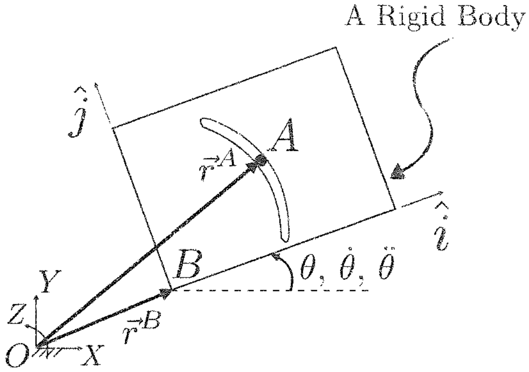


Figure 1: A Particle Sliding on a Translating and Rotating Rigid Body

(c) Show that the time derivative of the velocity of A relative to B denoted as

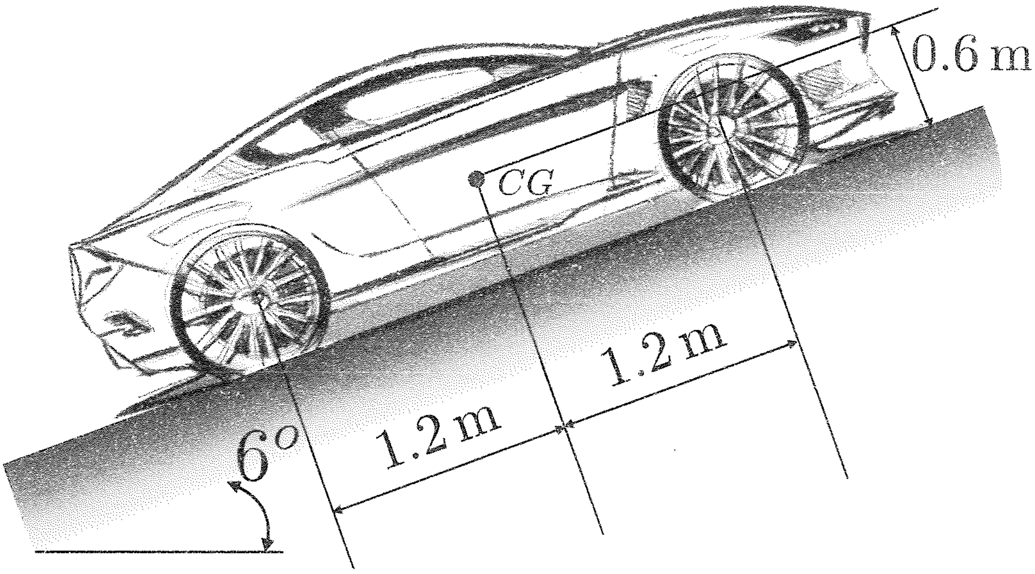
$$\frac{d\overrightarrow{v}_{rel}}{dt} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \overrightarrow{\omega} \times \overrightarrow{v}_{rel}.$$ (4 M)

Part C [Problem Solving Questions]

Answer the Question. The Question carries fifteen marks. (1Q=15M)

5. Volvo has designed a new automobile. Before manufacturing one of these cars it has decided to give you the task of analysing its performance when it travels up a 6^o slope. The car has a mass of 900 kilograms and reaches a speed of 10 m/s from rest over a distance of 70 meters up the slope. You are asked to check if the rear tyres, which experience friction, ever slip over the 70 meters. The coefficient of friction between the tyres and ground is experimentally determined to be at least 0.8.

(C. O. No. 2) [Application]



The End

Annexure I: Summary of Question Distribution [C. O. Wise and Bloom's Level Wise]

SCHOOL OF ENGINEERING

Sem & AY: Odd Sem, 2019-20	**Date**: 18.11.2019
Course Code: MEC 325	**Time**: 9:30 AM to 10:30 AM
Course Name: ENGINEERING DYNAMICS	**Max Marks**: 40
Program & Sem: B. Tech. (MEC) & VII (DE)	**Weightage**: 20%

Extract of Question Distribution [Outcome Wise and Level Wise]

Q. No.	C. O. No.	Module No. and Title	Memory Recall Type [Marks Allotted] Bloom's Level	Thought Provoking Type [Marks Allotted] Bloom's Level	Problem Solving Type [Marks Allotted] Bloom's Level	Total Marks
			K	C	A	
1	2	Dynamics of Particles and Systems of Particles	8	-	-	8
2	2		3	-	-	3
3	2		-	9	-	9
4	2		-	6	-	6
5	2		-	-	7	7
		Total Marks	11	15	14	40

