# Formulas and Calculations for Drilling Operations

Robello Samuel

WILEY

Scrivener

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**...An elegant Euler's formula wrapped with imaginary and real numbers resulting in nothing depicts the relationship between the Creator and human intellect...**

$$e^{i\pi} + 1 = 0$$

To

Cynthia, Nishanth and Sharon

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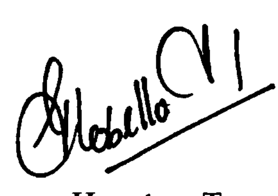
# Preface

This book is an introductory exposition for drilling engineers, students, lecturers, teachers, software programmers, testers, and researchers. The intent is to provide basic equations and formulas with the calculations for downhole drilling. This book may be a tutorial guide for students, to lecturers and teachers it may be a solution manual, and drilling engineers may find that it is a source for solving problems. Software programmers and testers may use it as a guide as they code, unit test, and validate their implementation, and researchers may use it as a source for further development. Of course, it is very difficult to cover all the aspects and areas of drilling, but this book aims to provide an introduction to exploring the vastness and complexity of drilling engineering. The readers are advised to refer to the books in the bibliography for more details regarding underlying theory. This book is a companion to my other books, *Drilling Engineering*, *Downhole Drilling Tools*, *Advanced Drilling Engineering*, and the upcoming *Applied Drilling Engineering Optimization*.

I am grateful to the contributors, the publisher, Phil Carmical, and copyeditor Brittyne Jackson and Mohana Sundaram from Exeter Premedia Services. Also, I thank Dr.João Carlos Plácido and Dr.Dali Gao for helping in formulating some problems. I thank them for their invaluable help. A work of this magnitude with many equations and numbers is bound to have errors even though painstaking efforts have been taken. Needless to say, I request that the readers send errors and comments in effort towards the improvement of this book.



**Houston, Texas**

*Formulas and Calculations for Drilling Operations*
by G. Robello Samuel


# 1

# Basic Calculations

This chapter focuses on different basic calculations such as buoyancy, weight, tension, etc.

## 1.1 Capacities

Capacities of the pipe, annular capacity, and annular volume can be calculated using the following equations.

The linear capacity of the pipe is

$$C_i = \frac{A_i}{808.5} \text{ bbl/ft}, \tag{1.1}$$

where $A_i$ is a cross-sectional area of the inside pipe in square inches and equals $0.7854 \times D_i^2$, and $D_i$ is the inside diameter of the pipe in inches.

Volume capacity is

$$V = C_i \times L \text{ bbl}, \tag{1.2}$$

where $L$ = the length of the pipe, ft.

Annular linear capacity against the pipe is

$$C_o = \frac{A_o}{808.5} \text{ bbl/ft,} \tag{1.3}$$

where $A_o$, a cross-sectional area of the annulus in square inches, is

$$0.7854 \times \left(D_h^2 - D_o^2\right), \tag{1.4}$$

$D_o$ = the outside side diameter of the pipe, in., and $D_h$ = the diameter of the hole or the inside diameter of the casing against the pipe, in.

Annular volume capacity is

$$V = C_o \times L \text{ bbl.} \tag{1.5}$$

## 1.2 Displacement

### 1.2.1 Displacement of the Pipe Based on the Thickness of the Pipe

Open-ended displacement volume of the pipe is

$$V_o = \frac{0.7854\left(D_o^2 - D_i^2\right)}{808.5} \text{ bbl/ft.} \tag{1.6}$$

$$\text{Displacement volume} = V_o \times L \text{ bbl.} \tag{1.7}$$

Close-ended displacement volume of the pipe is

$$V_c = \frac{0.7854\left(D_o^2\right)}{808.5} \text{ bbl/ft.} \tag{1.8}$$

$$\text{Displacement volume} = V_c \times L \text{ bbl.} \tag{1.9}$$

**Problem 1.1**

Calculate the drill pipe capacity, open-end displacement, closed end displacement, annular volume, and total volume for the following condition: 5,000 feet of 5″ drill pipe with an inside diameter of 4.276″ inside a hole of 8½″.

*Solution:*

Linear capacity of pipe, using equation 1.1, is

$$C_i = \frac{A_i}{808.5} = \frac{0.7854 \times D_i^2}{808.5} = \frac{0.7854 \times 4.276^2}{808.5} = 0.017762 \text{ bbl/ft.}$$

Pipe volume capacity = 0.017762 × 5000 = 88.81 bbl.
Open-end displacement of pipe, using equation 1.6, is

$$V_o = \frac{0.7854\left(D_o^2 - D_i^2\right)}{808.5} = \frac{0.7854\left(5^2 - 4.276^2\right)}{808.5} = 0.006524 \text{ bbl/ft.}$$

Close-end displacement of pipe, using equation 1.8 is

$$V_c = \frac{0.7854\left(D_o^2\right)}{808.5} = \frac{0.7854\left(5^2\right)}{808.5} = 0.024286 \text{ bbl/ft.}$$

Annular volume, using equation 1.5 is

$$V = C_o \times L = \frac{A_o}{808.5} \times L = \frac{0.7854}{808.5} \times \left(D_h^2 - D_o^2\right) \times L$$
$$= \frac{0.7854}{808.5} \times \left(8.5^2 - 5^2\right) \times 5000 = 229.5 \text{ bbl.}$$

Total volume = Pipe volume + Annular volume = 88.81 + 229.50 = 318.31 bbl.

## 1.3 Buoyancy, Buoyed Weight, and Buoyancy Factor (BF)

The calculations are based on one fluid.

$$\text{Buoyancy} = \frac{\text{Weight of material in air}}{\text{Density of material}} \times \text{Fluid density}. \tag{1.10}$$

$$\text{Buoyed weight} = \left( \frac{\text{Density of material} - \text{Fluid density}}{\text{Density of material}} \right) \times \text{Weight of material in air}. \tag{1.11}$$

$$\text{Buoyancy factor} = \left( \frac{\text{Density of material} - \text{Fluid density}}{\text{Density of material}} \right). \tag{1.12}$$

$$\text{Buoyancy factor} = \left( \frac{\rho_s - \rho_m}{\rho_s} \right) = \left( 1 - \frac{\rho_m}{\rho_s} \right), \tag{1.13}$$

where $\rho_s$ is the density of the steel/material, and $\rho_m$ is the density of the fluid/mud.

When the inside and outside fluid densities are different, the buoyancy factor can be given as

$$\text{Buoyancy factor (BF)} = \frac{A_o\left(1 - \frac{\rho_o}{\rho_s}\right) - A_i\left(1 - \frac{\rho_i}{\rho_s}\right)}{A_o - A_i}, \tag{1.14}$$

where $A_o$ is the external area of the component, and $A_i$ is the internal area of the component.

## 1.4 Effective Weight

Effective weight per unit length can be calculated using the following relation. Weight per foot in drilling mud is the weight per foot in air minus the weight per foot of the displaced drilling mud:

$$w_B = w_s + \rho_i A_i - \rho_o A_o, \tag{1.15}$$

$$A_o = \frac{\pi}{4}\left(0.95 \times D_o^2 + 0.05 \times D_{oj}^2\right), \tag{1.16}$$

$$A_i = \frac{\pi}{4}\left(0.95 \times D_i^2 + 0.05 \times D_{ij}^2\right). \tag{1.17}$$

Without tool joints, $A_i = 0.7854 \times D_i^2$, and $A_o = 0.7854 \times D_o^2$.
Using equation 1.15, $w_B = w_s + \rho_i A_i - \rho_o A_o$.
In the above equation, unit weight of the steel can be given as

$$w_s = \rho_s A_s. \tag{1.18}$$

When the inside and outside fluid densities are the same,

$$w_B = A_s\left(\rho_s - \rho_o\right) = A_s \rho_s \left(1 - \frac{\rho_o}{\rho_s}\right) = w_s \left(1 - \frac{\rho_o}{\rho_s}\right), \tag{1.19}$$

where $1 - \left(\rho_o / \rho_s\right)$ is the buoyancy factor, and where the following values are as follows:

- $D_o$ = outside diameter of component body
- $D_{oj}$ = outside diameter of tool joint
- $D_i$ = inside diameter of component body
- $D_{ij}$ = inside diameter of tool joint
- $A_s$ = cross-sectional area of the steel/material
- $\rho_o$ = annular mud weight at component depth in the wellbore
- $\rho_i$ = internal mud weight at component depth inside the component
- $\rho_s$ = density of the steel/material

**Problem 1.2**

Calculate the buoyancy factor and buoyed weight of 6,000 ft of 6 5/8″ 27.7 ppf E grade drill pipe in mud of density 10 ppg.

*Solution:*

Using equation 1.13 and a steel density of 65.4 ppg,

$$\text{Buoyancy factor} = \left(1 - \frac{\rho_\text{m}}{\rho_\text{s}}\right) = \left(1 - \frac{10}{65.4}\right) = 0.847.$$

Buoyed weight can be calculated using equation 1.11:

$$\begin{aligned}\text{Bouyed weight} &= 0.847 \times 27.7 \times 6000 \\ &= 140771.4 \text{ lbf} = 140.8 \text{ kips.}\end{aligned}$$

**Problem 1.3**

Calculate the buoyed weight of 5,000 ft of 20″ 106.5 ppf casing with drilling mud of density 9 ppg inside and 11 ppg cement outside the casing. Also, estimate the buoyed weight of the casing with the same drilling fluid inside and outside before pumping cement. Neglect the tool joint effects.

*Solution:*

After pumping cement with full cement behind the casing, the inside diameter of the casing is 18.98 in.

Using equation 1.14 and a steel density of 65.4 ppg,

$$\text{BF} = \frac{A_o\left(1 - \frac{\rho_\text{o}}{\rho_\text{s}}\right) - A_i\left(1 - \frac{\rho_\text{i}}{\rho_\text{s}}\right)}{A_o - A_i}$$

$$= \frac{0.7854 \times 20^2\left(1 - \frac{11}{65.4}\right) - 0.7854 \times 19^2\left(1 - \frac{9}{65.4}\right)}{0.7854 \times 20^2 - 0.7854 \times 18.98^2} = 0.5382.$$

Buoyed weight can be calculated using equation 1.11:

$$\text{Buoyed weight} = 0.5382 \times 106.5 \times 5000$$
$$= 286618 \text{ lbf} = 287 \text{ kips.}$$

Before pumping cement, the buoyed weight can be estimated using equation 1.13 and a steel density of 65.4 ppg:

$$\text{Buoyancy factor} = \left(1 - \frac{\rho_m}{\rho_s}\right) = \left(1 - \frac{9}{65.4}\right) = 0.8623.$$

Buoyed weight can be calculated using equation 1.11:

$$\text{Buoyed weight} = 0.8623 \times 106.5 \times 5000$$
$$= 459220.2 \text{ lbf} = 459.2 \text{ kips.}$$

**Problem 1.4**

Calculate the air weight, buoyed weight in drilling fluid, buoyed weight when cement is inside and drilling fluid is in the annulus, buoyed weight when cement is outside and drilling fluid is inside. Casing outside diameter is 9 5/8″, casing inside diameter is 8.681″, drilling fluid density is 10 ppg, cement slurry density is 12 ppg, and the depth of the well is 5,000 ft.

*Solution:*

$$\text{Air weight} = 47 \times 5000 = 235000 \text{ lbf} = 235 \text{ kips.}$$

$$\text{Buoyed weight with drilling fluid} = \left(1 - \frac{10}{65.4}\right) \times 5000 \times 47$$
$$= 199067 \text{ lbf} = 199 \text{ kips.}$$

Buoyed weight with cement inside and drilling fluid outside is

$$\mathrm{BF} = \frac{A_o\left(1-\frac{\rho_o}{\rho_s}\right) - A_i\left(1-\frac{\rho_i}{\rho_s}\right)}{A_o - A_i}$$

$$= \frac{0.7854\times 9.625^2\left(1-\frac{10}{65.4}\right) - 0.7854\times 8.681^2\left(1-\frac{12}{65.4}\right)}{0.7854\times 9.625^2 - 0.7854\times 8.681^2}$$

$$= 0.98,$$

$$= 0.98\times 5000\times 47 = 230406 \text{ lbf} = 230 \text{ kips.}$$

Buoyed weight with cement outside and drilling fluid inside is

$$\mathrm{BF} = \frac{A_o\left(1-\frac{\rho_o}{\rho_s}\right) - A_i\left(1-\frac{\rho_i}{\rho_s}\right)}{A_o - A_i}$$

$$= \frac{0.7854\times 9.625^2\left(1-\frac{12}{65.4}\right) - 0.7854\times 8.681^2\left(1-\frac{10}{65.4}\right)}{0.7854\times 9.625^2 - 0.7854\times 8.681^2}$$

$$= 0.6831,$$

$$= 0.6831\times 5000\times 47 = 160541 \text{ lbf} = 160 \text{ kips.}$$

## 1.5 Modulus of Elasticity

Modulus of elasticity is

$$E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L}\text{psi,} \tag{1.20}$$

where $\sigma$ = unit stress, psi, $\varepsilon$ = unit strain in inch per inch, $F$ = axial force, lbf, $A$ = cross sectional area, in$^2$, $\Delta L$ = total strain or elongation, in., and $L$ = original length, in.

## 1.6 Poisson's Ratio

$$v = \frac{\varepsilon_{lat}}{\varepsilon_{long}}, \tag{1.21}$$

where $\varepsilon_{lat}$ = lateral strain in inches, and $\varepsilon_{long}$ = longitudinal or axial strain in inches.

For most metals, Poisson's ratio varies from $\frac{1}{4} - \frac{1}{3}$.

Modulus of elasticity and shear modulus are related to Poisson's ratio as follows:

$$E = 2G(1+v). \tag{1.22}$$

Modulus of elasticity, shear modulus, and Poisson's ratio for common materials are given in Table 1.1.

## 1.7 Minimum Yield Strength

Yield strength is defined as the stress that will result in specific permanent deformation in the material. The yield strength can be conveniently determined from the stress strain diagram. Based on the test results, minimum and maximum yield strengths for the tubulars are specified.

**Table 1.1** Modulus of elasticity, shear modulus and Poisson's ratio at room temperature.

| Metal Alloy | Modulus of Elasticity | | Shear Modulus | | Poisson's Ratio |
|---|---|---|---|---|---|
| | Psi × $10^6$ | MPa × $10^6$ | Psi × $10^6$ | MPa × $10^6$ | |
| Aluminum | 10 | 6.9 | 3.8 | 2.6 | 0.33 |
| Copper | 16 | 11 | 6.7 | 4.6 | 0.35 |
| Steel | 30 | 20.7 | 12 | 8.3 | 0.27 |
| Titanium | 15.5 | 10.7 | 6.5 | 4.5 | 0.36 |
| Tungsten | 59 | 40.7 | 23.2 | 16 | 0.28 |

## 1.8 Ultimate Tensile Strength

The ultimate tensile strength (UTS) of a material in tension, compression, or shear, respectively, is the maximum tensile, compressive, or shear stress resistance to fracture or rupture. It is equivalent to the maximum load that can be applied over the cross-sectional area on which the load is applied. The term can be modified as the ultimate tensile, compressive, or shearing strength. Ultimate tensile strength of few API pipes are shown in Table 1.2.

## 1.9 Fatigue Endurance Limit

The endurance limit pertains to the property of a material and is defined as the highest stress or range of cyclic stress that a material can be subjected to indefinitely without causing failure or fracture. In other words, the endurance limit is the maximum stress reversal that can be indefinitely subjected a large number of times without producing fracture. The magnitude of the endurance limit of a material is usually determined from a fatigue test that uses a sample piece of the material.

**Table 1.2** API pipe properties.

| API | Yield Stress, psi | | Minimum Ultimate | Minimum |
|---|---|---|---|---|
| **Grade** | **Minimum** | **Maximum** | **Tensile, psi** | **Elongation (%)** |
| H-40 | 40,000 | 80,000 | 60,000 | 29.5 |
| J-55 | 55,000 | 80,000 | 75,000 | 24 |
| K-55 | 55,000 | 80,000 | 95,000 | 19.5 |
| N-80 | 80,000 | 110,000 | 100,000 | 18.5 |
| L-80 | 80,000 | 95,000 | 95,000 | 19.5 |
| C-90 | 90,000 | 105,000 | 100,000 | 18.5 |
| C-95 | 95,000 | 110,000 | 105,000 | 18.5 |
| T-95 | 95,000 | 110,000 | 105,000 | 18 |
| P-110 | 110,000 | 140,000 | 125,000 | 15 |
| Q-125 | 125,000 | 150,000 | 135,000 | 18 |

## 1.10 Twist

When a rod is subjected to torque it undergoes twist, which is given as

$$\theta = \frac{TL}{GJ} \text{ radians,} \tag{1.23}$$

where $\theta$ = angle of twist (radians) (can be $>2\pi$), $L$ = length of section, ft, $T$ = torque, ft-lbf, and $G$ = modulus of rigidity, psi.

$$G = \frac{E}{2(1+\nu)}, \tag{1.24}$$

$$J = \text{Polar moment of inertia (in.}^4) = \frac{\pi}{32}\left(D_o^4 - D_i^4\right), \tag{1.25}$$

and $E$ = modulus of elasticity, psi, and $\nu$ = Poisson's ratio.

**Problem 1.5**

Consider a pipe with the following dimensions carrying an applied tensile load of 5,000 lbs at the bottom. Calculate the maximum stress in the string. The pipe outside diameter = 5 in, the pipe inside diameter = 4 in, the pipe density = 490 lb/ft$^3$, and the pipe length = 30 ft.

*Solution:*

The cross sectional area of the pipe is

$$A = \frac{\pi}{4}\left(5^2 - 4^2\right) = 7.08 \text{ in}^2.$$

$$\text{Weight of the pipe} = \frac{\pi}{4}\left(5^2 - 4^2\right) \times 490 \times 30 \times 12 = 721.6 \text{ lbf}.$$

Total force acting at the top of the pipe:

$F$ = Weight of the pipe + Load applied,
$F = 721.6 + 5000 = 5721.6$ lbf.

$$\text{Maximum stress at the top of the pipe} = \sigma = \frac{F}{A} = \frac{5721.6}{7.08}$$
$$= 809 \text{ psi.}$$

**Problem 1.6**

Calculate the elongation of a cylindrical pipe of 5″ in outside diameter, 4.0 in inside diameter and 10,000 ft long when a tensile load of 20,000 lbf is applied. Assume that the deformation is totally elastic and modulus of elasticity = $30 \times 10^6$ psi.

*Solution:*

From equation $E = \dfrac{F/A}{\Delta L/L}$, the elongation can be written as follows:

$$\Delta L = \frac{F/A}{E/L} = \frac{L \times F}{E \times A} = \frac{L \times F}{E \times \frac{\pi}{4}\left(D_o^2 - D_i^2\right)} = \frac{4L \times F}{E \times \pi \times \left(D_o^2 - D_i^2\right)}.$$

Substituting the values,

$$= \frac{4 \times 10000 \times 12 \times 20000}{30 \times 10^6 \times \pi \times (5^2 - 4^2)} = 11.32 \text{ in.}$$

**Problem 1.7**

A downhole tool with a length of 30 ft, an outside diameter of 5.5 in., and an inside diameter of 4.75 in. is compressed by an axial force of 30 kips. The material has a modulus of elasticity 30,000 ksi and Poisson's ratio 0.3. Assume the tool is in the elastic range.

Calculate the following:

A. Shortening of tool
B. Lateral strain

C. Increase in outer diameter
D. Increase in inner diameter
E. Increase in wall thickness

*Solution:*

*A. Shortening of tool*

Using Hook's law,

$$\Delta L = \frac{\frac{F}{A}}{\frac{E}{L}} = \frac{L\times F}{E\times A} = \frac{30\times 12\times(-30000)}{30\times 10^6\times\frac{\pi}{4}\left(5.5^2-4.75^2\right)} = -0.05962 \text{ in.}$$

Negative sign shows shortening of the tool.

*B. Lateral strain*

Lateral strain can be obtained using the axial strain and Poisson's ratio:

$$\text{Axial strain} = \frac{\Delta L}{L} = \frac{-0.05962}{360} = -0.0001656,$$

$$\text{Lateral strain} = \nu\times\frac{\Delta L}{L} = \frac{0.05962}{360} = 0.3\times 0.0001656$$
$$= 4.968\times 10^{-5}.$$

*C. Increase in outer diameter*

Increase in outer diameter is the lateral strain times the outer diameter:

$$4.968\times 10^{-5}\times 5.5 = 0.000273 \text{ in.}$$

*D. Increase in inner diameter*

Increase in inner diameter is the lateral strain times the inner diameter:

$$4.968\times 10^{-5}\times 4.75 = 0.000236 \text{ in.}$$

*E. Increase in wall thickness*

Increase in wall thickness can be estimated in a similar way as diameters:

$$4.968 \times 10^{-5} \times \left( \frac{5.5 - 4.75}{2} \right) = 1.8632 \times 10^{-5} \text{ in.}$$

**Problem 1.8**

Calculate the angle of twist of a cylindrical pipe of 5″ in outside diameter, 4.0 in inside diameter, and 10,000 ft long when a torque of 3,000 ft-lbf is applied at the bottom of the pipe. Assume that the twisting is totally elastic and modulus of rigidity = 12 × 10⁶ psi.