K: Knowledge Level, C: Comprehension Level, A: Application Level

Note: While setting all types of questions the general guideline is that about 60% of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

Annexure II: Solutions and Scheme of Marking

SCHOOL OF ENGINEERING

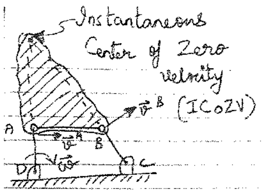
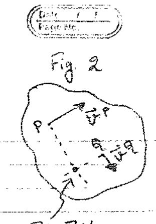
SOLUTION

Sem & AY: Odd Sem, 2019-20
Course Code: MEC 325
Course Name: ENGINEERING DYNAMICS
Program & Sem: B. Tech. (MEC) & VII (DE)

Date: 18.11.2019
Time: 9:30 AM to 10:30 AM
Max Marks: 40
Weightage: 20%

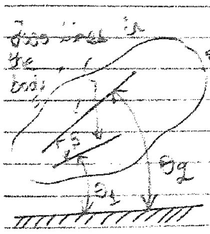
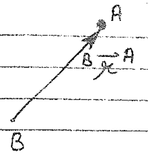
Part A

(2Q=11M)

Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
1	(a) False! An example: the velocities of any two points on a simple pendulum are different. (b) False! Points do not have angular accelerations. (c) True! This follows from the definition of absolute velocity. (d) False! Angular impulses and angular momenta are defined for particles and integration of the moment equation gives us the equation relating angular impulses and angular momenta.	✦ 1 M for each truth value. ✦ 1 M for justifying each truth value.	5 Minutes
2	 Fig. 1  Suppose we have a rigid body in plane motion. Consider any two arbitrary points on the body and sketch their velocity vectors. Draw lines perpendicular to the velocity vectors from these points. These points intersect and this intersection is the ICoZV. Fig. 1 shows this in the case of a 4-bar linkage and Fig. 2 shows this in a general case of a rigid body.	✦ 2 M for the figures. ✦ 3 M for the explanation.	5 Minutes

Part B

(2Q=14M)

Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
3. (a)	Particles are so small that it is hard to measure their dimensions. So it does not make sense to say a particle is "oriented" a certain way. Hence, there is no meaning in saying a particle has an angular velocity.	✦ 2 M for the explanation.	5 Minutes
3. (b)	 ← Rigid Body a constant! From the figure $\theta_2 = \theta_1 + \beta$. Then $\frac{d\theta_2}{dt} = \frac{d\theta_1}{dt} \Rightarrow \omega_2 = \omega_1 \triangleq \omega$. Furthermore, $\frac{d^2\theta_2}{dt^2} = \frac{d^2\theta_1}{dt^2} \Rightarrow \alpha_2 = \alpha_1 \triangleq \alpha$.	✦ 1 M for the figure with explanation. ✦ 3 M for the argument for angular velocities and angular accelerations.	10 Minutes
4	(a)  (b) ${}^B\vec{r}^A = x\hat{i} + y\hat{j}$ $\vec{v}_{rel} = \frac{d\,{}^B\vec{r}^A}{dt} = \dot{x}\hat{i} + \dot{y}\hat{j}$ (c) $\frac{d(\vec{v}_{rel})}{dt} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \dot{x}\frac{d\hat{i}}{dt} + \dot{y}\frac{d\hat{j}}{dt}$ $= \ddot{x}\hat{i} + \ddot{y}\hat{j} + \vec{\omega} \times \dot{x}\hat{i} + \vec{\omega} \times \dot{y}\hat{j}$ $= \ddot{x}\hat{i} + \ddot{y}\hat{j} + \vec{\omega} \times (\dot{x}\hat{i} + \dot{y}\hat{j})$ $= \ddot{x}\hat{i} + \ddot{y}\hat{j} + \vec{\omega} \times \vec{v}_{rel}$.	(a) 1 M for the free body diagram. (b) 3 M to determine the work done. (c) 1 M to determine the potential energy.	10 Minutes

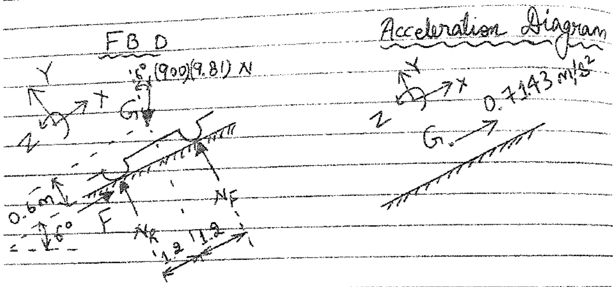
Part C

(1Q=15M)

<table>
<tr><th>Q. No.</th><th>Solution</th><th>Scheme of Marking</th><th>Maximum Time Needed for Each Question</th></tr>
<tr><td>5</td><td>

$m = 900\ kg,\ v(0) = 0,\ v(70) = 10\ m/s.$

Then $a = \frac{(10)^2 - (0)^2}{2(70)} = 0.7143\ m/s^2.$



$\Sigma F_x = ma_x : F - (900)(9.81)\sin 6^\circ = (900)(0.7143)$
$\Rightarrow F = 1565.5\ N$

$\Sigma F_Y = ma_Y^{\,0} : N_R + N_F - (900)(9.81)\cos 6^\circ = 0$
$\Rightarrow N_R + N_F = 8780.44$ — (1)

$\Sigma M_G = I_G \alpha^{\,0} : -1.2 N_R + (0.6)(1565.5) + 1.2 N_F = 0$
$\Rightarrow -N_R + N_F = -782.75$ — (2)

$N_R = \dfrac{\begin{vmatrix} 8780.44 & 1 \\ -782.75 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}} = 4781.595$

$\frac{F}{N_R} = 0.327 < 0.8 \Rightarrow$ The rear tyres do not slip.

</td><td>

- 2 M for the acceleration.
- 4 M for the free body diagram.
- 2 M for the acceleration diagram.
- 2 M for the friction force.
- 3 M for the normal reaction force on the rear wheel.
- 1 M to determine the required coefficient of friction.
- 1 M for the conclusion.

</td><td>15 Minutes</td></tr>
</table>

The End

Roll No.												

PRESIDENCY UNIVERSITY
BENGALURU

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

SCHOOL OF ENGINEERING

END TERM FINAL EXAMINATION

Semester: Odd Semester: 2019-20 | **Date**: 24 December 2019

Course Code: MEC 325 | **Time**: 9:30 AM to 12:30 PM

Course Name: ENGINEERING DYNAMICS | **Max Marks**: 80

Program & Sem: B. Tech. (MEC) & VII (DE-IV) | **Weightage**: 40%

Instructions:

(i) Answer all the questions.

Part A [Memory Recall Questions]

(4Q=25M)

Answer all the Questions.

1. The following statements are true. Explain why they are true.
   (a) The tangential acceleration of any two points on a rigid body in general plane motion are not equal in the general case. (3 M)(C. O. No. 2) [Comprehension]
   (b) The kinetic energy of a rigid body is given by the sum of half of the mass moment of inertia of the body about its centre of gravity times the square of its angular velocity and half of its mass times the square of the velocity of its centre of mass. (3 M) (C. O. No. 2) [Knowledge]
   (c) The acceleration term $\vec{\omega} \times \vec{\omega} \times \vec{r}$ is in the direction normal to the motion of the point in consideration. (3 M) (C. O. No. 2) [Knowledge]
   (d) The principle of linear impulse and linear momentum is applicable to particles, systems of particles and rigid bodies. (3 M) (C. O. No. 2) [Knowledge]
2. Suppose the angular acceleration of a rigid body is given by $\vec{\alpha} = \ddot{\theta}\hat{k}$. This body is rotating about a point O located on the body. Determine the tangential acceleration $\vec{\alpha} \times \vec{r}$ of a point on the body whose position from O is given by $\vec{r} = r\hat{e}_r$.
   (4 M)(C. O. No. 2) [Comprehension]
3. Complete the following statements by using as many words, mathematical symbols or equations as you need.
   (a) A difference between a particle and a rigid body is that the dimensions __________.
   (1 M)(C. O. No. 1) [Knowledge]
   (b) The unit vectors in the Cartesian coordinate system are fixed. The unit vectors in the normal and tangential coordinate system ______________.
   (1 M)(C. O. No. 1) [Knowledge]
   (c) The principle of work and energy for a rigid body states that ______________.
   (2 M)(C. O. No. 2) [Knowledge]