*Solution:*

Calculating the polar moment of inertial of the pipe, using equation 1.25,

$$J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right) = \frac{\pi}{32}\left(5^4 - 4^4\right) = 36.23 \text{ in}^4.$$

Using equation 1.23 and appropriate units,

$$\theta = \frac{TL}{GJ} = \frac{3000 \times 10000 \times 12 \times 12}{12 \times 10^6 \times 36.23} = 9.9374 \text{ rad}$$

$$= 9.9374 \times \frac{180}{\pi} = 569.38 \text{ deg.}$$

## 1.11 Composite Materials

For longitudinal directional ply and longitudinal tension, modulus can be given as

$$E = V_m E_m + V_f E_f, \tag{1.26}$$

where $E_m$ = elastic modulus of base pipe, psi, $E_f$ = elastic modulus of rubber attachment, $V_m$ = volume fraction of matrix, and $V_f$ = volume fraction of fiber attachment.

Also, $V_m + V_f = 1$, the same way Poisson's ratio can be calculated,

$$v = V_m v_m + V_f v_f. \tag{1.27}$$

When the stress is applied perpendicular to the fiber orientation, the modulus of the elasticity of the composite material can be given as

$$\frac{1}{E} = \frac{V_m}{E_m} + \frac{V_f}{E_f}. \tag{1.28}$$

**Problem 1.9**

Estimate the modulus of the composite shaft with 25 % of the total volume with fibers. Assume the modulus of elasticity for the fiber is $50 \times 10^6$ psi and the matrix 600 psi and the load is applied longitudinally as well as perpendicular to the fibers.

*Solution:*

When the load is applied longitudinally to the fibers,

$$E = V_m E_m + V_f E_f = 500 \times 0.75 + 25000000 \times 0.25 = 6250450 \text{ psi.}$$

When the load is applied perpendicular to the fibers,

$$\frac{1}{E} = \frac{V_m}{E_m} + \frac{V_f}{E_f} = \frac{.25}{500} + \frac{0.75}{25000000} = 0.00125.$$

Thus, $E = 800$ psi.

## 1.12 Friction

### 1.12.1 Coefficient of Friction

The coefficient of friction (COF) is defined as the ratio of the frictional force to the normal force acting at the point of contact. It is given as

$$\mu = \frac{F_f}{F_n}, \tag{1.29}$$

where $F_f$ = friction force, lbf, and $F_n$ = normal force, lbf.

The COF is a scalar dimensionless value that depends on the surface but is independent of the surface area. Table 1.3 shows typical COFs for various materials whereas Table 1.4 shows the friction factors when different fluids used.

### 1.12.2 Types of Friction

Static friction is

$$\mu_s = \frac{F_{sf}}{F_n}. \tag{1.30}$$

Kinetic friction is

$$\mu_k = \frac{F_{kf}}{F_n}. \tag{1.31}$$

A typical static and kinetic coefficient plot is shown in Figure 1.1.
Rolling friction is

$$\mu_r = \frac{F_{rf}}{F_n}. \tag{1.32}$$

Angle of friction is

$$\varphi = \tan^{-1} \mu_s. \tag{1.33}$$

Slide/roll friction is

$$\varphi = \tan^{-1} \mu_s.$$

The kinetic friction and the friction angle, by equation 1.32 are related as follows:

$$\mu_k = \tan\varphi - \frac{a_x}{g \sin\varphi}. \tag{1.34}$$

where $a_x$ = acceleration, $g$ = acceleration constant

**Table 1.3** Typical coefficient of Friction (Rabbat, 1985).

| Material 1 | Material 2 | Dry | | Lubricated | |
|---|---|---|---|---|---|
| | | Static | Sliding | Static | Sliding |
| Steel | Steel | 0.78 | 0.42 | 0.05–0.11 | 0.29–0.12 |
| Aluminium | Aluminum | 1.05–1.35 | 1.4 | 0.3 | |
| Aluminum | Mild steel | 0.61 | 0.47 | | |
| Copper–Lead | Steel | 0.22 | | | |
| Diamond | Diamond | 0.1 | | | |
| Diamond | Metal | 0.15 | | | |
| Steel | Concrete | 0.57–0.75 | 0.45 | | |
| Steel | EmbeddedSand | | 0.7 | | |

**Table 1.4** Range of friction factors.

| Fluid Type | Friction Factors | |
|---|---|---|
| | Cased Hole | Open Hole |
| Oil-based | 0.16–0.20 | 0.17–0.25 |
| Water-based | 0.25–0.35 | 0.25–0.40 |
| Brine | 0.30–0.4 | 0.3–0.4 |
| Polymer-based | 0.15–0.22 | 0.2–0.3 |
| Synthetic-based | 0.12–0.18 | 0.15–0.25 |
| Foam | 0.30–0.4 | 0.35–0.55 |
| Air | 0.35–0.55 | 0.40–0.60 |

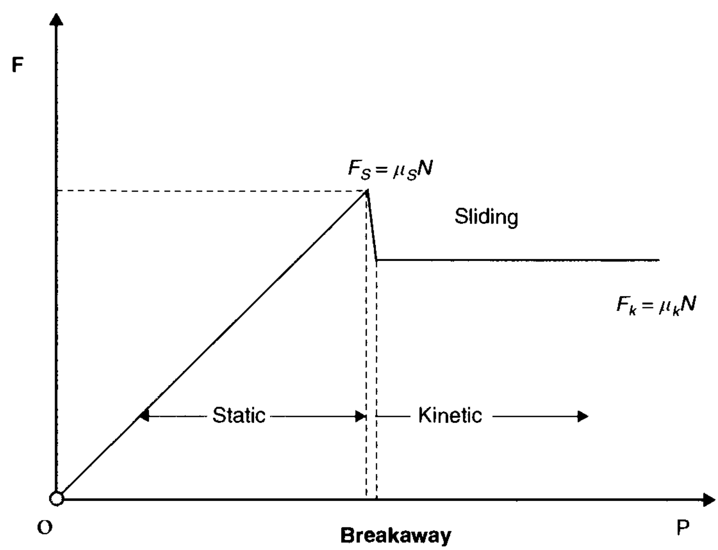



**Figure 1.1** Friction force as a function of pulling force.

### 1.12.3 Friction and Rotational Speed

The following empirical equation provides a good representation and coupling of the friction effects and drill string rotating speed as well as tripping speed:

$$\mu_v = \mu_s \times e^{-k|V_{rs}|}. \tag{1.35}$$

The resultant velocity, $V_{rs}$, of a contact point on the drill string is the vector sum of two components: circumferential velocity $V_C$ (caused by rotation) and axial velocity $V_{ts}$ (affected by drilling rate or tripping speed).

The friction factor, which has the dependency on the side force, kinematics, temperature, and geometrical parameters of the contacting surfaces, is given by

$$\mu_v = \frac{\mu_s}{1+\left(\frac{\mu_s \sigma_n}{k\Delta t}\right)|V_{rs}|}, \tag{1.36}$$

where $\sigma_n$ = normal stress at the contact, $\Delta t$ = average contact temperature, $|V_{ts}|$ = trip speed, $|V_{rs}|$ = resultant speed = $\sqrt{\left(V_{ts}^2 + \omega^2\right)}$, $|\omega|$ = angular speed = diameter $\times \pi \times (N/60)$, and $N$ = pipe rotational speed, rpm.

## 1.13 Gauge and Absolute Pressures

Pressure, expressed as the difference between the fluid pressure and that of the surrounding atmosphere, is relative to ambient or atmosphere.

Absolute pressure of a fluid is expressed relative to that of a vacuum and is given as

$$P_{abs} = P_{atm/ambient} + P_{gauge}. \tag{1.37}$$

The standard value of atmospheric pressure = 14.696 psi = 101.3 kPa = 29.92 inHg = 760 mmHg.

The relationship between gauge and absolute pressures is shown in Figure 1.2.

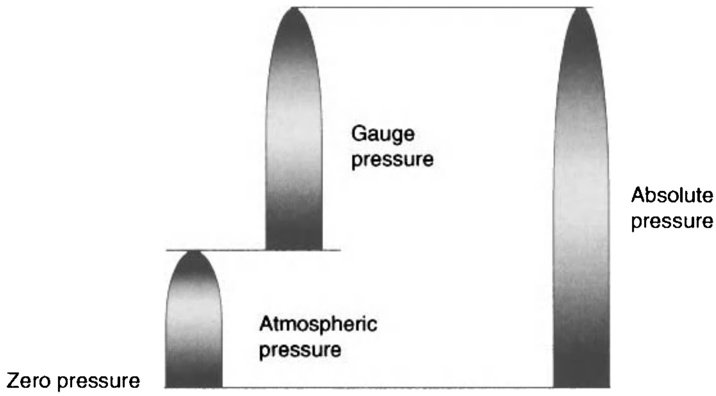



**Figure 1.2** Absolute-gauge-atmospheric pressures.

## 1.13.1 Hydrostatic Pressure

The hydrostatic pressure at a depth, $D$, column of mud having a density of $\gamma_m$ can be easily derived and is given by

$$P_h = k\rho_m D_v, \tag{1.38}$$

where $D_v$ = vertical depth of mud column, $\rho_m$ = mud weight, and $k$ = conversion unit factor.

If $P_h$ is in psi, $\rho_m$ is in ppg, and $D$ is in feet, then the value of $k = 0.052$.

## 1.13.2 Mud Gradient

Pressure inside the wellbore is expressed in terms of gradient and is expressed in psi/ft of depth. In oil field units,

$$\rho_g = 0.052 \times \rho_m \text{ psi/ft}, \tag{1.39}$$

where $\rho_m$ = mud weight in ppg.

$$0.052 = \frac{748 \text{ gal}}{144 \text{ in}^2}. \tag{1.40}$$

Hydrostatic pressure, $P_h$, at any measured depth in the wellbore can be calculated as

$$P_h = \rho_g \times D_v, \tag{1.41}$$

where $D_v$ is the corresponding true vertical depth at the measure depth in ft.

**Problem 1.10**

What would be the static mud density required to balance a formation pressure of 6,000 psi at a depth of 8,000 ft (vertical)?

*Solution:*

$$\rho_m = \frac{P_h}{0.052 \times D_v}$$

Substituting the appropriate values,

$$\rho_m = \frac{6000}{0.052 \times 8000} = 14.42 \text{ ppg}.$$

**Problem 1.11**

Convert 10 ppg to various units and vice versa.

*Solution:*

Converting 10 ppg to psi/ft = 0.0519 × 10 = 0.519 or 0.52 psi/ft.
Converting 10 ppg to psf/ft = 7.48 × 10 = 74.8 psf/ft.
Converting ppg to specific gravity (SG) = 8.3472 × 10 = 1.198.

Converting 0.519 psi/ft to mud weight lb/ft$^3$ = $\dfrac{0.519}{0.006944}$

$= 74.8$ lb/ft$^3$.

Converting 0.519 psi/ft to mud weight in SG = $\dfrac{0.519}{0.433} = 1.198.$

Converting 0.519 psi/ft to mud weight in ppg $= \dfrac{0.519}{0.0519} = 10$ ppg.

### 1.13.3 Measurement of Pressure

The pressure can be measured in the U-tube by applying the static fluid equations to both legs of the manometer. For example, for the manometer with different fluids it can be written as

$$P_2 = P_1 + 0.052 \times \rho_1 \times (h_2 + h_3),$$

$$P_3 = P_5 + 0.052 \times \rho_2 \times h_2 + 0.052 \times \rho_1 \times h_3.$$

Since $P_3 = P_2$,

$$P_5 = P_1 + 0.052 \times h_2 (\rho_1 - \rho_2). \tag{1.42}$$

**Problem 1.12**

Find the pressure, $P_6$, shown in Figure 1.3.

*Solution:*

Applying pressure balance in both the legs of the manometer at the points 2 and 3,

$$P_2 = P_1 + 0.052 \times \rho_1 \times h_1,$$

$$P_3 = P_6 + 0.052 \times \rho_1 \times (h_1 - h_2 - h_3) + 0.052 \times \rho_3 \times h_3 \\ + 0.052 \times \rho_2 \times h_2.$$

Since $P_3 = P_2$,

$$P_6 = P_1 + 0.052 \times h_2 (\rho_1 - \rho_2) + 0.052 \times h_3 (\rho_1 - \rho_3).$$

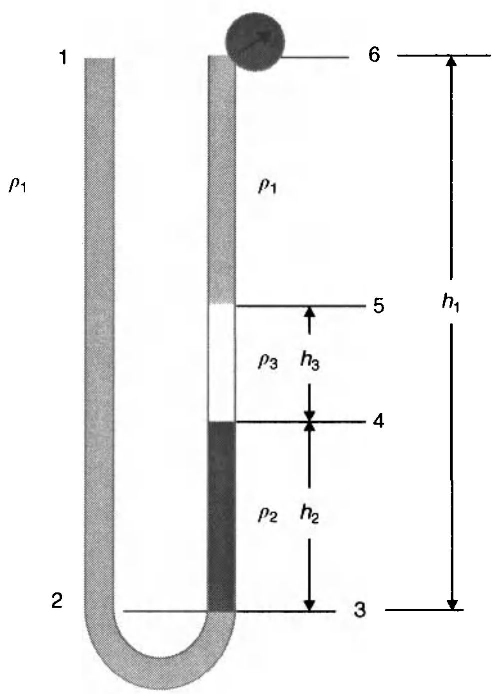



**Figure 1.3** Manometer – Problem 1.12.

## 1.14 Temperature

Geothermal gradient is given in deg F/100 ft. For normal wells, the gradient usually is 1.5°F/100 ft. The conversions used are as follows:

$$T\ ^\circ F = \frac{9}{5}(T\ ^\circ C) + 32, \tag{1.43}$$

$$T\ ^\circ C = \frac{5}{9}(T\ ^\circ F - 32). \tag{1.44}$$

Other conversions to Kelvin and Rankine are as follows:

$$T\ ^\circ K = T\ ^\circ C + 273.15, \tag{1.45}$$

$$T\ ^\circ K = \frac{5}{9}(T\ ^\circ F + 459.67), \tag{1.46}$$

$$T\ ^\circ K = \frac{5}{9} \times {}^\circ R, \tag{1.47}$$

$$T\ ^\circ R = (T\ ^\circ F + 459.67). \tag{1.48}$$

For offshore wells, the temperature profile inside the seawater is different depending on the depth, place, and time of the year. Usually below the themocline the temperature remains fairly constant, and the profile is similar to Figure 1.4. Approximately below the depth of 5,000 ft, the temperature of the seawater reaches close

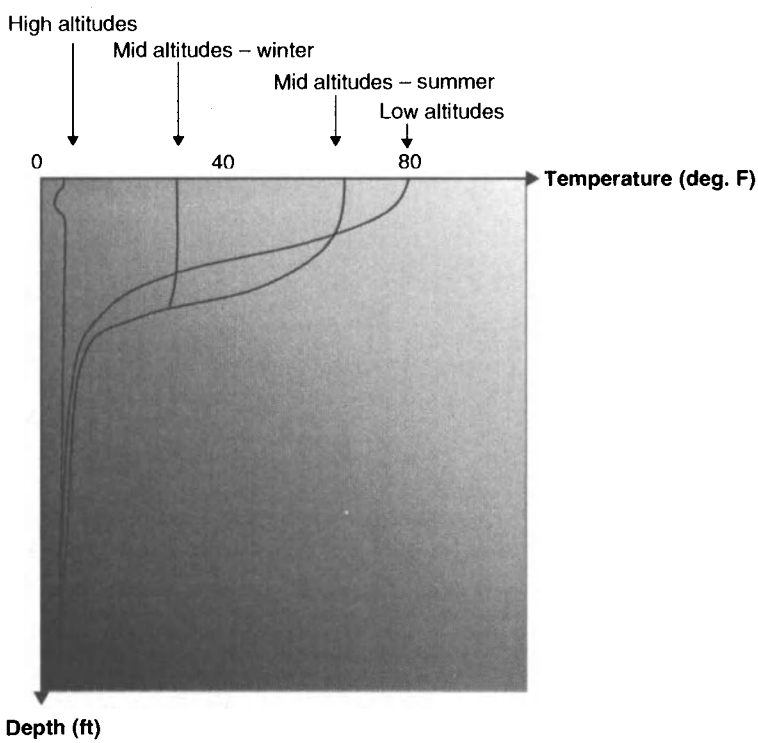



**Figure 1.4** Seawater temperature profile.

to 40~45 deg F and drops slowly as the depth increases. Deeper depths' temperature varies between 34° and 40° F.

**Problem 1.13**

Convert 80°F to Centigrade, Kelvin, and Rankine.

*Solution:*

Fahrenheit to Centigrade:

$$T\ °C = \frac{5}{9}(T\ °F - 32) = \frac{5}{9}(80 - 32) = 26.67\ °C.$$

Fahrenheit to Kelvin:

$$T\ °K = \frac{5}{9}(T\ °F + 459.67) = \frac{5}{9}(80 + 459.67) = 299.82\ °K.$$

Fahrenheit to Rankine:

$$T\ °R = (T\ °F + 459.67) = (80 + 459.67) = 539.7\ °R.$$

## 1.15 Horsepower

Mechanical horsepower is given as

$$HP = \frac{\text{force (lbf)} \times \text{ distance (ft)}}{550 \times \text{time (sec)}}. \tag{1.49}$$

It is also given as

$$HP = \frac{\text{force (lbf)} \times \text{velocity (ft/min)}}{33000}. \tag{1.50}$$

Hydraulic horsepower is given as

$$HHP = \frac{\text{flow rate (gpm)} \times \text{pressure (psi)}}{1714}. \tag{1.51}$$

Rotating horsepower is given as

$$\text{HP} = \frac{\text{torque (ft-lbf)} \times \text{speed (rpm)}}{5252} \tag{1.52}$$

One horsepower = 550 foot-pounds/second = 33,000 ft-lbf/minute, and 33,000 ft-lbf/6.2832 ft-lbf = 5252.

Other conversion factors are 1 horsepower = 0.0007457 megawatts = 0.7457 kilowatts = 745.7 watts.

**Problem 1.14**

The torque on a motor shaft follows a sinusoidal pattern with a maximum amplitude of 8,000 ft-lbf. The torque will always be positive through the cycle. Shaft speed is 200 rpm. Calculate the motor horsepower.

*Solution:*

Horsepower is given as

$$\text{HP} = \frac{2\pi NT}{3300}.$$

The torque is calculated using the sinusoidal pattern:

$$T = 2\int_0^{\pi} 8000 \sin x dx,$$

$$T = 2[8000 \cos x]_0^{\pi}.$$

Substituting the limits,

$$T = 16000[\cos \pi - \cos 0]_0^{\pi} = 32000 \text{ ft-lbf},$$

$$\text{HP} = \frac{2\pi \times 200 \times 32000}{33000} = 1219 \text{hp}.$$

## 1.16 Flow Velocity

Velocity is

$$V = \frac{\text{Flow rate}}{\text{Cross sectional area}} = \frac{Q}{A}. \tag{1.53}$$

When the flow rate is in gallons per minute and the cross sectional area is in sq.in,

$$V = \frac{19.25 \times Q}{A} \text{ feet/min.} \tag{1.54}$$

When the flow rate is in barrels per minute and the cross sectional area is in sq.in,

$$V = \frac{808.5 \times Q}{A} \text{ feet/min.} \tag{1.55}$$

**Problem 1.15**

Calculate the fluid velocity inside the pipe as well as in the annulus with the dimensions as follows for a flow rate of 350 gpm (4.762 bpm):

- Pipe inside diameter = 3 in
- Pipe outside diameter = 4.5 in
- Hole diameter = 8.5 in

*Solution:*

The velocity inside pipe using flow rate in gpm is

$$V_p = \frac{\text{Flow rate}}{\text{Cross sectional area}} = \frac{19.25 \times 200}{\frac{\pi}{4}(3^2)} = 544.7 \text{ fpm.}$$

The velocity in the annulus using flow rate in gpm is

$$V_a = \frac{19.25 \times 200}{\frac{\pi}{4}\left(8.5^2 - 4.5^2\right)} = 94.3 \text{ fpm.}$$

The velocity inside pipe using flow rate in bpm is

$$V_p = \frac{808.5 \times 4.762}{\frac{\pi}{4}\left(3^2\right)} = 544.7 \text{ fpm.}$$

The velocity in the annulus using flow rate in bpm is

$$V_a = \frac{808.5 \times 4.762}{\frac{\pi}{4}\left(8.5^2 - 4.5^2\right)} = 94.3 \text{ fpm.}$$

*Formulas and Calculations for Drilling Operations*
by G. Robello Samuel


# 2

# Rig Equipment

This chapter focuses on the different basic calculations involved in rig equipment and associated calculations as listed below:

- Rig capacity
- Engine calculations
- Mud pump sizing
- Pump output
- Rotary torque, HP
- Block line work and capacity
- Offshore environmental forces

## 2.1 Overall Efficiency of Engines

The overall efficiency of power generating systems may be defined as

$$\text{Efficiency (\%)} = \frac{\text{output power}}{\text{input power}} \times 100,$$

or

$$\eta_o = 100\frac{P_o}{P_i}. \tag{2.1}$$

The output power of an engine is

$$P_o = \frac{2\pi NT}{33000}, \tag{2.2}$$

where $T$ = output torque in ft-lbs, $N$ = engine rotary speed in revolution per minute (rpm), and $P_o$ = output power in horsepower, hp.