4. Match the terms in the first column to suitable ones in the second column.

(5 M)(C. O. No. 3) [Comprehension]

(a) $x(t), \theta(t)$	(i) May be a real or complex number
(b) $e^{\lambda t}$	(ii) Solutions to oscillatory motion in systems
(c) λ	(iii) Positions as functions of time
(d) $\sin t, \cos 3.2t, \sin(4t+\pi), \cdots$	(iv) Taylor's series expansion of $f(x)$ about the point x_o
(e) $f(x) = f(x_o) + \frac{df}{dx}\|_{x=x_o} \frac{(x-x_o)}{1!} + \frac{d^2f}{dx^2}\|_{x=x_o} \frac{(x-x_o)^2}{2!} + \cdots$	(v) Assumed solution to systems with no "inputs"
	(vi) A nonhomogeneous ordinary differential equation
	(vii) Numerical solution to the equation of motion of the spring-mass-damper system

Part B [Thought Provoking Questions]

(1Q=26M)

Answer the Question.

5. Answer the following questions on the spring-mass-damper system. You may use the fact that its equation of motion is $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)$ where m, c and k are the mass, damping constant and spring constant of the system, x is the position of the system at time t and $F(t)$ is the input force to the system.
   (a) State suitable S. I. units for m, c, k, x, t and F. (3 M)(C. O. No. 3) [Knowledge]
   (b) When is a mathematical function a solution to the equation of motion of the spring-mass-damper system? (2 M)(C. O. No. 3) [Comprehension]
   (c) Check if $x(t) = a, a \in \mathbb{R}$ can be a solution to the equation of motion. Also explain the conditions under which it can be a solution. (4 M)(C. O. No. 3) [Comprehension]
   (d) Distinguish between homogeneous and nonhomogeneous ordinary differential equations in the context of this system by explaining how they relate the physical conditions in the real world. (4 M)(C. O. No. 3) [Comprehension]
   (e) Suppose the spring-mass-damper system executes a motion without any input force. Name the different types of motion possible in this situation? Explain the motion with the help of mathematical expressions and sketches.
   (5 M)(C. O. No. 3) [Comprehension]
   (f) Show that $e^{\lambda t}$ is a solution to $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0$. Is this a solution to the system with the input force or without the input force?
   (4 M)(C. O. No. 3) [Application]
   (g) Discuss the kind of motion the system has when λ is a complex number with a real and an imaginary part. You must use illustrations of the motion in your discussion.
   (4 M)(C. O. No. 3) [Application]

Part C [Problem Solving Questions]

Answer all the Questions. (2Q=29M)

6. Answer the following questions on dynamics of the simple pendulum.

(a) Derive the equation of motion (EOM) $\ddot{\theta}(t) + \frac{g}{\ell}\sin\theta(t) = 0$ for a simple pendulum.

In the EOM $\theta(t)$ is the angular position of the pendulum at time t, $\ddot{\theta}(t) \triangleq \frac{d^2\theta}{dt^2}$, g is the local acceleration due to gravity and ℓ is the length of the simple pendulum. Your work must include a sketch showing the pendulum in its general configuration and sketches showing its free body diagram and acceleration diagram.

(4 M)(C. O. No. 1)[Application]

(b) *Mathematically* prove that the equation of motion is nonlinear.

(4 M)(C. O. No. 3)[Comprehension]

(c) Linearise the nonlinear EOM in a "crude," intuitive way without any sophisticated mathematical machinery about the point $\theta = 0$ radians.

(2M)(C. O. No. 3)[Comprehension]

(d) Explain the difference between the nonlinear and linear equations of motion of the simple pendulum in the context of region of validity of the two equations.

(2M)(C. O. No. 3)[Application]

(e) Determine the solution to the equation of motion linearised about the point $\theta = 0$ radians. (4M)(C. O. No. 3)[Application]

(f) Answer the following questions keeping in mind the solution you have obtained in the previous question. Each of the questions carry one mark.

(4M)(C. O. No. 3)[Application]

i. Is the solution oscillatory in nature or does it have constant amplitude?
ii. Is the solution underdamped, critically damped, overdamped or undamped?
iii. Identify the frequency of oscillations in your solution.
iv. Identify a region of validity of the solution to the linear equation of motion.