The input power is expressed as

$$P_i = \frac{Q_f H}{2545}, \tag{2.3}$$

where $Q_f$ = rate of fuel consumption in lbm/hr, $H$ = fuel heating value in BTU/lb, and $P_i$ = input power in horsepower, hp.

Fuel consumption can be given as

$$Q_f = 48.46\frac{NT}{\eta H}\text{ lb/hr}. \tag{2.4}$$

## 2.2 Energy Transfer

Efficiency transfer from the diesel engines to the hoisting system can be given as in Figure 2.1. Due to interrelated equipment, various efficiencies can be used:

- Engine Efficiency, $\eta_e$
- Electric motor Efficiency, $\eta_{el}$
- Drawworks Mechanical Efficiency, $\eta_m$
- Hoisting Efficiency, $\eta_h$
- Overall Efficiency $\eta_o = \eta_e \times \eta_{el} \times \eta_m \times \eta_h$

**Problem 2.1**

A diesel engine's output torque is estimated to be 1,870 ft-lbf at 1,100 rpm. Determine the output power and efficiency of the engine

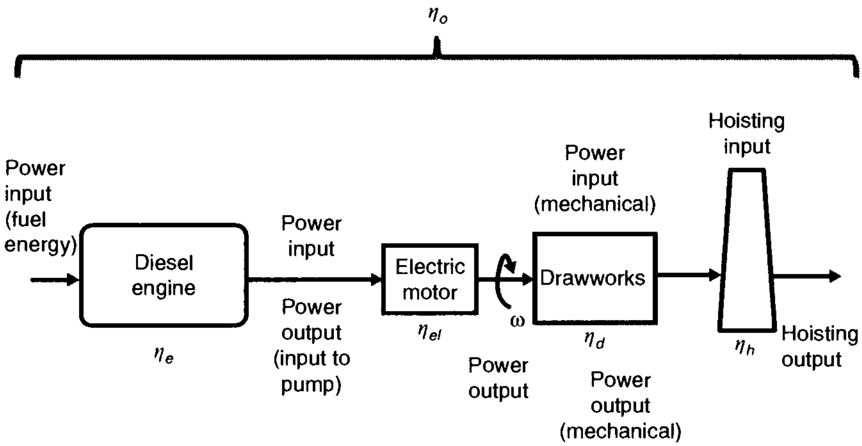



**Figure 2.1** Energy transfer.

if the fuel consumption rate is 30 gal/hr. The heating value of diesel oil is 1,9000 BTU/lbm.

*Solution:*

Fuel consumption = 30 (gal/hr) × 7.2 (lbm/gal) = 216 lbm/hr.

Using equation 2.2, the output power of an engine is

$$P_o = \frac{2\pi NT}{33000} = \frac{2\pi \times 1100 \times 1870}{33000} = 391.7\,\text{hp}.$$

Using equation 2.3, the input power is

$$P_i = \frac{Q_f H}{2545} = \frac{216 \times 19000}{2545} = 1612.6\,\text{hp}.$$

Using equation 2.1, efficiency is

$$\eta = 100\frac{P_o}{P_i} = \frac{391.7}{1612.6} \times 100 = 24.3\%.$$

**Problem 2.2**

A drilling rig has three diesel engines for generating the rig power requirement. Determine the total daily fuel consumption for an average engine running speed of 900 rpm, an average output torque of 1,610 ft-lb, and an engine efficiency of 40%. The heating value of diesel oil is 19,000 BTU/lbm.

*Solution:*

Given: $N = 900$ rpm, $\eta = 40\%$, $H = 19000$ BTU/lb, and $T = 1610$ ft-lbs
Using equation 2.4,

$$Q_f = 48.5\left(\frac{1610 \times 900}{40 \times 19000}\right) = 92.5\,\text{lb/hr},$$

or,

$$Q_f = \frac{92.5\,\text{lb}}{\text{hr}} \times \frac{24\,\text{hr}}{\text{day}} \times \frac{\text{gal}}{7.2\,\text{lb}} = 308\,\text{gal/day}.$$

For the three engines, $Q_f$ (total) = 308 × 3 = 924 gal/day.

**Problem 2.3**

Determine the fuel cost (dollars/day) to run an engine at 1,800 rpm with 3,000 ft-lb of output torque. The engine efficiency at the above rotary speed is 30%. The cost of diesel oil is $1.05/gal, weight is 7.14 lb/gal, and the heating value is 19,000 BTU/lb.

*Solution:*

Using equation 2.2, the output power for an engine is given as

$$P_o = \frac{2\pi NT}{33000} = \frac{2\pi \times 1800 \times 3000}{33000} = 1028.16\,\text{hp}.$$

Using equation 2.3, the input power is expressed in terms of the rate of fuel consumption, $Q_f$, and the fuel heating value, $H$:

$$P_i = \frac{Q_f H}{2545} = \frac{Q_f \times 19{,}000}{2545}\,\text{hp}.$$

Hence,

$$Q_f = \left(\frac{1028.16 \times 2545}{.30 \times 19000}\right) = 460\,\text{lb/hr},$$

or

$$Q_f = \frac{460\,\text{lb}}{\text{hr}} \times \frac{24\,\text{hr}}{\text{day}} \times \frac{\text{gal}}{7.2\,\text{lb}} = 1533\,\text{gal/day}.$$

Cost = 1533 × 1.05 = \$1610/day.

**Problem 2.4**

Engine speed is 1,500 rpm, the engine torque is 1800 ft-lbf, and the fuel consumption is 30 gal/hr.

Calculate the following:

A. Output power of the engine
B. Input power
C. Overall efficiency of the engine

*Solution:*

Using equation 2.2, the output power developed by the engine is

$$P_o = \frac{2\pi \times 1500 \times 1800}{33000} = 514\,\text{hp}.$$

Using equation 2.3, the input power is

$$P_i = \frac{216 \times 19000 \times 779}{33000 \times 60} = 1615\,\text{hp}.$$

Using equation 2.1, the overall efficiency of the engine is

$$\eta_o = \frac{514}{1615} \times 100 = 31.83\%.$$

## 2.3 Blocks and Drilling Line

The efficiency of block and tackle system is measured by

$$\eta = \frac{\text{power output}}{\text{power input}} = \frac{P_o}{P_i} = \frac{F_h v_{tb}}{F_f v_f}. \tag{2.5}$$

The output power is defined as

$$P_o = F_h v_{tb}, \tag{2.6}$$

where $F_h$ = load hoisted in pounds (buoyed weight of the string + travelling block, compensator, etc.), and $v_{tb}$ = traveling block velocity.

Input power from the drawworks to the fast line is given by

$$P_i = F_f v_f, \tag{2.7}$$

where $F_f$ is the load in fast line, and $v_f$ is the fast line speed.

The relationship between the travelling block speed and the fast line speed is

$$v_{tb} = \frac{v_f}{n}. \tag{2.8}$$

Therefore,

$$\eta = \frac{F_h}{nF_f}, \tag{2.9}$$

where $n$ is the number of lines strung between the crown block and traveling block.

## 2.4 Derrick Load

Static derrick load is calculated as

$$F_s = \frac{n+2}{n} \times F_h. \tag{2.10}$$

Dynamic fast line load is calculated as

$$F_f = \frac{F_h}{En}. \tag{2.11}$$

Dynamic derrick load is given by

$$F_f + F_h + F_{dl}. \tag{2.12}$$

Derrick load is given as

$$F_d = \left(\frac{1+E+En}{En}\right) \times F_h. \tag{2.13}$$

Maximum equivalent derrick load is given as

$$F_{de} = \left(\frac{n+4}{n}\right) \times F_h. \tag{2.14}$$

Derrick efficiency factor is calculated as

$$E_d = \left(\frac{F_d}{F_{de}}\right) = \frac{E(n+1)+1}{E(n+4)}, \tag{2.15}$$

where $F_{dl}$ = the dead line load.

### 2.4.1 Block Efficiency Factor

Overall block efficiency factor is given as

$$E = \frac{(\mu^n - 1)}{\mu^s n(\mu - 1)}, \tag{2.16}$$

where $\mu$ = the friction factor, ~1.04, and $n$ = number of rolling sheaves (usually $s = n$).

A simplified overall block efficiency factor can be given as

$$E = 0.9787^n. \tag{2.17}$$

### 2.4.2 Block Line Strength

The safety factor (SF) is calculated as

$$\text{SF} = \frac{\text{Breaking strength of rope}}{\text{fast line load}}. \tag{2.18}$$

A minimum safety factor of 3.5 is recommended for drilling, and 2.5 is recommended for casing running and fishing operations.

**Problem 2.5**

A rotary rig that can handle triples is equipped with 1200 hp drawworks. The efficiency of the hoisting system is 81%. Determine the time it takes to trip one stand of pipe at a hook load of 300000 lbs.

*Solution:*

Using equation 2.5, the efficiency is

$$\eta = \frac{\text{power output}}{\text{power input}} = \frac{P_o}{P_i} = 0.81 = \frac{P_o}{1200}, \text{ and } P_o = 927 \text{ hp.}$$

But, using equation 2.6, $P_o = v_{tb} F_h$.
Therefore,

$$v_{tb} = \frac{P_o}{F_h} = \frac{927 \text{ hp} \times 33000 \left( \frac{\text{ft} - \text{lb}}{\text{min}} / \text{hp} \right)}{300000} = 102 \text{ fpm,}$$

and

$$t = \frac{\text{length}}{v_{tb}} = \frac{90 \text{ ft}}{102 \text{ ft/min}} = 0.84 \text{ min} = 50.5 \text{ sec.}$$

**Problem 2.6**

Determine the required drawworks horsepower for a maximum hook load of 400,000 lbs and traveling block velocity of 2 ft/sec. Hoist efficiency is 80%, and drawworks efficiency is 70%.

*Solution:*

Hoisting efficiency is

$$\eta_h = \frac{\text{power output}}{\text{power input}} = \frac{P_o}{P_i} = 0.80 = \frac{2 \times 60 \times 400{,}000}{P_i \times 33{,}000}.$$

$P_i = 1455$ hp.

Output of the drawworks is the input to the hoisting system. So, the drawworks efficiency is

$$\eta_d = \frac{\text{power output}}{\text{power input}} = \frac{P_o}{P_i} = 0.70 = \frac{1455}{P_i}.$$

Therefore, the input horsepower for the drawworks = 2080 hp.

**Problem 2.7**

A trip is to be made from a depth of 20,000 ft. Determine the minimum time at which the first stand can be tripped out of the hole. Assume the following data:

- Rig: can handle triples
- Drawworks: 1000 hp, efficiency = 75%
- Drilling lines: 12; hoist efficiency = 80%
- Drill pipe: effective weight = 14 lb/ft
- Drill collar: effective weight = 90 lb/ft, length = 1000 ft
- Other suspended loads = 30000 lbs

*Solution:*

Well depth at which trip out is made = 20000 ft.
Drill collar length = 1000 ft.
Drill pipe length = 20000 – 1000 = 19000 ft.
Total load at the hook = 30000 + 14 × (20000 – 1000) + 90 × 1000 = 386000 lbf.

Using equation 2.9 or 2.11,

$$F_f = \frac{F_h}{n\eta} = \frac{386000}{12 \times 0.8} = 40208\,\text{lb},$$

$$v_f = \frac{1000 \times 0.75 \times 33000}{40208} = 615.5.$$

$$\text{Time to pull the first stand} = \frac{90 \times 12}{615.5} = 1.75\,\text{min}.$$

$$v_{tb} = \frac{P_o}{F_h} = \frac{1000\,\text{hp} \times 0.75 \times 33000 \dfrac{\text{ft}-\text{lb}}{\text{min}} / \text{hp} \times 0.8}{386000} = 51.28\,\text{fpm},$$

$$t = \frac{L_s}{v_{tb}} = \frac{90\,\text{ft}}{51.28\,\text{ft/min}} = 1.75\,\text{min}.$$

**Problem 2.8**

A hook is hoisted at a velocity of 40 ft/min. Calculate the velocity of the fast line, calculate line pull at the drawworks assuming

2.2% frictional losses per line, and calculate the horsepower of drawworks. Use the following data:

- Drill pipe: effective weight = 16.77 ppf
- Drill collar: effective weight = 94.6 ppf
- Length = 300 ft
- Depth of the well: 9000 ft
- Drilling lines: 10 lines

*Solution:*

Hook is hoisted at a velocity of 40 fpm.
Velocity of the fast line = $v_f = 40 \times 10 = 400$ fpm.
Hook load = $F_h = 94.6 \times 300 + 16.77 \times (9000 - 300) = 174279$ lbf.
2.2% frictional losses per line results in 78% efficiency:

$$F_f = \frac{174279}{10 \times 0.78} = 22343\,\text{lbf.}$$

$$\text{Horsepower of drawworks} = \frac{22343 \times 400}{33000} = 271\,\text{hp.}$$

**Problem 2.9**

A drilling rig has 10 lines strung between crown and traveling blocks. 9⅝″ - 47 ppf casing operation is planned for a depth of 10,000 feet. Neglecting the buoyancy effects, calculate the equivalent mast load, derrick efficiency factor. If an overpull margin of 30% casing weight for stuck conditions is allowed, calculate the equivalent mast load, derrick efficiency factor.

*Solution:*

Total hook load = $F_h = 47 \times 10000 = 470000$ lbf.
Using equation 2.10 and assuming an efficiency factor of 81%,

$$F_d = \left(\frac{1 + 0.81 + 0.81 \times 10}{0.81 \times 10}\right) \times 470000 = 575025\,\text{lbf.}$$

Using equations 2.14 and 2.15, the equivalent derrick load and derrick efficiency factor can be calculated, respectively, as

$$F_{de} = \left(\frac{n+4}{n}\right) \times F_h = \left(\frac{10+4}{10}\right) \times 470000 = 658000,$$

$$E_d = \frac{E(n+1)+1}{E(n+4)} = \frac{0.81(10+1)+1}{0.81(10+4)} = 87.4\%,$$

or

$$E_d = \frac{658000}{805035} = 87.4\%.$$

When an overpull margin of 30% is used, the hook load to be handled is $1.3 \times 470000 = 611000$ lbf, but the derrick efficiency factor does not change as seen with equation 2.12 because it is independent of the hook load.

**Problem 2.10**

Calculate the derrick load and the equivalent derrick load using the following data:

- Drawworks input power: 500 hp
- Hook load needed to be lifted: 400 kips
- Number of lines strung: 12

*Solution:*

Using equation 2.13, the derrick load is

$$F_d = \left(\frac{1+E+En}{En}\right)W = \left(\frac{1+.77+.77\times 12}{.77\times 12}\right)400,000 = 476.6\,\text{kips}.$$

Using equation 2.14, the equivalent derrick load is given as

$$F_{de} = \left(\frac{n+4}{n}\right)W = \left(\frac{16}{12}\right)400,000 = 533.3\,\text{kips}.$$

Derrick efficiency, using equation 2.15, is

$$\eta_d = \frac{476.6}{533.3} \times 100 = 89.39\%.$$

**Problem 2.11**

A well is being drilled with a drawworks that is capable of providing a maximum input power of 1000 hp. Maximum load expected to hoist is 250,000 lbf. Twelve lines are strung between the traveling block and crown block, and the deadline is anchored to a derrick leg. The drilling line has a nominal breaking strength of 51,200 lbf.

A. Calculate the dynamic tension in the fast line.
B. Calculate the maximum hook horsepower available.
C. Calculate the maximum hoisting speed.
D. Calculate the derrick load when upward motion is impending.
E. Calculate the maximum equivalent derrick load.
F. Calculate the equivalent derrick efficiency factor.
G. What is the maximum force that can be applied to free a stuck pipe that has a strength of 400000 lbf? Assume safety factors of 2.0 for the derrick, drill pipe, and drilling line.

*Solution:*

A. Using equation 2.3, the efficiency for 12 lines = 0.782.
Using equation 2.11, the dynamic fast line tension is

$$F_f = \frac{F_h}{En} = \frac{250000}{0.782 \times 12} = 26{,}641\,\text{lbf}.$$

B. Maximum hook horsepower available = $P_h$ = 0.782 × 1000 = 782 hp.

C. The maximum hoisting speed is

$$v_b = \frac{0.782 \times 782 \times 33000}{250000} = 80.7\,\text{fpm}.$$

D. The maximum derrick load is

$$F_d = \left(\frac{1 + 0.782 + 0.782 \times 12}{0.782 \times 12}\right) 250000 = 297{,}474\,\text{lbf}.$$

E. Using equation 2.14, the maximum equivalent derrick load is

$$F_{de} = \left(\frac{12 + 4}{12}\right) 250000 = 333{,}333\,\text{lbf}.$$

F. Using equation 2.15, the maximum equivalent derrick efficiency factor is

$$E_d = \left(\frac{297474}{333333}\right) = 89\%.$$

G. Checking for pipe, rope, and derrick failure,

$$\left(\frac{333333}{2}\times\frac{12}{16}\right) = 125000\,\text{lbf (lowest), based on the derrick,}$$

$$\left(\frac{400000}{2}\right) = 200{,}000\,\text{lbf, based on the pipe, and}$$

$$\left(\frac{51200}{2}\times 0.782\times 12\right) = 240230\,\text{lbf, based on the rope.}$$

**Problem 2.12**

Calculate the block efficiency for a block with 12 lines strung using simplified and a friction factor of 1.04.

*Solution:*

A simplified overall block efficiency factor is given as in equation 2.17:

$$E = 0.9787^n,$$

$$E = 77.23\%.$$

Using equation 2.17, the overall block efficiency factor is

$$E = \frac{(\mu^n - 1)}{\mu^s n(\mu - 1)} = \frac{(1.04^{12} - 1)}{1.04^{12}\times 12(12-1)} = 0.782,$$

$$E = 78.2\%.$$

**Problem 2.13**

A. Determine the cost of tripping out from 26,000 ft. Assume that the maximum allowed speed of pulling the pipe out is that of pulling the first stand.

B. Determine cost of tripping out with the maximum allowed speed of the first stand up to 20,000 ft and thereafter 1.25 ft/min. Use time to handle a drill collar stand, which is 2 minutes.
C. If rig can handle quadruples (124 ft), what would be the cost in part A?

Use the following data:

- Target well depth = 26000 ft
- Drill pipe effective weight = 20 lbs/ft
- Drill collar effective weight = 100 lbs/ft
- Maximum length of drill collar required = 1000 ft
- Additional suspended loads to drill string = 60000 lbs
- Number of lines strung between crown block and traveling block = 10 lines
- Drawwork maximum input power = 2000 horsepower
- Drawwork efficiency = 75%
- Rig can handle triples (93 ft)
- Time to handle a stand once out of hole = 1 minute
- Drilling cost = $15,200/day

*Solution:*

A. Given: $n = 10$, $P_i = 2000$, and $\eta_{dw} = .75$
Maximum length of the drill collar required = 1000 ft.
Total hook load = DP weight + DC weight + other hanging weight = 25000 × 20 + 1000 × 100 + 60000 = 660 kips.
Drawworks output is $P_o = 2000 \times 0.75 = 1500$ hp.

$$v_{tb} = \frac{P_o}{F_h} = \frac{0.81 \times 1500\,\text{hp} \times 33000 \frac{\text{ft}-\text{lb}}{\text{min}} / \text{hp}}{660000} = 60.75\,\text{fpm}.$$

$$t = \frac{L_s}{v_{tb}} = \frac{93\,\text{ft}}{60.75\,\text{ft/min}} = 1.53\,\text{min} = 93\,\text{sec}.$$

$$\text{Total number of stands} = \frac{26000}{93} = 280\,\text{stands}.$$

Total time consumed = 280 × 1.53 + 280 × 1 = 11.80 hrs.

$$\text{Total cost} = \frac{15200}{24} \times 11.80 = \$7474.$$

B. If the variable speed is used, the total cost will be \$6,968.78.
C. Time equals

$$t = \frac{L_s}{v_{th}} = \frac{124\,\text{ft}}{60.75\,\text{ft/min}} = 2.04\,\text{min} = 122.5\,\text{sec}.$$

$$\text{Total number of stands} = \frac{26000}{124} = 210\,\text{stands}.$$

Total time consumed = 210 × 2.04 + 210 × 1 = 10.64 hrs.
Total cost = \$6,738.66.

A savings of \$738.33 is realized for this particular trip.

For 26,000 feet, with the given efficiency and day rate, pulling out in quadruples may not save much compared to the cost of the rig to handle stands with four singles.

**Problem 2.14**

Using the following data, calculate the safety factor for a 1.5-inch block line with a breaking strength of 228,000 lbf. Assume a block efficiency of 82% for 12 lines in the sheave.

Mud weight = 9.9 ppg, casing length is 9,500 ft, 29 ppf, and block weight = 35 kips.

*Solution:*

$$\text{Buoyed weight of the casing} = \left(1 - \frac{9.9}{65.5}\right) \times 29 = 24.62\,\text{ppf}.$$

Total weight of the casing = 24.62 × 9,500 = 233890 lb.
Total hook load = 233890 + 35000 = 268890 lb.