7. The sheave E of a hoisting rig shown in Figure 2 has a mass of 25 kg and a centroidal radius of gyration of 250 mm. The 40-kg load D which is carried by the sheave has an initial downward velocity of $v_1 = 1.25$ m/s at the instant when a clockwise torque is applied to the hoisting drum A to maintain essentially a constant force $F = 380$ N in the cable B. Compute the angular velocity ω_2 of the sheave 5 seconds after the torque is applied to the drum and find the tension T in the cable at O during the interval. Neglect all friction.

(9 M)(C. O. No. 2) [Application]

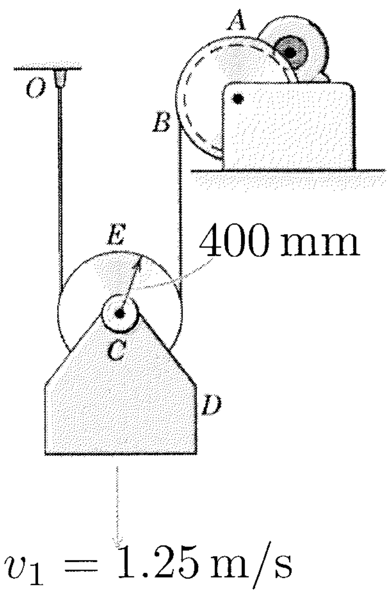


Figure 2: A Hoisting Rig

The End

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

END TERM FINAL EXAMINATION

Extract of Question Distribution [Outcome Wise and Level Wise]

The numbers and titles of the modules in this Course are:
1. Unit 1: Dynamics of Particles and Systems of Particles,
2. Unit 2: Dynamics of Rigid Bodies,
3. Unit 3: Linearisation and Solutions to Equations of Motion.

Q. No.	C. O. No.	Module No. and Title	Memory Recall Type [Marks Allotted] Bloom's Level	Thought Provoking Type [Marks Allotted] Bloom's Level	Problem Solving Type [Marks Allotted] Bloom's Level	Total Marks
			K	C	A	
1	2	Unit 2	9	3	-	12
2			-	4	-	4
3	1, 2	Units 1 and 2	4	-	-	4
4	3	Unit 3	-	5	-	5
5			3	15	8	26
6			-	6	14	20
7	2	Unit 2	-	-	9	9
		Total Marks	16	33	31	80

K: Knowledge Level, C: Comprehension Level, A: Application Level

Note: While setting all types of questions the general guideline is that about 60% of the questions must be such that even a below average students must be able to attempt, About 20% of the questions must be such that only above average students must be able to attempt and finally 20% of the questions must be such that only the bright students must be able to attempt.

Faculty Signature:

Reviewer Commend:

Annexure II: Solutions and Scheme of Marking

SCHOOL OF ENGINEERING

SOLUTION

Sem & AY: Odd Sem, 2019-20
Course Code: MEC 315
Course Name: TRIBOLOGY AND BEARING DESIGN
Program & Sem: B. Tech. (MEC) & V (DE-II)

Date: 23.12.2019
Time: 9:30 AM to 12:30 PM
Max Marks: 80
Weightage: 40%

Part A

(4Q=25M)

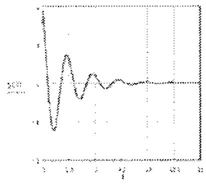
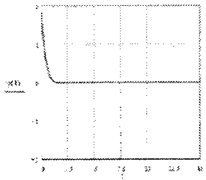
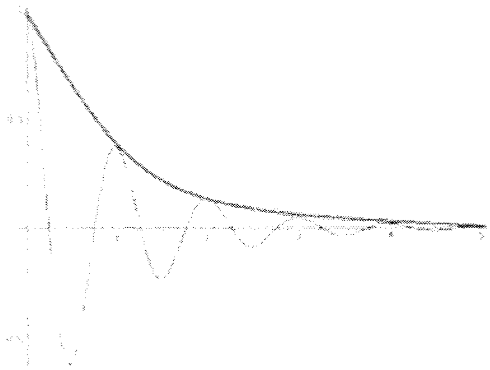
Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
1 (a)	Let the angular acceleration of a rigid body be $\vec{\alpha}$. Let the position of the i-th point on the body be given by $\vec{r}_i$. The tangential acceleration of this point is given by $\vec{\alpha} \times \vec{r}_i$. Since the position of each point is unique, the tangential acceleration of each point is also unique.	✦ 3 M for the correct explanation in each sub-question	20 minutes to answer 1 (a) thru 1 (d)
1 (b)	The kinetic energy of a rigid body is the sum of its translational kinetic energy and its rotational kinetic energy. Its translational kinetic energy is given by $\frac{1}{2}mv_{\text{CG}}^2$ and its rotational kinetic energy is given by $\frac{1}{2}I_{\text{CG}}\omega^2$.		
1 (c)	The cross product $\vec{v} = \vec{\omega} \times \vec{r}$ is in the direction of motion. The cross product $\vec{\omega} \times \vec{v}$ is normal to the direction of motion.		
1 (d)	The principle of linear impulse and momentum is the first integral of Newton's second law of motion, which is applicable to particles, systems of particles and rigid bodies. Hence, the principle if applicable to all of them.		

Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
2	$\vec{\alpha} \times \vec{r} = \begin{vmatrix} \hat{e}_r & \hat{e}_\theta & \hat{k} \\ 0 & 0 & \ddot{\theta} \\ r & 0 & 0 \end{vmatrix}$ $= \hat{e}_r \cdot 0 - \hat{e}_\theta \cdot (-r\ddot{\theta}) + \hat{k} \cdot 0$ $= r\ddot{\theta}\hat{k}.$	✦ 2 M for setting up the determinant ✦ 2 M for the final answer	10 Minutes
3 (a)	… of a particles are negligible.	✦ 1 M for the right answer	5 minutes to answer these three questions.
3 (b)	… can translate and rotate.	✦ 1 M for the right answer	
3 (c)	… the total work done in moving a particles from one point to another equals the corresponding change in kinetic energy.	✦ 1 M for the right answer	
4	(a)→(iii), (b)→(v), (c)→(i), (d)→(ii), (e)→(iv)	✦ 1 M for each correct match	10 Minutes

Part B

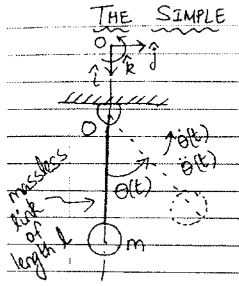
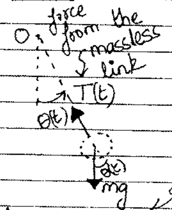
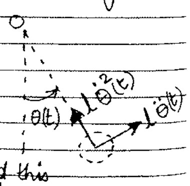
(1Q=26M)

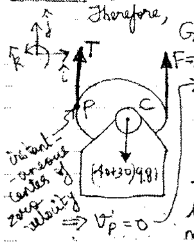
Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
5 (a)	Acceptable S. I. for mass, damping constant, spring constant, displacement, time and force are kg, N·s/m, N/m, m, s and N.	✦ $\frac{1}{2}$ M for each correct unit	5 Minutes
5 (b)	A function $x(t)$ is a solution to the mathematical model $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)$ of the spring-mass-damper system if it satisfies the model.	✦ 2 M for the right answer	5 Minutes

5 (d)	The model $m\ddot{x}(t) + c\dot{x}(t)+kx(t) = F(t)$ is referred to as a non homogeneous ordinary differential equation because it has the input force $F(t)$. When the system does not have an input the right hand side of the equation is zero and is referred to as a homogeneous differential equation.	✦ 2 M for the right explanation for each type of differential equation	5 Minutes
5 (e)	The system can execute a motion that is oscillatory in nature or non-oscillatory in nature. These motions are shown in the figures below.  	✦ 2 M for naming the possible motions ✦ 3 M for the figures	10 Minutes
5 (f), (g)	When λ is a complex number the solution $e^{\lambda t}$ can be written as $e^{\alpha t}e^{i\beta t}$. The real function $e^{\alpha t}$ forms the amplitude of oscillations produced by the complex function $e^{i\beta t}$. So we have decaying oscillations as shown in the figure. 	✦ 2 M for the explanation ✦ 2 M for the figure	10 Minutes

Part C

(2Q=29M)