From equation 2.8, fast line is

$$F_f = \frac{F_h}{En} = \frac{268890}{0.82 \times 12} = 27326\,\text{lbf}.$$

From equation 2.18, the safety factor is

$$\text{SF} = \frac{\text{Breaking strength of rope}}{\text{fast line load}} = \frac{228{,}000}{27326} = 8.3.$$

## 2.5 Ton-Miles (TM) Calculations

Round-trip ton miles is calculated as follows:

$$T_f = \frac{L_h\left(L_s + L_h\right)W_{dp} + 4L_h\left(W_b + \frac{1}{2}W_1 + \frac{1}{2}W_2 + \frac{1}{2}W_3\right)}{10{,}560{,}000}\ \text{ton-miles,} \tag{2.19}$$

where $L_h$ = measured depth of the hole or trip depth, ft, $L_s$ = length of the stand, ft, $W_{dp}$ = buoyed weight of the drill pipe per foot, ppf, $W_b$ = weight of the block, hook, etc., lbs, $W_1$ = excess weight of drill collar in mud, lbs (buoyed drill collar weight in mud – buoyed drill pipe weight of same length in mud), $W_2$ = excess weight of heavy weight pipe in mud, lbs (buoyed heavy weight pipe in mud – buoyed drill pipe weight of same length in mud), and $W_3$ = excess weight of miscellaneous drilling tools in mud, lbs (buoyed miscellaneous drilling tools in mud – buoyed drill pipe weight of same length in mud).

### 2.5.1 Drilling Ton-Miles Calculations

When a hole is drilled only once without any reaming,

$$T_d = 2\left(T_{i+1} - T_i\right) \tag{2.20}$$

When a hole is drilled with one time reaming,

$$T_d = 3\left(T_{i+1} - T_i\right) \tag{2.21}$$

When a hole is drilled with two times reaming,

$$T_d = 4\left(T_{i+1} - T_i\right), \tag{2.22}$$

where $T_i$ = round trip ton-mile calculated, using equation 2.16 from depth $i$.

### 2.5.2 Coring Ton-Miles Calculations

When a hole is drilled only once without any reaming,

$$T_d = 2\left(T_{i+1} - T_i\right) \tag{2.23}$$

### 2.5.3 Casing Ton-Miles Calculations

$$T_f = \frac{1}{2}\left(\frac{L_h(L_s + L_h)W_c + 4W_b L_h}{10{,}560{,}000}\right)\text{ton-miles,} \qquad (2.24)$$

where $W_c$ = buoyed weight of the casing per foot, ppf.

**Problem 2.15**

Calculate the ton-mile for a round trip from 8,000 feet with 450 feet of drill collar with a weight of 83 ppf. The weight of the drill pipe is 19.5 ppf. The mud density is 9.6 ppg. Assume the stand length to be 93 feet and the total block weight is 40,000 lbs.

*Solution:*

Use the following data:

- $L_h = 8000$ ft
- $L_s = 90$ ft
- $W_b = 40000$ lbs

Buoyancy factor = $\left(1 - \dfrac{9.6}{65.5}\right) = 0.853.$

$W_{dp} = 19.5 \times 0.853 = 16.64$ ppf.

Buoyed weight of drill collar – buoyed weight of drill pipe = $0.853 \times 450(83 - 19.5) = 24387$ lbs.

Substituting the values in equation 2.19,

$$T_f = \frac{8000(93 + 8000)16.64 + 4 \times 8000(40000 + 24387)}{10{,}560{,}000} = 297\,\text{TM}.$$

**Problem 2.16**

Compute the ton-miles required to drill from 15,000 to 15,900 feet for the following conditions:

- Drill pipe = 5 in × 19.5 ppf
- Drill collars = 6 ½- in × 2 ½- in – 660 ft

- Heavy weight drill pipe = 5 in × 3 in × 50 ppf – 270 ft
- Expected bit performance = 300 ft/bit
- Mud used is 12.5 ppg for 15,000 ft, 12.7 ppg for 15,300 ft, 12.9 ppg for 15,600 ft and 13.0 ppg for 15,900 ft
- Traveling block weight = 40 kips
- Average length of the stand = 93 ft
- Additionally, a 15 ft core was cut from 15,900 ft

*Solution:*

*Total operations:*

- $T_1$. Drilling from 15,000 ft to 15,300 ft with mud weight 12.5 ppg
- $T_2$. Since the bit's performance is 350 ft, it can drill another 50 ft with 12.7 ppg mud
- $T_3$. Pull out the bit/change the bit and run in to 15,350 for drilling with 12.7 ppg mud
- $T_4$. While running in reaming one time from 15,097 to 15,127 ft with 12.7 ppg mud
- $T_5$. Drilling from 15,350 ft to 15,600 ft with mud weight 12.7 ppg
- $T_6$. Pull out the bit/change the bit with core bit and run in to 15,600 for coring with a mud weight of 12.7 ppg
- $T_7$. Drilling from 15,600 ft to 15,900 ft with mud weight 13 ppg
- $T_8$. Round trip from 15,900 ft to run the core barrel in 13 ppg mud
- $T_9$. Coring from 15,900 ft to 15,915 ft with 13 ppg mud

1. Round trip ton-mile from 15,300 ft with 12.7 ppg:

$$T_f = \frac{L_h\left(L_s + L_h\right)W_{dp} + 4L_h\left(W_b + \frac{1}{2}W_1 + \frac{1}{2}W_2 + \frac{1}{2}W_3\right)}{10{,}560{,}000},$$

$$\text{BF} = \left(1 - \frac{12.5}{65.5}\right) = 0.809,$$

The effective weight of the drill collar is

$$\left(\frac{\frac{\pi}{4}\times\left(6.5^2-2.5^2\right)}{0.2945}\right)\times \mathrm{BF} = 77.67 \text{ ppf.}$$

Effective weight of drill pipe = 19.5 × BF = 15.78 ppf.
Effective weight of heavy weight drill pipe = 50 × BF = 40.45 ppf.

$$T_{15300} = \frac{15300\,(93+15300)15.78 + 4\times 15300\left(40000+\frac{1}{2}6663+\frac{1}{2}40857\right)}{10{,}560{,}000}$$

$$= 722\,\text{TM.}$$

$$T_{15000} = \frac{15000\,(93+15000)15.78 + 4\times 15000\left(40000+\frac{1}{2}6663+\frac{1}{2}40857\right)}{10{,}560{,}000}$$

$$= 701\,\text{TM.}$$

1. Drilling from 15,000 ft to 15,300 ft with mud weight 12.5 ppg:
Drilling ton-mile for 15,000 ft to 15,300 ft = 2(722 – 701) = $T_1$ = 42 TM.

$$T_{15350} = 723 \text{ TM; } T_{15300} = 720 \text{ TM.}$$

2. Drilling another 50 ft to 15,350 ft with 12.7 ppg mud:
Drilling to 15,350 ft with 12.7 ppg = 2(723 – 720) $T_2$ = 7 TM.
3. Round trip with 12.7 ppg mud:
Round trip ton-mile from 15,350 ft with mud weight 12.7 ppg $T_3$ = 723.
4. While running in reaming one time from 15,097 to 15,127 ft with 12.7 ppg mud:
Reaming ton-mile $T_4$ = 6 TM.
5. Drilling from 15,350 to 15,600 ft with mud weight 12.7 ppg:

$$T_{15600} = 741 \text{ TM; } T_{15350} = 723 \text{ TM.}$$

Drilling ton-mile = 2(741 – 723) = $T_5$ = 35 TM.

6. Pull out the bit/change the bit with core bit and run in to 15,600 for coring with a mud weight of 12.7 ppg:
Round trip ton-mile from 15600 ft $T_6 = 741$ tm.

$$T_{15900} = 760 \text{ TM}; \ T_{15600} = 741 \text{ TM}.$$

7. Drilling from 15,600 ft to 15,900 ft with mud weight 13 ppg:
Drilling from 15,600 ft to 15,900 ft = 2(760 – 741) = $T_7$ = 39 TM.
8. Round trip from 15,900 ft to run the core barrel in 13 ppg mud:
Round trip from 15,900 feet for coring is

$$\text{Round trip ton-mile } T_8 = 760 \text{ TM}.$$

9. Coring from 15,900 ft to 15,915 ft with 13 ppg mud:
Coring from 15,600 to 15,615 ft with mud weight 13 ppg is

$$T_{15915} = 760.4 \text{ TM}; \ T_{15900} = 760.3 \text{ TM}.$$

Coring ton-mile = 2(760.4 – 760.3) = $T_9$ = 0.2 TM.
Total ton miles to drill, ream and core from 15,000 to 15,915 = 2353 TM.

**Problem 2.17**

Determine the casing ton-miles for a casing run of 7 inch 29 ppf casing to 9,500 ft in a mud of 9.9 ppg. Travelling block weight is 35 kips.

*Solution:*

$$\text{Buoyed weight of the casing} = \left(1 - \frac{9.9}{65.5}\right) \times 29 = 24.62 \text{ ppf}.$$

Assuming 40 ft of casing, casing ton-miles can be calculated using equation 2.19:

$$T_f = \frac{1}{2}\left(\frac{9500(40 + 9500) \times 24.62 + 4 \times 35000 \times 9500}{10{,}560{,}000}\right)$$
$$= 169 \text{ ton-miles}$$

## 2.6 Crown Block Capacity

The crown block capacity required to handle the net static hook-load capacity can be calculated using the following formula:

$$R_c = \frac{(H_L + S)(n + 2)}{n}. \tag{2.25}$$

where $R_c$ = required crown block rating (lbs), $H_L$ = net static hook load capacity (lbs), $S$ = effective weight of suspended equipment (lbs), and $n$ = number of lines strung to the traveling block.

**Problem 2.18**

What minimum drawworks horsepower is required to drill a well using 10,000 ft of 4 ½" OD 16.6# drill pipe and 50,000 lbs of drill collars? Efficiency is 65%.

*Solution:*

Drill string weight (air weight) = 50,000 lbs + (10,000 ft) (16.6 lb/ft) = 216,000 lbs.

$$\text{Hook horsepower} = \frac{(216{,}000 \text{ lbs})\ (100 \text{ ft/min})}{33{,}000} = 655 \text{ hp.}$$

$$\text{Required minimum drawworks horsepower rating} = \frac{655}{0.65} =$$

1,007 hp.

This method neglects the effect of buoyancy and the weight of the block and hook.

## 2.7 Line Pull Efficiency Factor

Hoisting engines should have a horsepower rating for intermittent service equal to the required drawworks horsepower rating divided by 85% efficiency. Drawworks also have a line pull rating efficiency depending on the number of lines strung between crown block and traveling block. See Table 2.1 below.

**Table 2.1** Line pull efficiency factor.

| No. of lines | Efficiency factor |
|---|---|
| 6 | 0.874 |
| 8 | 0.842 |
| 10 | 0.811 |
| 12 | 0.782 |

**Problem 2.19**

What line pull is required to handle a 500,000 lb casing load with 10 lines strung?

*Solution:*

$$\text{Line Pull} = \frac{500{,}000}{(10)\,(.810)} = 61{,}728 \text{ lbs.}$$

## 2.8 Rotary Power

Rotary horsepower can be calculated as follows:

$$H_{rp} = \frac{2\pi NT}{33000}, \tag{2.26}$$

where $N$ = rotary table speed (RPM), and $T$ = torque (ft-lbs).

**Problem 2.20**

The drill pipe must transmit rotating power to the bottomhole assembly (BHA) and the bit. The following example illustrates the calculation of the horsepower that the drill pipe can transmit without torsion failure. For example, if a drill pipe has a maximum recommended make-up torque of 20,000 ft-lbs, then what is the rotary horsepower that can be transmitted at 100 rpm?

*Solution:*

Using equation 2.26,

$$H_{rp} = \frac{(2\pi \times 100 \times 20{,}000)}{33{,}000} = 381\,\text{hp}.$$

The empirical relationship to estimate the rotary horsepower requirements is

$$H_{rp} = F \times N,$$

where $F$ = torque factor, ft-lbf, and $N$ = rotary speed (RPM).
The torque factor ($F$) is generally estimated as follows:

- $F$ = 1.5 to 1.75 for shallow holes less than 10,000 ft with light drill string.

- $F = 1.75$ to 2.0 for 10,000–15,000 ft wells with average conditions.
- $F = 2.0$ to 2.25 for deep holes with heavy drill string
- $F = 2.0$ to 3.0 for high-torque

The above empirical estimates are subject to many variables but have proved to be reasonable estimates of rotary requirements. However, for highly deviated wells, torque/$H_{rp}$ requirements must be closely calculated using available computer software programs.

**Problem 2.21**

Derive the equations to calculate the length, $L$, in feet of the drill line in a drawworks reel drum (Fig. 2.2) with the following dimensions:

- Reel diameter: $D_R$ in.
- Reel width: $W$ in.
- Core diameter: $D_c$ in.
- Diameter of the drill line: $d$ in.
- Wrapping types: offset and inline

A drilling engineer is planning to estimate the number of times the drill line can be slipped before cutting. Establish an equation to estimate the number of times 200 ft of drill line can be slipped. The initial drill line length on the drum is m% of the drawworks drum capacity. The maximum drill line length is not to exceed n% of the drawworks drum capacity.

*Solution:*

Inline:

$$\text{First layer} = L_1 = \frac{\pi}{12}(D_c + d)\frac{W}{d}\text{ ft.}$$

$$\text{Second layer} = L_2 = \frac{\pi}{12}(D_c + 3d)\frac{W}{d}\text{ ft.}$$

$$n^{\text{th}}\text{ layer} = L_n = \frac{\pi}{12}[D_c + (2n-1)d]\frac{W}{d}\text{ ft,}$$

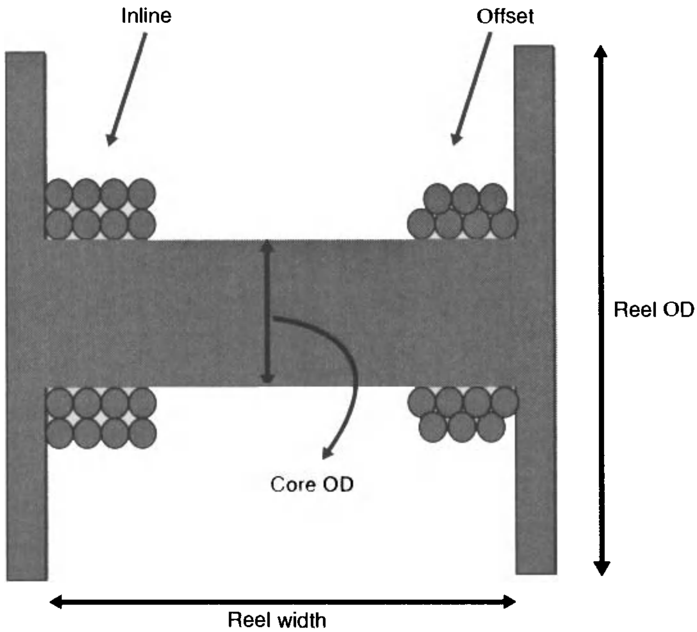



**Figure 2.2** Problem 2.21.

where $n$ = the total number of layers on the drum.

$$L_T = L_1 + L_2 \ldots + L_n = \frac{\pi}{12}\left[D_c + d + D_c + 3d \ldots . + D_c + (2n-1)d\right]\frac{W}{d}$$
$$= \frac{\pi}{12}\frac{W}{d}\left(nD_c + n^2 d\right)\text{ft.}$$

From the geometry, the number of laps is

$$n = \frac{D_r - D_c}{2d}.$$

Offset:

$$h' = \sqrt{d^2 - \left(\frac{d}{2}\right)^2} = \frac{\sqrt{3}}{2}d.$$

$$\text{First layer} = L_1 = \frac{\pi}{12}(D_c + d)\frac{W}{d}\text{ ft.}$$

$$\text{Second layer} = L_2 = \frac{\pi}{12}\left(D_c + d + \sqrt{3}d\right)\left(\frac{W}{d} - 1\right)\text{ft.}$$

$$3^{\text{rd}}\text{ layer} = L_3 = \frac{\pi}{12}\left(D_c + d + 2\sqrt{3}d\right)\frac{W}{d}\text{ft.}$$

$$n^{\text{th}}\text{ layer} = L_n = \frac{\pi}{12}\left[D_c + d + (n-1)\sqrt{3}d\right]\left(\frac{W}{d} - f(n)\right)\text{ft,}$$

where $n$ is the total number of layers on the drum.
$f(n) = 0$ if $n$ is odd, and $f(n) = 1$ if $n$ is even.

$$L_T = L_1 + L_2 \ldots + L_n = \frac{\pi}{12}\left[\left(D_c + d\right)\frac{W}{d} + \left(D_c + d + \sqrt{3}d\right)\left(\frac{W}{d} - 1\right)\ldots\right.$$
$$\left. + \frac{\pi}{12}\left[D_c + d + (n-1)\sqrt{3}d\right]\left(\frac{W}{d} - f(n)\right)\right]\text{ft.}$$

$f(n) = 0$ if $n$ is odd, and $f(n) = 1$ if $n$ is even.

Adding and simplifying using

$$1 + 2 + 3\ldots = \frac{(n-1)n}{2};\ 1 + 3 + 5\ldots = (n-2)^2.$$

$$L_T = \frac{\pi}{12}\frac{W}{d}\left(nd + nD_c + \frac{(n-1)n}{2}\sqrt{3}d\right)$$
$$-\frac{\pi}{12}\left[(n-2)d + (n-2)D_c + (n-2)^2\sqrt{3}d\right.$$
$$\left. + \left(d + D_c + (n-1)\sqrt{3}d\right)f(n)\right]\text{ft.}$$

From the geometry,

$$\frac{D_r - D_c}{2} = d + (n-1)\frac{\sqrt{3}}{2}d.$$

Therefore,

$$n = \left(D_r - D_c - 2d\right)\frac{1}{d\sqrt{3}} + 1.$$

Slip and cut:

Usually three or four times the drill line is slipped into the drum before cutting and discarding certain amount of drill line from the drum so that the lap points are removed.

$$\text{Number of slips} = \left(\frac{(n-m)\times L_T}{200\times 100}\right)\text{(expressed in integer).}$$

**Problem 2.22**

Determine the total time it takes to make a trip at a depth of 11,700 ft. Use the following data:

- Rig can handle triples.
- Drill pipe: length = 10,700 ft; effective weight = 15 lb/ft
- Drill collar: effective weight = 80 lb/ft
- Maximum available drawworks horsepower = 1200 hp
- Drawworks consists of 12 lines strung between crown block and traveling block
- Block and tackle efficiency = 85%
- Drawworks efficiency = 75%
- Rig crew can break a joint or connect a joint in average time of 18 seconds
- Assume average stand length = 90 feet

*Solution:*

Given: $n = 12$, $\eta = .85$, $P_i = 1200$ hp, and $\eta_{dw} = .75$
Total hook load = DP weight + DC weight + other hanging weight = $10{,}700 \times 15 + 1000 \times 80 = 240{,}500$ lbs.
Drawworks output is $P_o = 2000 \times 0.75 = 1500$ hp.

$$v_{tb} = \frac{P_o}{F_h} = \frac{0.85\times 1{,}500\,\text{hp}\times 33{,}000\,\frac{\text{ft}-\text{lb}}{\text{min}}/\text{hp}}{240{,}500} = 175\,\text{fpm}.$$

$$t = \frac{L_s}{v_{tb}} = \frac{90\,\text{ft}}{175\,\text{ft/min}} = .51\,\text{min} = 31\,\text{sec}.$$

$$\text{Total number of stands} = \frac{11{,}700}{90} = 130\,\text{stands.}$$

$$\text{Total time consumed} = 130 \times 0.51 + 130 \times \frac{18}{60} \approx 106\,\text{min.}$$

**Problem 2.23**

Determine engine fuel cost (\$/day) given the following information:

- Total well depth = 20,000 ft.
- Drill collar weight = 150 lb/ft
- Drill collar total length = 1,000 ft
- Drill pipe weight = 20 lb/ft
- Average drag force on drill string while being tripped out at 60 ft/mm. = 3.5 lb/ft
- Mechanical efficiency between power generating engine output and drawworks output = 70%
- Ten lines strung between crown and traveling blocks
- Diesel engine mechanical efficiency = 50%
- Diesel oil heating value = 20000 BTU/lb
- Diesel oil weight = 7.2 lb/gal.
- Diesel cost = \$2.00/gal.

*Solution:*

$$\text{Input } H_{HP} = \frac{QP}{1714} = \frac{360 \times 3400}{1714 \times 0.85} = 840\,\text{hp}$$

or

$$= \frac{QP}{1714} = \frac{324 \times 3400}{1714 \times 0.85 \times 0.9} = 840\,\text{hp},$$

$$H_{HP} = \frac{840}{0.55} = 1527\,\text{hp.}$$

$$Q_f = \frac{1527 \times 2545 \times 24}{20000 \times 0.5 \times 7.2} = 1177\,\text{gal/day.}$$

Consider the following:

Engine input (fuel energy) → engine output (input to pump)
→ mechanical pump output (input to hydraulic)
→ hydraulic output.

Overall efficiency can also be used to find the engine input to the pump output horsepower.

## 2.9 Mud Pumps

The following are calculations for theoretical volume of fluid displaced.

For single-acting pump,

$$V_t = \left(\frac{\pi}{4} D_L^2 L_s\right) N_c. \tag{2.27}$$

For double-acting pump,

$$V_t = \frac{\pi}{4} N_c L_s \left(2D_L^2 - D_r^2\right). \tag{2.28}$$

Actual flow rate is

$$Q_a = Q_t \eta_v. \tag{2.29}$$

Pump hydraulic horsepower is

$$HHP_p = \frac{P_p Q}{1714} \tag{2.30}$$

where $D_L$ = liner or piston diameter, in, $L_s$ = stroke length, in, $N_c$ = number of cylinders, 2 for duplex and 3 for triplex, $D_r$ = rod diameter, in, and $\eta_v$ = volumetric efficiency.

### 2.9.1 Volumetric Efficiency

$$\eta_v = \frac{Q_a}{\text{displacement volume} \times \text{speed}} \times 100, \tag{2.31}$$

where $Q_a$ = actual flow rate.

$$\eta_v = \frac{Q_t - \Delta Q}{Q_t} \times 100, \tag{2.32}$$

where $\Delta Q$ = leakage losses, and $Q_t$ = theoretical flow rate = displacement volume × pump speed.

## 2.9.2 Pump Factor

The pump factor is the pump displacement per cycle and is given as *PF* in bbl/stroke or gal/stroke.

Duplex pump factor is given by

$$2\left(V_{fs} + V_{bs}\right) = PF_d, \tag{2.33}$$

$$PF_d = \frac{\pi}{2} L_s \left(2D_L^2 - D_r^2\right). \tag{2.34}$$

Triplex pump factor is given by

$$3V_{fs} = PF_t, \tag{2.35}$$

$$PF_t = \left(\frac{3\pi}{4} D_L^2 L_s\right). \tag{2.36}$$

Volumetric efficiency is given by

$$\eta_v = \frac{PF_d}{PF_t} \times 100, \tag{2.37}$$

where $PF_a$ = actual pump factor, and $PF_t$ = theoretical pump factor.

For duplex pumps,

$$V_t = \frac{\pi}{4} N_c L_s \left(2D_L^2 - D_r^2\right)\eta_v, \tag{2.38}$$

$$V_t = \frac{N_c L_s \left(2D_L^2 - D_r^2\right)\eta_v}{42 \times 294} \text{ bbl/stroke}. \tag{2.39}$$

For triplex pumps (single-acting, three cylinders),

$$V_t = \frac{L_s\left(D^2\right)\eta_v}{42 \times 98.03}\,\text{bbl/stroke.} \tag{2.40}$$

## 2.10 Energy Transfer

Efficiency transfer from the diesel engines to the mud pump can be given as in Figure 2.3. Due to interrelated equipment, various efficiencies can be used:

- Engine efficiency, $\eta_e$
- Electric motor efficiency, $\eta_{el}$
- Mud pump mechanical efficiency, $\eta_m$
- Mud pump volumetric efficiency, $\eta_v$
- Overall efficiency, $\eta_o = \eta_e \times \eta_{el} \times \eta_m \times \eta_v$

Engine input (fuel energy) → engine output (input to pump)
→ mechanical pump output (input to hydraulic)
→ hydraulic output. (2.41)

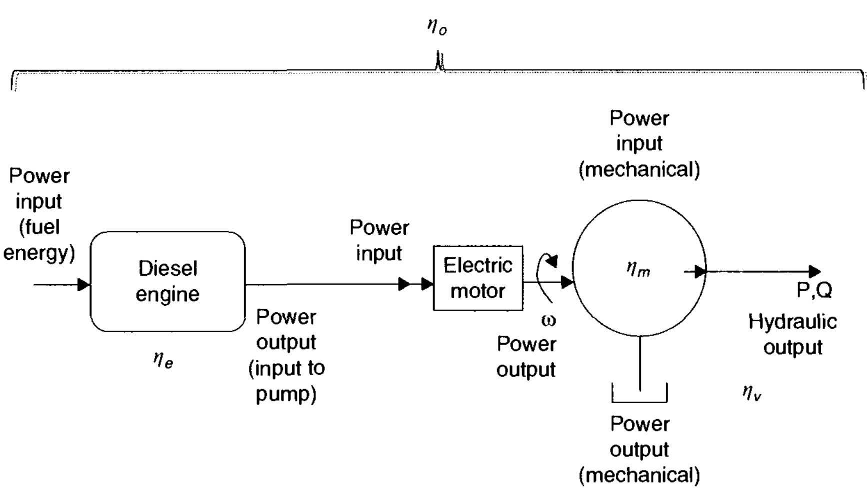



Figure 2.3 Energy transfer.

**Problem 2.24**

Estimate the liner size required for a double-acting duplex pump given the following pump details:

- Rod diameter = 2.5 in
- Stroke length = 20 in stroke
- Pump speed = 60 strokes/min.

The maximum available pump hydraulic horsepower is 1,000 hp. For optimum hydraulics, the pump recommended delivery pressure is 3,500 psi.

*Solution:*

Using equation 2.39, theoretical pump displacement for a duplex pump is

$$V_t = \frac{\pi}{4} N_c L_s \left(2D_L^2 - D_r^2\right) = \frac{2 \times 20\left(2D_L^2 - 2.5^2\right)}{294} \text{gal/stroke.}$$

The theoretical flow rate of the pump operating at 60 strokes/min is

$$V_t = \frac{2 \times 20\left(2D_L^2 - 2.5^2\right)}{294} \times 60 \text{gal/stroke,}$$

$$V_t = \left(16.33D_L^2 - 51.02\right) \text{gpm.}$$

Using equation 2.31, the volumetric relationship follows:

$$Q_a = Q_t \eta_v,$$

$$490 = \left(16.33D_L^2 - 51.02\right) \times 0.9 \text{gpm,}$$

$$D_L = 6.03 \text{ in.}$$

Therefore, the liner size that can be used is 6 in.

**Problem 2.25**

There is a single-acting triplex pump with a stroke length of 12 in., a rod diameter of 1.5 in., and a pump factor of 6.0 at 100% volumetric efficiency. The pump was operated at 100 strokes/mm. and

3428 psi for 4 mm. The amount of mud collected at the flowline was 2040 gal.

A. Determine the pump input hydraulic horsepower.
B. Determine the liner diameter.

*Solution:*

Using equation 2.30,

$$HHP_p = \frac{P_p Q}{1714} = \frac{3428 \times 2040/4}{1714} = 1020\,\text{hp}.$$

Pump factor of 6 gal/stroke at 100 efficiency is $Q_t = 6 \times 100 = 600$ gpm.

The actual flow rate is

$$Q_a = \frac{2040}{4} = 510\,\text{gpm}.$$

$$\text{Volumetric efficiency} = \frac{510}{600} = 0.85.$$

Therefore, using the power factor equation, the diameter of the liner is

$$D_L = \sqrt{\frac{294 \times 6}{12 \times 3}} = 7\,\text{in}.$$

Alternatively, the flow rate equation can also be used.

**Problem 2.26**

A double-acting duplex pump (liner diameter = 6.5″; rod diameter = 35% of liner diameter; stroke length = 16″) is being used while drilling a well at 15,000 ft. At 70 strokes/min., the pump is delivering 350 gal/min of mud at a pressure of 4000 psi. The pump mechanical efficiency is 70%.

A. What must be the input horsepower to the pump?
B. For an engine efficiency of 55%, how many gallons of diesel fuel are being consumed per day?

Use the following data:

- Heating value of diesel oil = 19000 BTU/lbm.
- Density = 7.2 ppg
- Cost = \$2.16/gal

Determine the total cost of diesel oil.

*Solution:*

A. Volume per stroke $= \dfrac{32\left(2\times 6.5^2 - 5.175625\right)}{294} = 8.63$ gal/stroke.

Actual hydraulic horsepower is

$$HHP_p = \frac{P_p Q}{1714} = \frac{350\times 4000}{1714} = 816.8\,\text{hp}.$$

The theoretical flow rate is $Q_t = 8.63\times 70 = 604$ gpm.

Therefore, volumetric efficiency is

$$\eta_v = \frac{Q_a}{Q_t}100 = \frac{350}{604}\times 100 = 58\%.$$

Input horsepower = 816.8/0.58 = 1408 hp.

Considering the mechanical efficiency, the pump horsepower required = 1408/0.7 = 2012 hp.

B. The amount of diesel consumed for a single engine is

$$Q_f = \frac{2010\times 2545\times 24}{19000\times 0.55\times 7.2} = 1633\,\text{gal/day}.$$

The cost of the diesel consumption = \$3227.

**Problem 2.27**

It was decided to circulate and clean the hole before pulling out for a logging run. Calculate the number of strokes and time in minutes to displace the annulus and drill string for the wellbore configuration.

Wellbore details:

- Total depth: 12,000 ft
- Last intermediate casing: 9⅝″ × 8.681″ set at depth 11200 ft.
- Open hole size: 8 ½″

Drill string details:

- Drill pipe: 5″ × 4.276″
- HWDP: 5″ × 2 13⁄16″; length = 300 ft
- Drill collar: 6 ½″ × 3″; length = 600 ft

Pump Details:

- Pump: 7″ liner
- Type: duplex, double-acting
- Stroke length: 17″
- Rod diameter: 2 ½″
- Strokes per minute: 60
- Volumetric efficiency: 90%

*Solution:*

Theoretical pump displacement for a duplex pump is

$$V_t = \frac{2\times 17\left(2\times 7^2 - 2.5^2\right)}{294}\left(\frac{\text{gal}}{\text{stroke}}\right)\left(\frac{\text{bbl}}{42.09\text{ gal}}\right)$$
$$= 0.2520\,\text{bbl/stroke}.$$

$$1\text{ bbl} = 42.09\text{ gal}.$$

*Pipe Capacities:*

$$\text{Capacity of drill pipe} = \frac{4.276^2}{1029.4} = 0.017762\,\text{bbl/ft}.$$

Total volume of drill pipe = 0.017762 × (12000 − 600 − 300) = 197.158 bbl.

Capacity of heavyweight drill pipe = $\dfrac{2.8125^2}{1029.4}$ = 0.0076842 bbl/ft.

Total volume of heavyweight drill pipe = 0.0076842 × 300 = 2.305272 bbl.

Capacity of drill colar = $\dfrac{3^2}{1029.4}$ = 0.008743 bbl/ft.

Total volume of drill colar = 0.008743 × 600 = 5.245774 bbl.

Total pipe volume = 195.3817 + 2.305272 + 5.245774 = 204.6871 bbl.

*Annular Capacities:*

Annular capacity in open hole against the drill pipe=

$$\frac{\left(8.681^2 - 5^2\right)}{1029.4} = 0.0489215\,\text{bbl/ft.}$$

Annular volume in casing against the drill pipe =
0.0489215 × 11100 = 543.03 bbl.

Annular volume in casing against the heavyweight drill pipe =
0.0489215 × 100 = 4.89 bbl.

Annular capacity in open hole against the drill pipe =

$$\frac{\left(8.5^2 - 5^2\right)}{1029.4} = 0.0459005\,\text{bbl/ft.}$$

Annular volume in open hole against the heavyweight drill pipe =
0.0459005 × 200 = 9.1801 bbl.

Annular capacity in open hole against the drill collar=

$$\frac{\left(8.5^2 - 6.5^2\right)}{1029.4} = 0.0291432\,\text{bbl/ft.}$$

Annular volume against the drill collar =
0.0291432 × 600 = 17.486 bbl.

Total annular volume = 543.03 + 4.89 + 9.1801 + 17.486 = 574.5861 bbl.

Total strokes needed to displace string volume =

$$\frac{204.6871}{.227} = 902\,\text{strokes.}$$

Total strokes needed to displace annular volume =

$$\frac{574.5865}{.227} = 2532\,\text{strokes.}$$

Total number of strokes = 902 + 2532 = 3434 strokes.

$$\text{Total time required} = \frac{3434\ (\text{strokes})}{60\left(\dfrac{\text{strokes}}{\text{min}}\right)} = 57.23\,\text{min}.$$

**Problem 2.28**

A duplex pump (liner diameter = 6.5″; rod diameter = 35%; stroke length = 18″) is being used while drilling a well at 15,000 ft. At 50 strokes/min, the pump is delivering 350 gal/min of mud at a pressure of 4000 psi. The pump volumetric efficiency is 70%.

A. What must be the input horsepower to the pump?
B. If the dynamic pressure in the circulating system is 2200 psi, what percent of the horsepower calculated in part (A) is wasted due to friction pressure losses?

*Solution:*

The theoretical flow rate is

$$\frac{Q_a}{\eta_v} = \frac{350}{0.7} = 500\,\text{gpm}.$$

The theoretical hydraulic horsepower is

$$HHP_p = \frac{P_p Q}{1714} = \frac{500 \times 4000}{1714} = 1,167\,\text{hp}.$$

Since the horsepower is directly proportional to pressure drop, the waste due to frictional pressure losses can be given as

$$\%\ \text{change} = \frac{4000 - 2200}{4000} = 45\%.$$

**Problem 2.29**

There is a single-acting triplex mud pump powered by a diesel engine. The pump data follows:

- Rod size = 2 ½″
- Liner size = 7″;

- Stroke length = 12″
- Operating delivery pressure = 3400 psi at 60 strokes/minute
- Volumetric efficiency = 90%
- Mechanical efficiency = 85%

For an engine efficiency of 55%, how many gallons of diesel fuel are being consumed per day?
Given: Heating value of diesel oil = 19000 BTU/lb.; density = 7.2 ppg

*Solution:*

For a single-acting triplex pump, the flow rate is calculated as

$$Q_t = \frac{\pi}{4} D_L^2 L_S N_c = \frac{\pi}{4} \times 7^2 \times 12 \times 3 \left( \frac{\text{in}^3}{\text{stroke}} \right) \times 60 \left( \frac{\text{stroke}}{\text{min}} \right) \times \left( \frac{1\ \text{ft}^3}{12\ \text{in}^3} \right) \times \left( \frac{7.48\ \text{gal}}{1\ \text{ft}^3} \right) = 360\ \text{gpm}$$

Actual flow rate is

$$Q_a = Q_t \eta_v = 360 \times 0.90 = 324\ \text{gpm}.$$

Hydraulic horsepower is

$$HHP = \frac{P_p Q}{1714} = \frac{324 \times 3400}{1714} = 642\,\text{hp}.$$

Output mechanical horsepower is

$$\text{Input } H_{HP} = \frac{QP}{1714} = \frac{360 \times 3400}{1714 \times 0.85} = 840\,\text{hp},$$

$$= \frac{QP}{1714} = \frac{324 \times 3400}{1714 \times 0.85 \times 0.9} = 840\,\text{hp}.$$

$$HH_{HP} = \frac{840}{0.55} = 1527\,\text{hp}.$$

$$Q_f = \frac{1527 \times 2545}{19000} = 682\,\text{gal/day}.$$

Consider the following:

Engine input (fuel energy) → engine output (input to pump)
→ mechanical pump output (input to hydraulic)
→ hydraulic output.

Overall efficiency can also be used to find the engine input to the pump output horsepower.

**Problem 2.30**

An 8 ½″ hole is being drilled at a depth of 8,000 ft with 4 ½″ OD drill pipe. Calculate the strokes and time required to displace the string and annulus with the following details:

Duplex pump details:

- Liner diameter = 6″
- Stroke length = 15″
- Rod diameter = 2″
- Volumetric efficiency = 90%
- Pump strokes = 50 spm

Pipe and hole details:

- 4 ½″ DP capacity = 0.01422 bbl/ft
- 4 ½″ and 8 ½″ annulus capacity = 0.05 bbl/ft

Volumetric output is

$$V_t = \frac{\pi}{4} N_c L_s \left(2D_L^2 - D_r^2\right),$$

$$V_a = \frac{2 \times 20\left(2 \times 6^2 - 2^2\right)}{294} \times .90 = 8.32\,\text{gal/stroke},$$

$$1\text{ bbl} = 42.09\text{ gal}.$$

Total flow rate is

$$8.32\frac{\text{gal}}{\text{stroke}} \times \frac{1}{42.09}\left(\frac{\text{bbl}}{\text{gal}}\right) = 0.1976717\,\text{bbl/stroke}.$$

Drill string capacity is =

$$0.01422 \frac{\text{bbl}}{\text{ft}} \times 8000\,\text{ft} = 113.76\,\text{bbl}.$$

Annulus capacity is =

$$0.05 \frac{\text{bbl}}{\text{ft}} \times 8000\,\text{ft} = 400\,\text{bbl}.$$

Drill string capacity + annulus capacity = 513.76 bbl.

$$\text{Number of strokes} = \frac{513.76}{0.1976717} = 2600\,\text{strokes}.$$

Total time required to displace the string and annulus =

$$\left(\frac{2600}{50}\right)\left(\frac{\text{stroke}}{\dfrac{\text{strokes}}{\text{min}}}\right) = 52\,\text{min}.$$

**Problem 2.31**

A double-acting duplex pump with a 6.5″ liner, 2.5″ rod diameter, and 18″ stroke is operated at 3000 psi and 40 strokes per minute for 5 minutes with the suction tank isolated from the return mud flow. The mud level in the suction tank, which is 7′ wide and 30′ long, was observed to fall 1 feet during this period. Determine the volumetric efficiency and hydraulic horsepower of the pump.

*Solution:*

Using the data given, the theoretical flow rate can be calculated using equation 2.39:

$$Q_t = \frac{\pi}{4} \times 2 \times 18 \times \frac{\left(2 \times 6.5^2 - 2.5^2\right)}{294} \times 40 = 301\,\text{gpm}.$$

The amount of fluid collected for five minutes = 301 × 5 = 1505 gal.

The total amount of fluid from the tank is the actual flow during the same amount of time:

$$Q_a = 30 \times 7 \times 1 \times 7.48 = 1570.8\,\text{gal}.$$

Using the theoretical flow and the actual flow, volumetric efficiency can be calculated as

$$\eta = \frac{Q_a}{Q_t} = \frac{1505}{1570.8} = 96\%.$$

Hydraulic horsepower is

$$HHP = \frac{P_p Q_a}{1714} = \frac{3000 \times 3016}{1714 \times 0.96} = 550\,\text{hp}.$$

**Problem 2.32**

There is a single-acting triplex pump with a stroke length of 12 in. and a pump factor, $F_p$, of 6.0 gal/stroke at $\eta_v = 100\%$. The pump was operated at 100 strokes/min and 2428 psi for 3 minutes.

Calculate the increase in the mud level in the trip tank (8 ft. wide and 8 ft. long) if the pump volumetric efficiency is 93%. Determine the pump hydraulic horsepower and the liner diameter.

Trip tank: collection or supply tank at the surface

*Solution:*

Pump factor for triplex pump can be given as $PF_t = N_c L_s D_L^2 \eta_v$.

The pump factor is

$$PF_t = \frac{3 \times 12 \times D_L^2 \times 1}{294} = 6\,\text{gal/stroke}.$$

Solving results in a liner, size of $D_L = 7$ in.
Theoretical flow rate $= Q_t = 6 \times 100 = 600$ gpm.

Actual flow rate for a volumetric efficiency of 93% is

$$Q_a = Q_t \eta_v = 600 \times 0.93 = 558\text{ gpm}.$$

The increase in mud level in the trip tank can be calculated as

$$H = \frac{558 \times 3}{8 \times 8 \times 7.48} = 3.5\text{ ft}.$$

Hydraulic horsepower of the pump is

$$HHP = \frac{P_p Q_a}{1714} = \frac{2428 \times 558}{1714} = 790\text{ hp}.$$

**Problem 2.33**

A drilling engineer is planning to conduct a simple test to calculate the volumetric efficiency of the mud pump by placing a sample of calcium carbide in the drill string during the pipe connection. Calcium carbide reacts with mud to form acetylene gas and is detected at the surface after pumping 450 strokes. Drill string consists of 800 ft. of 8″ OD and 3″ ID drill collars and is at bottom of the well that has an open hole diameter of 12 ¼″.

Pump Details:

- Type: single-acting triplex pump
- Pump factor of 6.0 gal/stroke at 100% volumetric efficiency

Neglect gas slip and fluid mixing. Estimate the volumetric efficiency of the pump. Assume uniform open hole size from surface to bottom.

*Solution:*

Pump factor = 6 × 0.0322 = 0.19358 bbl/stroke.
Total inside volume = 6.99 bbl.
Total outside volume = 66.88 bbl.
Total strokes needed for inside volume = 36.
Total strokes needed for outside volume = 345.
Total strokes = 36 + 345 = 381.
Volume efficiency = 85%.
Drill string capacity is

$$0.01422\frac{\text{bbl}}{\text{ft}}\times 8000\text{ ft} = 113.76\,\text{bbl}.$$

Annulus capacity is

$$0.05\frac{\text{bbl}}{\text{ft}}\times 8000\text{ ft} = 400\,\text{bbl}.$$

Drill string capacity + annulus capacity = 513.76 bbl.

$$\text{Number of strokes} = \frac{513.76}{0.1976717} = 2600\,\text{strokes}.$$

Total time required to displace the string and annulus =

$$\left(\frac{2600}{50}\right)\left(\frac{\text{stroke}}{\frac{\text{strokes}}{\text{min}}}\right) = 52\,\text{min}.$$

**Problem 2.34**

A 1,200 hp single-acting duplex pump has a volumetric efficiency of 90%. A delivery pump pressure of 3000 psi is required to drill the last one thousand feet of the hole. What liner size would you specify if the pump has a 20 in. stroke, a 2.5 in. rod diameter, and operates at 60 cycles/min?

*Solution:*

Actual flow rate is $Q_a = 1714 \times 1200/3000 = 685.6$ gpm.