Q. No.	Solution	Scheme of Marking	Maximum Time Needed for Each Question
6 (a), (b)	THE SIMPLE PENDULUM  Here the motion of the pendulum is described by θ(t). So our "equation of motion for the simple pendulum" must have θ(t) as the unknown. FBD  Acceleration Diagram  we discussed this in class! Let's be "clever" now. Let's write $\Sigma\vec{F}=m\vec{a}$ along the two components of acceleration of the mass instead of doing so along $\hat{i}$ and $\hat{j}$. We have: $T(t) - mg\cos\theta(t) = m\ell\dot{\theta}^2(t)$, —③ $-mg\sin\theta(t) = m\ell\ddot{\theta}(t)$. —④ Equations ③ and ④ have 2 unknowns, T(t) and θ(t). Interestingly, equation ④ has only one unknown, θ(t), which is the function that describes the motion of the simple pendulum. Solving ④ for θ(t) will give us the motion of the pendulum. Hence, it is natural to call ④ as the equation of motion of the simple pendulum.	✦ 2 M for the figure ✦ 2 M for the derivation of the equation of motion ✦ 4 M for proving the equation is nonlinear	25 Minutes
6 (c)	The equation of motion $\ddot{\theta}(t) + \frac{g}{\ell}\sin\theta(t) = 0$ has one nonlinear term $\sin\theta(t)$. We know that $\sin\theta(t) \approx \theta(t)$. We can replace $\sin\theta(t)$ with $\theta(t)$ in the equation to get $\ddot{\theta}(t) + \frac{g}{\ell}\theta(t) = 0$.	✦ 2 M for the explanation	5 Minutes

<table>
<tr>
<td>6 (d)</td>
<td>The nonlinear equation is valid for all values of θ while the linear equation is valid in small neighbourhoods of the point of linearisation.</td>
<td>✦ 2 M for the explanation</td>
<td>5 Minutes</td>
</tr>
<tr>
<td>6 (e), (f)</td>
<td>i. The solutions is oscillatory in nature.
ii. The solution is undamped.
iii. The frequency of oscillations is $\sqrt{\frac{g}{\ell}}$ rad/s.
iv. The solution is valid is a small neighbourhood of $\theta = 0$ radians.</td>
<td>✦ 1 M for teach correct answer</td>
<td>15 Minutes</td>
</tr>
<tr>
<td>7</td>
<td>SOLUTION: The initial velocity is 1.2 m/s downward.
Therefore,
$G_1 = -(40)(1.2) - 30(1.2) = -84$ kg·m/s.


Since $v_P = 0$ we have $v_C = v_P^{\nearrow 0} + \omega_E(0.375) \Rightarrow \frac{1.2}{0.375} = (\omega_E)_1$.
Hence, $(H_1)_G = (30)(0.25)^2\left(\frac{1.2}{0.375}\right) = 6$ N·m·s
−ve! ω_E is ↻
We have the linear and angular momenta at the start.

We now determine the linear and angular impulses as follows:
$\int_{t_1}^{t_2} \Sigma F\, dt = \int_0^5 (T + \overbrace{380 - (40+30)9.81}^{-306.7})dt = (T - 306.7)(t)\Big|_0^5$
$= 5T - 1533.5,$
$\int_{t_1}^{t_2} (\Sigma M)_G\, dt = \int_0^5 [-T(0.375) + \overbrace{F(0.375)}^{712.5}]dt$
$= -1.875T + 712.5.$

Let's determine the final linear and angular momenta.
$G_2 = +40(0.375\,\omega_{E_2}) + (30)(0.375\,\omega_{E_2})$
$= 26.25\,\omega_{E_2}$
$(H_2)_G = \overbrace{(30)(0.25)^2}^{1.875}\omega_{E_2}.$
The two impulse-moment equations are:
$-84 + 5T - 1533.5 = 26.25\,\omega_{E_2}$
$5T - 26.25\,\omega_{E_2} = 1617.5,$ —— (1)
$-6 - 1.875T + 712.5 = 1.875\,\omega_{E_2},$
$-1.875T - 1.875\,\omega_{E_2} = -706.5,$
$T + \omega_{E_2} = $ ~~383.2~~ 376.8, —(2)
Solving (1) and (2):
$\omega_{E_2} = \frac{\begin{vmatrix} 5 & 1617.5 \\ 1 & 376.8 \end{vmatrix}}{\begin{vmatrix} 5 & -26.25 \\ 1 & 1 \end{vmatrix}} = 8.528$ rad/s,
and from (2) $T = 376.8 - 8.528 = 368.27$ N. ▨</td>
<td>✦ 4 M for the free body diagram.
✦ 5 M in total for application of the two principles of impulse and momentum
✦ 2 M for the two equations to be solved
✦ 2 M the right solution</td>
<td>25 Minutes</td>
</tr>
</table>

The End