With the volumetric efficiency of 90%, the theoretical flow rate can be given as $Q_t = 685.6/0.9 = 761.7$ gpm.

For triplex single-acting pump, the theoretical flow rate can be given as

$$Q_t = \frac{nN_cLD_L^2}{294} = 761.7 = \frac{60 \times 2 \times 20 \times D_L^2}{294}.$$

Solving for liner diameter,

$$D_L = \sqrt{\frac{761.7 \times 294}{60 \times 2 \times 20}} = 9.66 \approx 10\,in.$$

**Problem 2.35**

Determine the daily cost of running a diesel engine to power a duplex pump under the following conditions:

- Pump: single-acting duplex
- Stroke length = 18 inches
- Liner size = 8 inches

- Delivery pressure = 1000 psi at 40 spm
- Volumetric efficiency = 90%
- Mechanical efficiency = 85%
- Diesel engine: efficiency = 50%
- Diesel oil: weight = 7.2 lb/gal
- Heating value = 19000 BTU/lb
- Cost = $1.15/gal

*Solution:*

Consider the following:

Engine input (fuel energy) → engine output (input to pump)
→ mechanical pump output (input to hydraulic)
→ hydraulic output.

The theoretical flow rate for a single-acting duplex pump is

$$Q_t = \frac{\pi}{4} 8^2 \times 18 \times 2\left(\frac{7.48}{12^3}\right) \times 40 = 313.3 \,\text{gpm}.$$

The actual flow rate with the given volumetric efficiency of 90% is

$$Q_a = 313.3 \times 0.90 = 282 \text{ gpm}.$$

The hydraulic horsepower of the mud pump is

$$HHP_p = \frac{P_p Q}{1714} = \frac{1000 \times 282}{1714} = 164 \,\text{hp}.$$

The input horsepower of the pump can be calculated using the mechanical efficiency of 85%:

$$HP_m = \frac{164}{0.85} = 193.56 \,\text{hp}.$$

The engine horsepower is

$$H_{HP} = \frac{193.56}{0.50} = 387 \,\text{hp}.$$

The input power is expressed in terms of the rate of fuel consumption, $Q_f$, and the fuel heating value, $H$:

$$P_i = \frac{Q_f H}{2545} = \frac{Q_f \times 19{,}000}{2545}.$$

Hence,

$$Q_f = \left( \frac{387 \times 2545}{19000} \right) = 51.84\,\text{lb/hr},$$

or

$$Q_f = \frac{51.84\ \text{lb}}{\text{hr}} \times \frac{24\ \text{hr}}{\text{day}} \times \frac{\text{gal}}{7.2\ \text{lb}} = 172.8\,\text{gal day}.$$

Cost = 172.8 × 1.15 = \$198.7/day.

## 2.11 Offshore Vessels

Offshore vessels' sides and the types of vessel motions encountered are shown in Figure 2.4 and Figure 2.5, respectively.

Draft is the depth of the vessel in the water, whereas the freeboard is the distance above the water. They are shown in Figure 2.6.

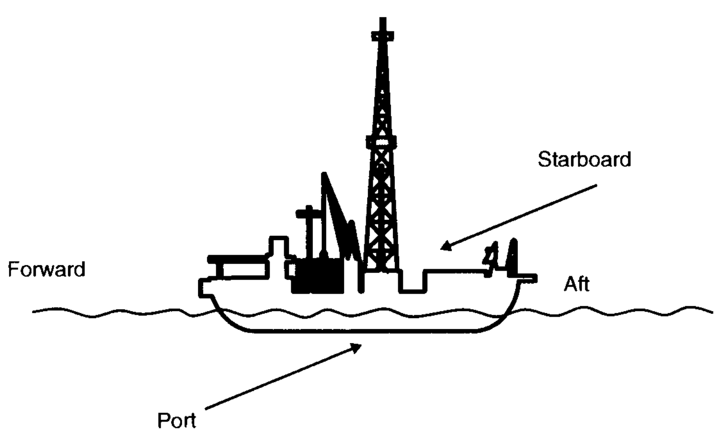



**Figure 2.4** Vessel sides.

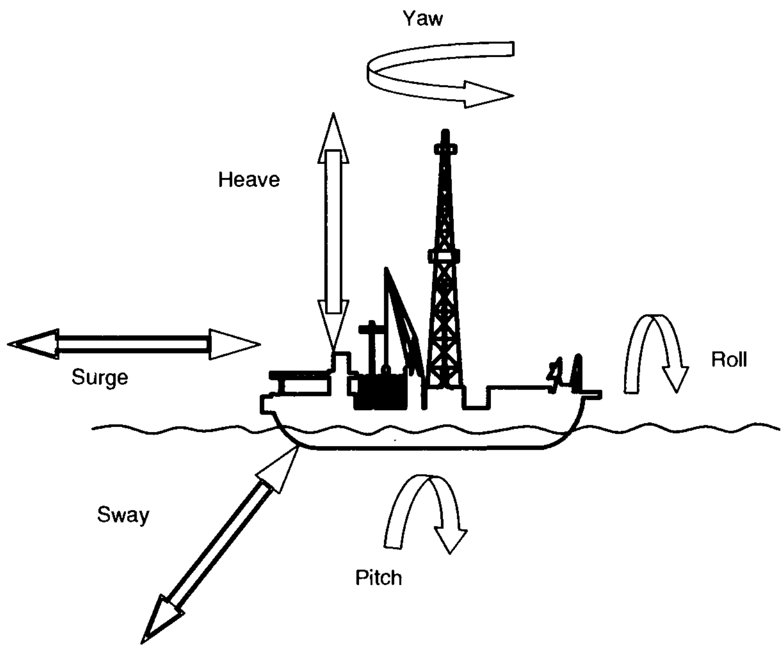



**Figure 2.5** Vessel motion.

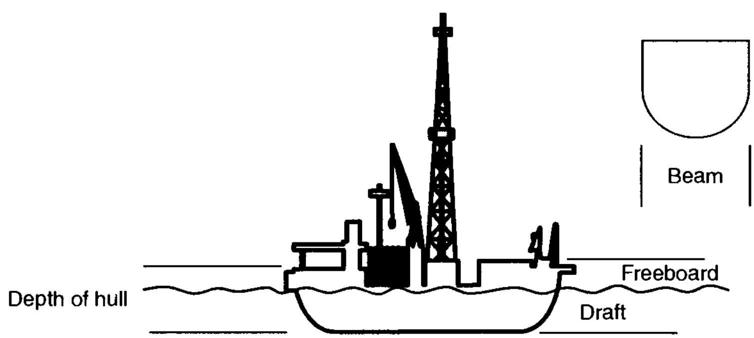



**Figure 2.6** Draft and freeboard.

## 2.11.1 Environmental Forces

Environmental forces are shown in Figure 2.7. These forces include the following:

- Wind force
- Wave force
- Current force

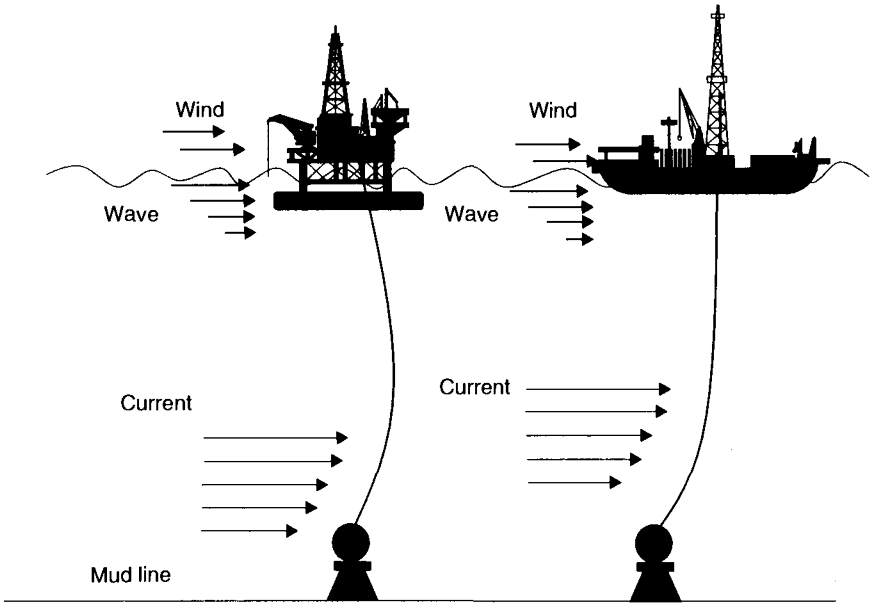



**Figure 2.7** Environmental forces.

Wind force is given by

$$F_w = 0.00338\, V_w^{\,2}\, C_h\, C_s\, A, \tag{2.42}$$

where $F_A$ = wind force, lbf, $V_A$ = wind velocity, knots, $C_s$ = shape coefficient, $C_h$ = height coefficient, and $A$ = projected area of all exposed surfaces, sq. ft.

Shape coefficients can be estimated from Table 2.2, and height coefficients can be estimated from Table 2.3.

Current force is calculated from

$$F_c = g_c\, V_c^2\, C_s\, A, \tag{2.43}$$

where $F_c$ = current drag force, lbf, $V_c$ = current velocity, ft/sec, $C_s$ = drag coefficient same as wind coefficient, and $A$ = projected area of all exposed surfaces, sq. ft.

The following are facts regarding the nautical mile:

- One nautical mile is one minute of latitude.
- A speed of one nautical mile per hour is termed the knot.

**Table 2.2** Shape coefficients.

| Shape | Cs |
|---|---|
| Cylindrical shapes | 0.5 |
| Hull (surface type) | 1.0 |
| Deck House | 1.0 |
| Isolated structural shapes (cranes, beams..etc) | 1.0 |
| Under Deck areas (smooth surfaces) | 1.0 |
| Under Deck areas (exposed beams, girders) | 1.3 |
| Rig derrick (each face) | 1.25 |

**Table 2.3** Height coefficients.

| From to (ft) | Ch |
|---|---|
| 0–50 | 1.0 |
| 50–100 | 1.1 |
| 100–150 | 1.2 |
| 150–200 | 1.3 |
| 200–250 | 1.37 |
| 250–300 | 1.43 |
| 300–350 | 1.48 |
| 350–400 | 1.52 |
| 400–450 | 1.56 |
| 450–500 | 1.6 |

- Determined by latitude (not longitude)
- 1 minute of latitude = 1 nautical mile – 6076 feet
- 1 degree of latitude = 60 NM

Wave force is calculated for various conditions.

Bow forces are under different conditions, depending on the wave period.

$$\text{If } A > 0.332\sqrt{L}, \tag{2.44}$$

$$F_{bow} = \frac{0.273 H^2 B^2 L}{T^4}. \tag{2.45}$$

$$\text{If } A < 0.332\sqrt{L},$$

$$F_{bow} = \frac{0.273 H^2 B^2 L}{\left(0.664\sqrt{L} - A\right)^4}, \tag{2.46}$$

where $F_{bow}$ = bow force, lbf, $A$ = wave period, sec, $L$ = vessel length, ft, $H$ = significant wave height, ft, and $B$ = vessel beam length, ft.

Beam forces are as follows:

$$\text{If } T > 0.642\sqrt{B + 2D},$$

$$F_{beam} = \frac{2.10 H^2 B^2 L}{A^4}. \tag{2.47}$$

$$\text{If } A < 0.642\sqrt{B + 2D},$$

$$F_{beam} = \frac{2.10 H^2 B^2 L}{\left(1.28\sqrt{B + 2D} - A\right)^4}, \tag{2.48}$$

where $F_{beam}$ = bow force, lbf, $A$ = wave period, sec, $L$ = vessel length, ft, $H$ = significant wave height, ft, $B$ = vessel beam length, ft, and $D$ = vessel draft, ft.

## 2.11.2 Riser Angle

The riser angle is measured relative to the vertical. Riser angles are measured with respect to the x and y axis, and the resultant riser angles follow.

The exact equation is

$$\theta = \tan^{-1}\sqrt{\tan^2\theta_x + \tan^2\theta_y}. \tag{2.49}$$

The following is an approximate equation:

$$\theta \cong \sqrt{\theta_x^2 + \theta_y^2}, \tag{2.50}$$

where $\theta_x$ = riser angle in x- direction, deg, $\theta_y$ = riser angle in y- direction, deg, and $\theta$ = resultant angle, deg.

**Problem 2.36**

Calculate the percentage error in using the approximate equation to calculate the resultant riser angle when the riser angle in x and y directions are 4° and 5°, respectively.

*Solution:*

Using exact equation 2.49,

$$\theta = \tan^{-1}\sqrt{\tan^2\theta_x + \tan^2\theta_y} = \tan^{-1}\sqrt{\tan^2 4 + \tan^2 5} = 6.39^o.$$

Using approximate equation 2.50,

$$\theta \cong \sqrt{\theta_x^2 + \theta_y^2} = \sqrt{4^2 + 5^2} = 6.40^o.$$

$$\text{Percentage error} = \frac{6.39^o - 6.40^o}{6.39^o} = 0.2\%.$$

**Problem 2.37**

With the following information regarding the floater, calculate the wind and current forces.

Floater details:

- Draft: 45 ft
- Freeboard: 50 ft
- Length: 450 ft
- Width: 80 ft

Substructure: 40 ft × 25 ft

Rig Derrick:

- Bottom section: 40 ft × 90 ft height and 20 ft width at the top
- Top section: 20 ft × 20 ft height and 10 ft width at the top

Heliport Truss: 200 $ft^2$ area of wind path
Wind velocity: 50 mph (towards port side perpendicular to floater)
Current velocity: 3 ft/sec(towards port side perpendicular to floater)

*Solution:*

$$\text{Rig wind force} = 0.00338 \times 43.45^2 \times 1.1 \times 1.25 \times \left[\frac{1}{2}(20+10)20 + \frac{1}{2}(40+20)90\right] = 26322\,\text{lbf.}$$

Helipad = $0.00338 \times 43.45^2 \times 1.1 \times 1.25 \times 200 = 1754$ lbf.
Hull = $0.00338 \times 43.45^2 \times 1 \times 1 \times 450 \times 50 = 143575$ lbf.
Total wind force = 180 kips.
Current force = $1 \times 1 \times 3^2 \times 450 \times 45 = 182250$ lbf (smooth surface assumed)

*Formulas and Calculations for Drilling Operations*
by G. Robello Samuel


# 3

# Well Path Design

This chapter focuses on different basic calculations involved in designing a well path as well as in monitoring well trajectory while the well is drilled.

## 3.1 Average Curvature – Average Dogleg Severity (DLS)

The equations commonly used to calculate the average curvature of a survey interval are

$$\bar{\kappa} = \sqrt{\left(\frac{\Delta\alpha}{\Delta L}\right)^2 + \left(\frac{\Delta\phi}{\Delta L}\right)^2 \sin^2\bar{\alpha}}, \tag{3.1}$$

and

$$\bar{\kappa} = \frac{\beta}{\Delta L}, \tag{3.2}$$

where $\alpha$ = inclination (°), $\phi$ = azimuth or direction (°), $\bar{\alpha}$ = average inclination angle (°), $\bar{\kappa}$= average borehole curvature, and $\beta = a\cos\left(\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\Delta\phi\right)$.

## 3.2 Vertical and Horizontal Curvatures

Vertical and horizontal curvatures can be calculated, respectively, using the following equations:

$$\kappa_V = \kappa_\alpha, \tag{3.3}$$

$$\kappa_V = \frac{d\alpha}{dL} = \kappa_\alpha;$$

$$\kappa_H = \frac{\kappa_\phi}{\sin\alpha}, \tag{3.4}$$

$$\kappa_H = \frac{d\phi}{dS} = \frac{d\phi}{dL\sin\alpha} = \frac{\kappa_\phi}{\sin\alpha}.$$

where $\kappa_V$ = the curvature of the wellbore trajectory in a vertical position plot, °/30 m or °/100 ft, $\kappa_H$ = the curvature of the wellbore trajectory in a horizontal projection plot, °/30 m or °/100 ft, $\Delta L$ = $L_2 - L_1$, $\Delta L$ = curved section length, m or ft, and S = arc length in the azimuthal direction, m or ft.

## 3.3 Borehole Curvature

The general formula for borehole curvature can be given as

$$\kappa = \sqrt{\kappa_\alpha{}^2 + \kappa_\phi{}^2 \sin^2\alpha}, \tag{3.5}$$

The general formula for borehole curvature with the vertical and horizontal curvatures can be given as

$$\kappa = \sqrt{\kappa_V{}^2 + \kappa_H{}^2 \sin^4\alpha}, \tag{3.6}$$

where $\kappa_V$ = the curvature of wellbore trajectory in a vertical expansion plot, °/30 m or °/100 ft, and $\kappa_H$ = the curvature of wellbore trajectory in a horizontal projection plot, °/30 m or °/100 ft.

## 3.3.1 Borehole Radius of Curvature

There are two ways of deducing the applied formula for borehole curvature and torsion. The corresponding curvature radius and torsion radii can be calculated as follows:

$$R = \frac{180 C_\kappa}{\pi \kappa}, \tag{3.7}$$

$$\rho = \frac{180 C_\kappa}{\pi \tau}, \tag{3.8}$$

where $C_\kappa$ = the constant related to the unit of borehole curvature.

If the unit for borehole curvature is °/30 m and °/30 ft, then $C_\kappa = 30$ and 100, respectively.

$\kappa$ = the curvature of wellbore trajectory, °/30 m, (°)/100 ft, and $\tau$ = the torsion of wellbore trajectory, °/30 m, (°)/100 ft.

**Problem 3.1**

Calculate the radius curvature for a build section with a build rate of 2°/100 ft.

*Solution:*

Using equation 3.7, the radius of curvature is

$$R = \frac{180 C_\kappa}{\pi \kappa},$$

where $C_\kappa = 100$.

With the given data, the radius of the curvature is

$$R = \frac{180 \times 100}{\pi \times 2} = 2{,}863.63 \text{ ft}.$$

**Problem 3.2**

Calculate the buildup rate of a build section with a radius of curvature of 2291.83 ft.

*Solution:*

Using equation 3.7, the radius of curvature is

$$R = \frac{180 C_\kappa}{\pi \kappa},$$

where $C_\kappa = 100$.

With the given data, the buildup rate is

$$R = \frac{180 C_\kappa}{\pi \kappa} = \frac{180 \times 100}{\pi \times 2291.83} = 2.5°/100\,\text{ft}.$$

## 3.4 Bending Angle

With the given inclination angles and directions, the borehole bending angle can be given as

$$\cos\beta = \cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\Delta\phi, \tag{3.9}$$

where $\Delta\phi = \phi_2 - \phi_1$, $\beta$ = bending angle, (°), $\Delta\phi$ = section increment of azimuth angle, (°), $\alpha_1$ = the inclination angle at survey point 1, (°), and $\alpha_2$ = the inclination angle at survey point 2, (°).

## 3.5 Tool Face Angle

With the given inclination angles and directions, the tool face rotation angle is

$$\gamma = arc\cos\left(\frac{\cos\alpha_1 \cos\beta - \cos\alpha_2}{\sin\alpha_2 \sin\beta}\right). \tag{3.10}$$

**Problem 3.3**

Calculate the dogleg severity (DLS) with the following survey data:

| Measured Depth (ft) | Angle (deg) | Direction (deg) |
|---|---|---|
| 15000 | 16.5 | 150 |
| 15045 | 17.5 | 148 |

*Solution:*

Using equation 3.1, DLS between the first two survey stations is

$$\bar{\kappa} = \sqrt{\left(\frac{17.5-16.5}{45}\right)^2 + \left(\frac{148-150}{45}\right)^2 \sin^2 \frac{16.5+17.5}{2}}$$

$$= 25.4 \text{ deg}/100 \text{ ft.}$$

**Problem 3.4**

Calculate the dogleg severity (DLS) for the following data:

| MD | Angle (deg) | Direction (deg) |
|---|---|---|
| 5000 ft | 5.5 | 150 |
| 5120 ft | 7.5 | 148 |

*Solution:*

Using equation 3.9, the overall angle change is given as

$$\beta = arc\cos(\cos 2 \sin 7.5 \sin 5.5 + \cos 5.5 \cos 7.5),$$

$$\beta = 2.01,$$

$$DLS = \frac{2.01}{120} \times 100 = 1.67° / 100 \text{ ft.}$$

**Problem 3.5**

Using the following data, calculate the hole curvature between the following survey stations:

| MD | Angle (deg) | Direction (deg) |
|---|---|---|
| 5100 | 3.5 | N 150 E |
| 5226 | 6.5 | N 250 E |

*Solution:*

Given: $\Delta\phi = 10°$

Using equation 3.9,

$$\beta = arc\cos\left(\cos\Delta\phi\sin\alpha_{n}\sin\alpha + \cos\alpha\cos\alpha_{n}\right),$$

$$\beta = arc\cos(\cos 10\sin 6.5\sin 3.5 + \cos 3.5\cos 6.5),$$

$$\beta = 2.47°/100.$$

**Problem 3.6**

A whipstock was set to change the direction from N85E to S88E. After drilling a sidetrack of 90 feet, it was found that the inclination has changed from 12 deg to 14.5 deg. Compute the dogleg severity.

*Solution:*

Azimuth change is from 85 deg (N85E) to 92 deg (S88E). Total azimuth change is 7 deg.

Using equation 3.9, the overall angle change is given as

$$\beta = arc\cos(\cos 7\sin 12\sin 14.5 + \cos 12\cos 14.5),$$

$$\beta = 2.97,$$

$$DLS = \frac{2.97}{90} \times 100 = 3.29°/100.$$

**Problem 3.7**

Calculate the toolface angle for the following data:

| MD | Angle (deg) | Direction (deg) |
|---|---|---|
| 5000 ft | 5.5 | 150 |
| 5120 ft | 7.5 | 148 |

*Solution:*

The bending angle $\beta$ can be calculated as shown in Problem 3.6 and is 2.01 deg/100 ft.

With other given values, $\alpha_1 = 5.5$ deg and $\alpha_2 = 7.5$ deg, and using equation 3.10, the toolface angle is

$$\gamma = arc\cos\left[\frac{(\cos 2.01\cos 5.5 - \cos 7.5)}{(\sin 7.5\sin 2.01)}\right] = 43.270°.$$

**Problem 3.8**

While drilling a directional well, a survey shows the original inclination of 32 deg and an azimuth of 112 deg. The inclination and directional curvatures are to be maintained at 4 deg/100 ft and 7 deg/100 ft, respectively, for a course length of 100 ft. Calculate the new inclination angle and final direction. Also, determine the curvature, vertical curvature, and horizontal walk curvatures.

*Solution:*

Given data: $\alpha_1$ = 32 deg; $\phi_2$= 112 deg; $\kappa_\alpha$ = 4 deg/100 ft; $\kappa_\phi$ =7 deg/100 ft; $\Delta L$ = 100 ft

The new inclination angle is

$$\alpha_2 = \alpha_1 + \kappa_\alpha (L_2 - L_1) = 32 + \frac{4}{100}\times 100 = 36°.$$

The new direction is

$$\phi_2 = \phi_1 + \kappa_\phi (L_2 - L_1) = 112 + \frac{7}{100}\times 100 = 119°.$$

The average inclination angle is

$$\bar{\alpha} = \frac{\alpha_1 + \alpha_2}{2} = \frac{32+36}{2} = 34°.$$

The average curvature is

$$\kappa = \sqrt{4^2 + 7^2 \sin^2 34} = 5.596°/100 \text{ ft}.$$

Using the bending or dogleg angle,

$$\cos\beta = \cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\Delta\phi,$$

$$\bar{\kappa} = \frac{arc\cos(\cos 32\cos 36 + \sin 32\sin 36\cos 7)}{100} = 5.591°/100 \text{ ft}.$$

It can be seen that a small difference is observed between the two calculation methods.

Vertical curvature is

$$\kappa_V = \kappa_\alpha = 4°/100 \text{ ft.}$$

Horizontal curvature is

$$\kappa_H = \frac{7}{\sin 34} = 12.518°/100 \text{ ft.}$$

## 3.6 Borehole Torsion

With the given inclination angles and directions, the borehole bending angle can be given as

$$\tau = \frac{\kappa_\alpha \dot{\kappa}_\phi - \kappa_\phi \dot{\kappa}_\alpha}{\kappa^2} \sin\alpha + \kappa_\phi \left(1 + \frac{\kappa_\alpha{}^2}{\kappa^2}\right) \cos\alpha \tag{3.11}$$

where $\tau$ = the torsion of wellbore trajectory, °/30m or °/100 ft, $\dot{\kappa}_\alpha$= the first derivative of inclination change rate, viz., the second derivative of inclination angle, and $\dot{\kappa}_\phi$ = the first derivative of azimuth change rate, viz., the second derivative of azimuth angle.

### 3.6.1 Borehole Torsion – Cylindrical Helical Method

When the wellbore curvature equals zero, the wellpath is a straight line that will result in zero torsion. When $\kappa \neq 0$, the torsion equation for the cylindrical helical model is

$$\tau = \kappa_H \left(1 + \frac{2\kappa_V{}^2}{\kappa^2}\right) \sin\alpha \cos\alpha. \tag{3.12}$$

**Problem 3.9**

The original hole inclination is 22°. To reach the limits of the target, it is desired to build an angle of 26° in a course length of 100 ft. The directional change is −5°. What is the resulting curvature and torsion that will achieve the desired objective?

*Solution:*

Given data: $\alpha_1 = 22°$; $\alpha_2 = 26°$; $\Delta\phi = -5°$; $\kappa_\alpha = 4°/100$ ft; $\kappa_\phi = 7°/100$ ft; $\Delta L = 100$ ft.

Using equation 3.3, the vertical curvature is

$$\kappa_\alpha = \frac{\alpha_2 - \alpha_1}{(L_2 - L_1)} = \frac{4}{100} \times 100 = 4°/100 \text{ ft.}$$

Using equation 3.4, the directional curvature is

$$\kappa_\phi = \frac{\phi_2 - \phi_1}{(L_2 - L_1)} = \frac{-10}{100} \times 100 = -10°/100 \text{ ft.}$$

The average inclination angle is

$$\bar{\alpha} = \frac{\alpha_1 + \alpha_2}{2} = \frac{22 + 26}{2} = 24°,$$

$$\kappa_\alpha = \kappa_V = 4°/100 \text{ ft.}$$

$$\kappa_H = \frac{\kappa_\phi}{\sin\alpha} = \frac{-10}{\sin 24} = -12.294°/100 \text{ ft.}$$

Using equation 3.16, the wellbore curvature can be calculated as

$$\kappa = \sqrt{\kappa_V{}^2 + \kappa_H{}^2 \sin^4\alpha} = 4.487(°)/100 \text{ ft.}$$

Using equation 3.12 to calculate the torsion,

$$\tau = -12.294\left(1 + \frac{2 \times 4^2}{4.487^2}\right)\sin 24 \cos 24 = -11.827°/100 \text{ ft.}$$

## 3.7 Wellpath Length Calculations

Circular arc:

$$\text{Arc length} = L_c = \frac{\alpha_2 - \alpha_2}{\text{BRA}} \tag{3.13}$$

where BRA = build rate angle (deg/100 ft).
Vertical distance is

$$V = R_b(\sin\alpha_2 - \sin\alpha_1), \tag{3.14}$$

where $R_b$ is the radius of the build, or

$$R_b = \frac{180}{\pi \times \text{BRA}}. \tag{3.15}$$

$$\text{Horizontal distance} = H = R_b(\cos\alpha_1 - \cos\alpha_2) \tag{3.16}$$

*Tangent section:*
Length of tangent = $L_t$.

$$\text{Vertical distance} = V = L_t \cos\alpha. \tag{3.17}$$

$$\text{Horizontal distance} = H = L_t \sin\alpha. \tag{3.18}$$

**Problem 3.10**

Determine the build rate radius given the following data for a build-and-hold pattern type well:
Build rate angle = 30 deg/100 ft

*Solution:*

The radius of the build is

$$R_b = \frac{1}{\text{BRA}}\left(\frac{180}{\pi}\right) = \frac{100}{3}\left(\frac{180}{\pi}\right) = 1910\,\text{ft}.$$

## 3.7.1 Wellpath Trajectory Calculations from Survey Data

### *3.7.1.1 Minimum Curvature Method*

The coordinates $x_i$, $y_i$, and $z_i$, at survey station $i$, which represent the west to east, south to north, and vertical depth, respectively, are given below:

$$\Delta N_i = \lambda_i(\sin\alpha_{i-1}\cos\phi_{i-1} + \sin\alpha_i\cos\phi_i), \tag{3.19}$$

$$\Delta E_i = \lambda_i \left(\sin\alpha_{i-1}\sin\phi_{i-1} + \sin\alpha_i \sin\phi_i\right), \tag{3.20}$$

$$\Delta H_i = \lambda_i \left(\cos\alpha_{i-1} + \cos\alpha_i\right). \tag{3.21}$$

where,

$$\lambda_i = \frac{180}{\pi}\frac{\Delta L_i}{\varepsilon_i}\tan\frac{\beta_i}{2}, \tag{3.22}$$

$$\cos\varepsilon_i = \cos\alpha_{i-1}\cos\alpha_i + \sin\alpha_{i-1}\sin\alpha_i\cos\Delta\phi_i. \tag{3.23}$$

$\beta_i$ = angle change between stations *i*, and *i–1* and is given by the following:

For the *i*-th measured section at the survey station *i*, the coordinate increment can be calculated and the coordinate of the next measured point also can be obtained. So,

$$N_i = N_{i-1} + \Delta N_i \tag{3.24}$$

$$E_i = E_{i-1} + \Delta E_i \tag{3.25}$$

$$H_i = H_{i-1} + \Delta H_i \tag{3.26}$$

### 3.7.1.2 *Radius of Curvature Method*

From survey data at two consecutive stations *i–1* and *i*, the coordinates can be determined by the following equations:

$$\Delta N_i = r_i \left(\sin\phi_i - \sin\phi_{i-1}\right), \tag{3.27}$$

$$\Delta E_i = r_i \left(\cos\phi_{i-1} - \cos\phi_i\right), \tag{3.28}$$

$$\Delta H_i = R_i \left(\sin\alpha_i - \sin\alpha_{i-1}\right), \tag{3.29}$$

where,

$$R_i = \frac{180}{\pi}\frac{\Delta L_i}{\Delta\alpha_i}, \quad r_i = \frac{180}{\pi}\frac{R_i}{\Delta\phi_i}\left(\cos\alpha_{i-1} - \cos\alpha_i\right), \tag{3.30}$$

$$\Delta a_i = \alpha_i - \alpha_{i-1}, \tag{3.31}$$

and where $R$ = the radius of curvature in the vertical plane, (°)/30 m or (°)/100 ft., and $r$ = the radius of curvature on the horizontal projection, (°)/30 m or (°)/100 ft.

Obviously, if either $\Delta\alpha_i$ or $\Delta\phi_i$ is zero, then it is not possible to calculate $R_i$ or $r_i$. Following are the general formulas that are applicable for all the cases:

$$\Delta H_i = \begin{cases} \Delta L_i \cos\alpha_i, & \text{if } \Delta\alpha_i = 0 \\ R_i(\sin\alpha_i - \sin\alpha_{i-1}), & \text{if } \Delta\alpha_i \neq 0 \end{cases}, \tag{3.32}$$

$$\Delta S_i = \begin{cases} \Delta L_i \sin\alpha_i, & \text{if } \Delta\alpha_i = 0 \\ R_i(\cos\alpha_{i-1} - \cos\alpha_i), & \text{if } \Delta\alpha_i \neq 0 \end{cases}, \tag{3.33}$$

$$\Delta N_i = \begin{cases} \Delta S_i \cos\phi_i, & \text{if } \Delta\phi_i = 0 \\ r_i(\sin\phi_i - \sin\phi_{i-1}), & \text{if } \Delta\phi_i \neq 0 \end{cases}, \tag{3.34}$$

$$\Delta E_i = \begin{cases} \Delta S_i \sin\phi_i, & \text{if } \Delta\phi_i = 0 \\ r_i(\cos\phi_{i-1} - \cos\phi_i), & \text{if } \Delta\phi_i \neq 0 \end{cases}, \tag{3.35}$$

where,

$$R_i = \frac{180}{\pi}\frac{\Delta L_i}{\Delta a_i}, \text{ and } r_i = \frac{180}{\pi}\frac{\Delta S_i}{\Delta\phi_i} \tag{3.36}$$

**Problem 3.11**

Data at two survey depths, 1015 ft and 1092.6 ft, are given below.

Depth 1015 ft: inclination 12°, azimuth 123°

Depth 1092.6 ft: inclination 14°, azimuth 119°

Calculate the incremental TVD, incremental northing, and incremental easting. Also, compute the dogleg severity.

*Solution:*

$$R_i = \frac{180}{\pi}\frac{(1092.6 - 1015)}{2} = 2223.07.$$

$$r_i = \frac{180}{\pi}\frac{2223.07}{4} = 250.03.$$

$$\Delta N_i = 250.03(\sin 119 - \sin 123) = 8.98 \text{ ft}.$$

$$\Delta E_i = 250.03(\cos 119 - \cos 123) = 14.96 \text{ ft}.$$

$$\Delta H_i = 2223.06(\sin 14 - \sin 12) = 75.6 \text{ ft}.$$

Alternatively, the displacements can be calculated as follows:

$$\begin{aligned}\Delta H_i &= \frac{\Delta MD_i\,(\sin\alpha_i - \sin\alpha_{i-1})}{\alpha_i - \alpha_{i-1}}\left(\frac{180}{\pi}\right)\\ &= \frac{(1092.6-1015)\,(\sin 14 - \sin 12)}{14-12}\left(\frac{180}{\pi}\right)\\ &= 78.53 \text{ ft},\end{aligned}$$

$$\begin{aligned}\Delta L_{(N/S)i} &= \frac{(1092.6-1015)\,(\cos 14 - \cos 12)\,(\sin 119 - \sin 123)}{(14-12)\,(123-119)}\left(\frac{180}{\pi}\right)^2\\ &= 9.33 \text{ ft},\end{aligned}$$

$$\begin{aligned}\Delta L_{(E/W)i} &= \frac{(1092.6-1015)\,(\cos 14 - \cos 12)\,(\cos 119 - \cos 123)}{(14-12)\,(123-119)}\left(\frac{180}{\pi}\right)^2\\ &= 15.53 \text{ ft}.\end{aligned}$$

The dogleg angle is

$$\beta = arc\cos(\cos 4 \sin 14 \sin 12 + \cos 12 \cos 12) = 2.19^\circ.$$

The dogleg severity is

$$\frac{\beta}{\Delta L_i}\times 100 = \frac{2.19}{(1092.6-1015)}\times 100 = 2.83 \text{ deg}/100 \text{ ft}$$

### 3.7.2 Natural Curve Method

The course coordinates are given by

$$\Delta N_i = \frac{1}{2}\left[F_C\left(A_{P,i}, \kappa_{P,i}, \Delta L_i\right) + F_C\left(A_{Q,i}, \kappa_{Q,i}, \Delta L_i\right)\right], \quad (3.37)$$

$$\Delta E_i = \frac{1}{2}\left[F_S\left(A_{P,i}, \kappa_{P,i}, \Delta L_i\right) - F_S\left(A_{Q,i}, \kappa_{Q,i}, \Delta L_i\right)\right], \quad (3.38)$$

$$\Delta H_i = F_S\left(\alpha_{i-1}, \kappa_{\alpha,i}, \Delta L_i\right), \quad (3.39)$$

where,

$$\begin{cases} \kappa_{\alpha,i} = \dfrac{\Delta\alpha_i}{\Delta L_i} \\ \kappa_{\phi,i} = \dfrac{\Delta\phi_i}{\Delta L_i} \end{cases} \quad (3.40)$$

$$\begin{cases} A_{P,i} = \alpha_{i-1} - \phi_{i-1} \\ A_{Q,i} = \alpha_{i-1} + \phi_{i-1} \end{cases} \quad (3.41)$$

$$\begin{cases} \kappa_{P,i} = \kappa_{\alpha,i} - \kappa_{\phi,i} \\ \kappa_{Q,i} = \kappa_{\alpha,i} + \kappa_{\phi,i} \end{cases} \quad (3.42)$$

$$F_C(\theta, \kappa, \lambda) = \begin{cases} \lambda \sin\theta & \text{if } \kappa = 0 \\ \dfrac{180}{\pi\kappa}\left[\cos\theta - \cos(\theta + \kappa\lambda)\right] & \text{if } \kappa \neq 0 \end{cases} \quad (3.43)$$

$$F_S(\theta, \kappa, \lambda) = \begin{cases} \lambda \cos\theta & \text{if } \kappa = 0 \\ \dfrac{180}{\pi\kappa}\left[\sin(\theta + \kappa\lambda) - \sin\theta\right] & \text{if } \kappa \neq 0 \end{cases} \quad (3.44)$$

### 3.7.3 Constant Tool Face Angle Method

Using the constant tool face angle method, course coordinates can be calculated using the following equations:

$$\Delta N_i = \begin{cases} R_i\left(\cos\alpha_{i-1} - \cos\alpha_i\right)\cos\phi_i, & \text{when } \alpha_{i-1} = 0 \text{ or } \alpha_i = 0 \\ r_i \sin\alpha_i\left(\sin\phi_i - \sin\phi_{i-1}\right), & \text{when } \Delta\alpha_i = 0 \\ \int_{L_1}^{L_2} \sin\alpha(L)\cos\phi(L)dL, & \text{for rest of the conditions} \end{cases} \tag{3.45}$$

$$\Delta E_i = \begin{cases} R_i\left(\cos\alpha_{i-1} - \cos\alpha_i\right)\sin\phi_i, & \text{if } \alpha_{i-1} = 0 \text{ or } \alpha_i = 0 \\ r_i \sin\alpha_i\left(\cos\phi_{i-1} - \cos\phi_i\right), & \text{if } \Delta\alpha_i = 0 \\ \int_{L_1}^{L_2} \sin\alpha(L)\sin\phi(L)\,dL, & \text{for rest of the conditions} \end{cases} \tag{3.46}$$

$$\Delta H_i = \begin{cases} \Delta L_i \cos\alpha_i, & \text{if } \Delta\alpha_i = 0 \\ R_i\left(\sin\alpha_i - \sin\alpha_{i-1}\right), & \text{if } \Delta\alpha_i \neq 0' \end{cases} \tag{3.47}$$

where,

$$R_i = \frac{180}{\pi}\frac{\Delta L_i}{\Delta\alpha_i}, \text{ and } r_i = \frac{180}{\pi}\frac{\Delta L_i}{\Delta\phi_i} \tag{3.48}$$

$$\tan\omega_i = \frac{\pi}{180}\frac{\Delta\phi_i}{\ln\dfrac{\tan\frac{\alpha_i}{2}}{\tan\frac{\alpha_{i-1}}{2}}}, \tag{3.49}$$

$$\alpha(L) = \alpha_{i-1} + \frac{\Delta\alpha_i}{\Delta L_i}\left(L - L_{i-1}\right), \tag{3.50}$$

and

$$\phi(L) = \phi_{i-1} + \frac{180}{\pi}\tan\omega_i \cdot \ln\frac{\tan\frac{\alpha(L)}{2}}{\tan\frac{\alpha_{i-1}}{2}}. \tag{3.51}$$

**Problem 3.12**

Original hole inclination is 22°. To reach the limits of the target, it is desired to build an angle of 26° in a course length of 100 ft. The directional change is –5°. What is the resulting curvature and torsion that will achieve the desired objective?

*Solution:*

Given data: $\alpha_1 = 22°$; $\alpha_2 = 26°$; $\Delta\phi = -5°$; $\kappa_\alpha = 4°/100$ ft; $\kappa_\phi = 7°/100$ ft; and $\Delta L = 100$ ft

The vertical curvature is

$$\kappa_\alpha = \frac{\alpha_2 - \alpha_1}{(L_2 - L_1)} = \frac{4}{100} \times 100 = 4°/100 \text{ ft.}$$

The horizontal curvature is

$$\kappa_\phi = \frac{\phi_2 - \phi_1}{(L_2 - L_1)} = \frac{-10}{100} \times 100 = -10°/100 \text{ ft.}$$

The average inclination angle is

$$\bar{\alpha} = \frac{\alpha_1 + \alpha_2}{2} = \frac{22 + 26}{2} = 24.$$

Average curvature is

$$\kappa = \sqrt{4^2 + (-10)^2 \sin^2 24} = 4.487°/100 \text{ ft.}$$

Using equation 3.11 to calculate the torsion,

$$\tau = -10\left(1 + \frac{4^2}{4.487^2}\right)\cos 24 = -8.197°/100 \text{ ft.}$$

**Problem 3.13**

Calculate the course coordinates with the following data:

- Initial direction and azimuth are 35° and 182°, respectively.
- $\kappa_\alpha = 40/100$ ft.
- $\kappa_\phi = 00/100$ ft.
- The tangent length = 200 ft.

*Solution:*

The new angle and azimuth are

$$\alpha = 35 + \frac{4}{100} \times 200 = 43°$$

and

$$\phi = 182 + \frac{0}{100} \times 200 = 182°.$$

$$\kappa_V = 4°/100\,\text{ft.}$$

$$\kappa_H = \frac{0}{\sin 43} = 0\ \text{deg}/100\ \text{ft.}$$

$$\kappa = \sqrt{4^2 + 0} = 4°/100\ \text{ft.}$$

Calculating the intermediate variables,

$$\begin{cases} A_P = 35 + 182 = 217 \\ A_Q = 35 - 182 = -147 \end{cases} \text{and} \begin{cases} \kappa_P = 4 + 0 = 4\ \text{deg}/100\ \text{ft} \\ \kappa_Q = 4 - 0 = 4\ \text{deg}/100\ \text{ft}. \end{cases}$$

$$\begin{aligned} F_S(217,4,200) &= \frac{180 \times 100}{\pi \times 4}\left[\sin\left(217 + \frac{4}{100} \times 200\right) - \sin 217\right] \\ &= -150.8\ \text{ft.} \end{aligned}$$

$$\begin{aligned} F_c(217,4,200) &= \frac{180 \times 100}{\pi \times 4}\left[\cos 217 - \sin\left(217 + \frac{4}{100} \times 200\right)\right] \\ &= -131\ \text{ft.} \end{aligned}$$

$$\begin{aligned} &F_S(-147,4,200) \\ &= \frac{180 \times 100}{\pi \times 4}\left[\sin\left(-147 + \frac{4}{100} \times 200\right) - \sin(-147)\right] \\ &= -159.6\ \text{ft.} \end{aligned}$$

$$F_c(-147,4,200)$$
$$= \frac{180\times100}{\pi\times4}\left[\cos(-147)-\sin\left(-147+\frac{4}{100}\times200\right)\right] = -120.26 \text{ ft.}$$

$$F_S(35,4,200) = \frac{180\times100}{\pi\times4}\left[\sin\left(35+\frac{4}{100}\times200\right)-\sin(35)\right]$$
$$= 155.3 \text{ ft.}$$

$$F_c(35,4,200) = \frac{180\times100}{\pi\times4}\left[\cos(35)-\sin\left(35+\frac{4}{100}\times200\right)\right]$$
$$= 125.76 \text{ ft.}$$

The course coordinates are

$$\Delta N = \frac{1}{2}\left[F_C(217,4,200)+F_C(-147,4,200)\right] = -125.68 \text{ ft,}$$

$$\Delta E = \frac{1}{2}\left[F_S(-147,4,200)-F_S(-147,4,200)\right] = -4.4 \text{ ft,}$$

$$\Delta H = F_S(50,4,200) = 155 \text{ ft,}$$

and

$$\Delta S = F_C(50,4,200) = 125.76 \text{ ft.}$$

**Problem 3.14**

Using SI units, Problem 3.13 is worked out below:

The new angle and azimuth are

$$\alpha = 35+\frac{4}{30.48}\times60.96 = 43°$$

and

$$\phi = 182+\frac{0}{100}\times200 = 182.$$

Therefore,

$$\kappa = \sqrt{4^2 + 0} = 4°/30.48 \text{ m}.$$

Calculating the intermediate variables,

$$\begin{cases} A_P = 35 + 182 = 217° \\ A_Q = 35 - 182 = -147° \end{cases} \text{and} \begin{cases} \kappa_P = 4 + 0 = 4 \text{ deg}/30.48 \text{ m} \\ \kappa_Q = 4 - 0 = 4 \text{ deg}/30.48 \text{ m} \end{cases}$$

$$F_S(217, 4, 60.96)$$
$$= \frac{180 \times 30.48}{\pi \times 4}\left[\sin\left(217 + \frac{4}{30.48} \times 60.96\right) - \sin 217\right] = -45.9 \text{ m}.$$

$$F_c(217, 4, 60.96)$$
$$= \frac{180 \times 30.48}{\pi \times 4}\left[\cos 217 - \sin\left(217 + \frac{4}{30.48} \times 60.96\right)\right] = -39.96 \text{ m}.$$

$$F_S(-147, 4, 60.96)$$
$$= \frac{180 \times 30.48}{\pi \times 4}\left[\sin\left(-147 + \frac{4}{30.48} \times 60.96\right) - \sin(-147)\right]$$
$$= -48.64 \text{ m}.$$

$$F_c(-147, 4, 60.96)$$
$$= \frac{180 \times 30.48}{\pi \times 4}\left[\cos(-147) - \sin\left(-147 + \frac{4}{30.48} \times 60.96\right)\right]$$
$$= -36.65 \text{ m}.$$

$$F_S(35, 4, 60.96)$$
$$= \frac{180 \times 30.48}{\pi \times 4}\left[\sin\left(35 + \frac{4}{30.48} \times 60.96\right) - \sin(35)\right]$$
$$= 47.33 \text{ m}.$$

$$F_c(35,4,60.96) = \frac{180 \times 30.48}{\pi \times 4}\left[\cos(35) - \sin\left(35 + \frac{4}{30.48} \times 60.96\right)\right] = 38.33 \text{ m}.$$

The course coordinates are

$$\Delta N = \frac{1}{2}\left[F_C(217,4,60.96) + F_C(-147,4,60.96)\right] = -38.30 \text{ m},$$

$$\Delta E = \frac{1}{2}\left[F_S(-147,4,60.96) - F_S(-147,4,200)60.96\right] = -1.33 \text{ m},$$

$$\Delta H = F_S(35,4,60.96) = 47.33 \text{ m},$$

and

$$\Delta S = F_C(35,4,60.96) = 38.33 \text{ m}.$$

## 3.8 Types of Designs

Different types of wellpath designs are shown in Figure 3.1.

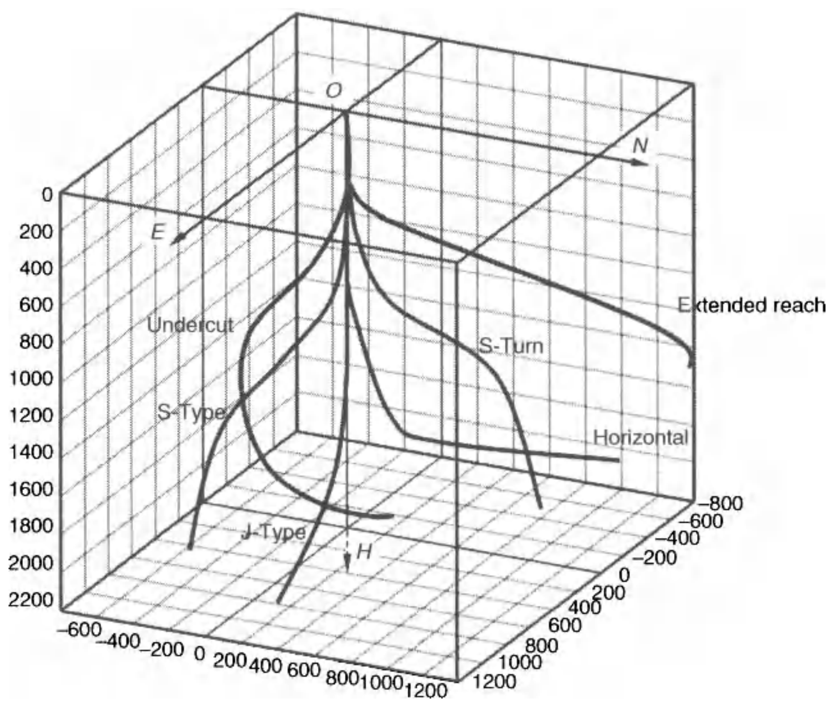



**Figure 3.1** Wellpath designs.

## 3.9 Tool Face Angle Change

If the inclination $\alpha$, new inclination $\alpha_n$, and the dogleg severity $\beta$ are known, then the toolface angle can be calculated using the following relationship:

$$\gamma = arc\cos\left(\frac{\cos\alpha\cos\beta - \cos\alpha_n}{\sin\alpha\sin\beta}\right). \tag{3.52}$$

If the new inclination $\alpha_n$, azimuth change $\Delta\phi$, and the dogleg severity $\beta$ are known, then the toolface angle can be calculated using the following relationship:

$$\gamma = arc\sin\left(\frac{\sin\alpha_n\sin\Delta\phi}{\sin\beta}\right). \tag{3.53}$$

If the inclination $\alpha$, the dogleg severity $\beta$, and the toolface angle $\gamma$ are known, the new inclination angle can be calculated using the following relationship:

$$\alpha_n = arc\cos\left(\cos\alpha\cos\beta - \sin\beta\sin\alpha\cos\gamma\right). \tag{3.54}$$

If the inclination $\alpha$, the dogleg severity $\beta$, and the toolface angle $\gamma$ are known, the change in the azimuth can be calculated using the following relationship:

$$\Delta\phi = arc\tan\left(\frac{\tan\beta\sin\gamma}{\sin\alpha + \tan\beta\cos\alpha\cos\gamma}\right) \tag{3.55}$$

**Problem 3.15**

Derive an equation to find the overall angle change in terms of $\Delta\phi$, $\alpha$, and $\gamma$, the toolface rotation angle.

*Solution:*

Starting from the azimuth change,

$$\Delta\phi = arc\tan\frac{\tan\beta\sin\gamma}{\sin\alpha + \tan\beta\cos\alpha\cos\gamma},$$

$$\tan\Delta\phi = \frac{\tan\beta\sin\gamma}{\sin\alpha + \tan\beta\cos\alpha\cos\gamma}.$$

Rearranging the equation,

$$\tan\Delta\phi\sin\alpha+\tan\Delta\phi\tan\beta\cos\alpha\cos\gamma-\tan\beta\sin\gamma=0,$$

$$\tan\Delta\phi\sin\alpha+\tan\beta\left(\tan\Delta\phi\cos\alpha\cos\gamma-\sin\gamma\right)=0.$$

Solving the above equation to get the overall angle change $\beta$,

$$\beta=arc\tan\frac{\tan\Delta\phi\sin\alpha}{\sin\gamma-\tan\Delta\phi\cos\alpha\cos\gamma}.$$

**Problem 3.16**

A drilling engineer plans to kick off the well from the surface with an initial holding inclination of $\alpha_1$. The complete path planning parameters and the well configuration (straight inclined build-hold-drop-hold) are depicted in Figure 3.2.

Prove that $\tan\frac{\alpha_3}{2}=\frac{H_0-\sqrt{H_0^{\,2}+A_0^{\,2}-R_0^{\,2}}}{R_0-A_0}$.

Also find out the condition at which $\alpha_3=2\tan^{-1}\left(\frac{A_0}{2H_0}\right)$.

*Solution:*

From the figure, it can be written as $R_0=R_2+R_4$.
The following relationship can be obtained:

$$A_0=\Delta L_3\sin\alpha_3+R_0\left(1-\cos\alpha_3\right)$$
$$H_0=\Delta L_3\cos\alpha_3+R_0\sin\alpha_3$$

Multiplying the second equation by $\cos\alpha_3$ and the third equation by $\sin\alpha_3$ and some manipulation will result in $H_0\sin\alpha_3+\left(R_0-A_0\right)\cos\alpha_3=R_0$.

Using the identity $\sin\alpha_3=\dfrac{2\tan\frac{\alpha_3}{2}}{1+\tan^2\frac{\alpha_3}{2}}$, $\cos\alpha_3=\dfrac{1-\tan^2\frac{\alpha_3}{2}}{1+\tan^2\frac{\alpha_3}{2}}$,

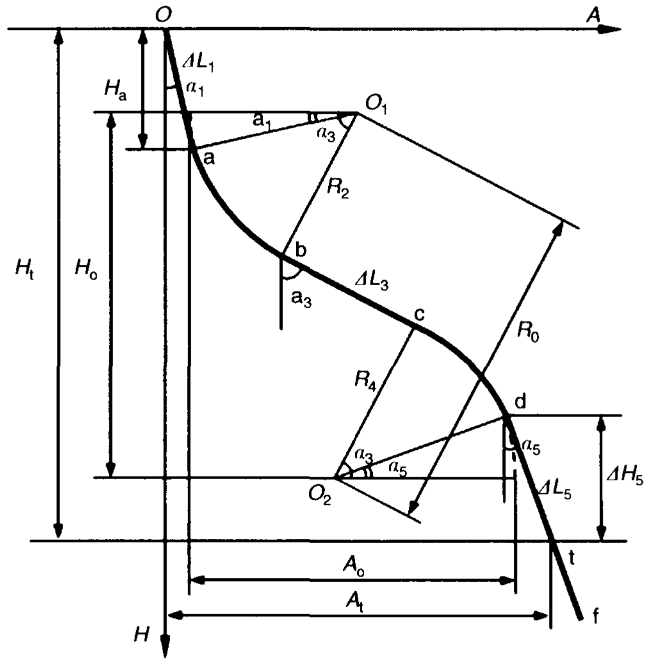



**Figure 3.2** Wellpath design for Problem 3.16.

$$(2R_0 - A_0)\tan^2\frac{\alpha_3}{2} - 2H_0\tan\frac{\alpha_3}{2} + A_0 = 0.$$

Solving the quadratic equation and using the positive root will result in

$$\tan\frac{\alpha_3}{2} = \frac{H_0 \pm \sqrt{H_0^2 + A_0^2 - 2R_0A_0}}{2R_0 - A_0}.$$

In the result above, if $(2R_0 - A_0) = 0$, then

$$\tan\frac{\alpha_3}{2} = \frac{A_0}{2H_0},$$

i.e., $2R_0 = A_0$.

Based on the equations and condition derived, similar derivations can be done with the condition:

$$\begin{cases} \Delta L_3\cos\alpha_3 + R_0\sin\alpha_3 = H_0 \\ \Delta L_3\sin\alpha_3 - R_0\cos\alpha_3 = A_0 \end{cases},$$

$$\tan\frac{\alpha_3}{2} = \frac{H_0 - \sqrt{H_0^{\,2} + A_0^{\,2} - R_0^{\,2}}}{R_0 - A_0},$$

Therefore,

$$\alpha_3 = 2\tan^{-1}\left(\frac{A_0}{2H_0}\right).$$

**Problem 3.17**

A sidetracking was planned using a whipstock after a futile attempt to retrieve a fish. The survey data at the planned sidetrack depth are an inclination angle of 10° and an azimuth of N25W. The whipstock selected will produce a total angle change of 4°/100 ft. The new angle desired is 11°. Calculate the toolface setting of the whipstock with reference to the high side of the wellbore for a course length of 30 ft. Also, calculate the new direction.

*Solution:*

Using the dogleg severity, the build angle is

$$\beta = \frac{4}{100}30 = 1.2°.$$

The toolface angle can be calculated using equation 3.44:

$$\gamma = arc\cos\left(\frac{\cos 10 \cos 1.2 - \cos 11}{\sin 10 \sin 1.2}\right) 35.4°.$$

The new direction is calculated using equation 3.47:

$$\begin{aligned}\Delta\phi &= 335° + arc\tan\left(\frac{\tan 1.2 \sin 35.4}{\sin 10 + \tan 1.2 \cos 10 \cos 35.4}\right) \\ &= 338.6° \text{ (N21.6W)}.\end{aligned}$$

**Problem 3.18**

Survey was taken at a depth of 4100 ft and found to be 5 deg inclination and direction N15W. The directional driller desires to make a course correction using a bent sub and a motor. The bent sub is expected to result in an angle change of 3 deg per 100 ft. The tool face setting is 35 deg to the right from the high side. Calculate the new direction and the angle at 4190 ft. What would be the bent sub angle response needed if the final inclination angle is 7 deg.

*Solution:*

$$\beta = \frac{3}{100} \times 90 = 2.7°.$$

The azimuth change is

$$\Delta\phi = 345° + arc\tan\left(\frac{\tan 2.7 \sin 35}{\sin 5 + \tan 2.7 \cos 5 \cos 35}\right)$$
$$= 357.2°\,(\text{N}2.8\text{W}),$$

$$\alpha_n = arc\cos(\cos 5 \cos 2.7 - \sin 2.7 \sin 5 \cos 35) = 7.38,$$

$$\delta = \frac{arc\cos(\cos 12.2 \sin 7.38 \sin 5 + \cos 5 \cos 7.38)}{90} \times 100$$
$$= 2.6°/100\text{ ft}.$$

Also, you can find it iteratively.

## 3.10 Horizontal Displacement

On the horizontal plane, the displacement of the target point can be calculated as

$$H_t = \sqrt{(N_t - N_o)^2 + (E_t - E_o)^2}, \tag{3.56}$$

where $N_t$ = northing of target, ft, $N_o$ = northing of slot, ft, $E_t$ = easting of target, ft, and $E_o$ = easting of slot, ft.

Target bearing is given as

$$\varphi_t = \tan^{-1}\left(\frac{E_t - E_o}{N_t - N_o}\right). \tag{3.57}$$

**Problem 3.19**

With the slot coordinates 1200.5 ft N, 700 ft E and target coordinates 3800 ft N, 4520 ft E, calculate the horizontal displacement of the well. Also, calculate the target bearing.

*Solution:*

Given: $N_t = 3800$ ft, $N_o = 1200.5$ ft, $E_t = 4520$ ft, and 700 ft

Using equation 3.56, the horizontal displacement is

$$H_t = \sqrt{(3800 - 1200.5)^2 + (4520 - 700)^2} = 4620.6\,\text{ft}.$$

Using equation 3.57, the target bearing is

$$\varphi_t = \tan^{-1}\left(\frac{4520 - 700}{3800 - 1200.5}\right) = 55.76°, \text{ or N55.76E}.$$

**Problem 3.20**

A well will be drilled in a 500-acre lease land. The surface location is pegged at 1000 ft from the north line and 2200 ft from the east line. The target location is 500 ft from the south line and 300 ft from the east line. Estimate the horizontal departure.

*Solution:*

The surface and the bottom location can be shown in the Figure 3.3

*Solution:*

1 acre = 43,560 sq. ft

Therefore, the total lease area = 500(acre) × 43560 (sq. ft/acre) = 21780000 sq. ft.

Assuming a square lease area, the side of the lease area is $\sqrt{21780000} = 4666.9$ ft.

Using equation 3.51, the horizontal displacement is

$$H_t = \sqrt{(4666.9 - 1000 - 500)^2 + (22200 - 300)^2} = 3693.14 \text{ ft}.$$

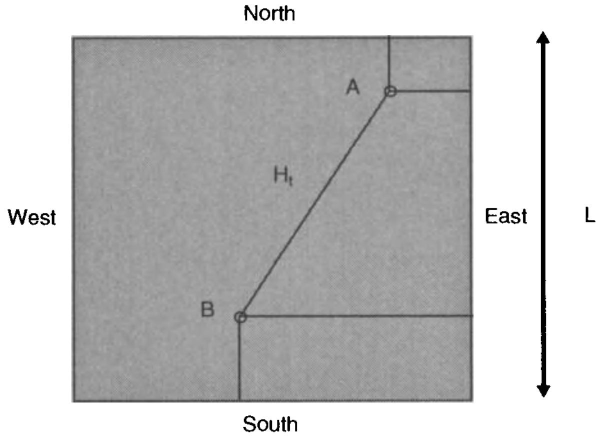



**Figure 3.3** Surface location for Problem 3.20.

## 3.11 Tortuosity

Wellbore tortuosity is given as

$$T = \frac{\sum_{i=1}^{m} \alpha_{n-1} + \Delta D \times \beta_i}{D_i - D_{i-1}}. \tag{3.58}$$

### 3.11.1 Absolute and Relative Tortuosity

$$\Gamma_{(abs)_n} = \left( \frac{\sum_{i=1}^{i=n} \alpha_{adj}}{D_n + \Delta D_n} \right), \tag{3.59}$$

where, $\alpha_{adj} = \alpha_i + \Delta D_i \times \beta_i$ and is the dogleg adjusted, summed total inclination angle.

$$\Gamma_{(rel)_n} = \Gamma^{tor}_{(abs)_n} - \Gamma^{notor}_{(abs)_n} \text{deg}/100\text{ ft}. \tag{3.60}$$

The period of the sine wave should not be of $\frac{2}{n}(n = 1,2,3...)$.

Different methods include 1) the sine wave method, 2) the helical method, 3) random inclination and the azimuth method, and 4) the random inclination dependent azimuth method.

## 3.11.2 Sine Wave Method

$$\Delta\alpha = \sin\left(\frac{D}{P}\times 2\pi\right)\times M \tag{3.61}$$

where $D$ = measured depth (ft), $P$ = period, and $M$ = magnitude.

If the measured depth of the survey point is an exact integer multiple of the period, then

$$\Delta\alpha = \sin\left(\frac{MD}{P}2\pi\right) = 0.$$

In addition, the inclination angle is modified so that it does not become less than zero, since negative inclination angles are not allowed.

If the $\Delta\alpha$ is negative and $\phi_n = \phi + \Delta\alpha$ is negative, then $\phi_n = 360 + \Delta\alpha$ :

$$\alpha_n = \alpha + \Delta a, \tag{3.62}$$

$$\phi_n = \phi + \Delta\alpha + \psi_{xvc}, \tag{3.63}$$

where $\psi_{xvc}$ = cross vertical correction.

If $\alpha_n < 0$, then cross vertical correction = 180°.

$$\alpha_n = |\alpha_n|.$$

If $\alpha_n \geq 0$, then cross vertical correction = 0°

**Problem 3.21**

Measured depth = 3900 ft, Inclination = 5.15°, and Azimuth = 166°.

Measured depth = 3725 ft, Inclination = 3.25°, and Azimuth = 165°.

Calculate the tortuosity using the sine wave method.

*Solution:*

$$\Delta\alpha = \sin\left(\frac{3725}{1000}2\pi\right)\times 1 = -0.99°$$

$$\phi_n = 165 - 0.99 + 0 = 164.01°.$$

$$\alpha_n = 3.25 - 0.99 = 2.26°.$$

If $\alpha_n < 0$, then cross vertical correction = 180°.

$$\alpha_n = |\alpha_n|.$$

If $\alpha_n \geq 0$, then cross vertical correction = 0°.

### 3.11.3 Helical Method

The helical method modifies the inclination and azimuth of the survey points by superimposing a helix along the wellbore path using the magnitude (radius of the cylinder in the parametric equation) and period (pitch) specified. This method uses the circular helix defined as $f(u) = a\cos(u) + a\sin(u) + bu$.

The generalized parametric set of equations for the helix used to superimpose the wellbore path is given by

$$\begin{cases} x(u) = M\cos(u) \\ y(u) = M\sin(u). \\ z(u) = \dfrac{P}{2\pi}u \end{cases} \tag{3.64}$$

### 3.11.4 Random Inclination Azimuth Method

$$\alpha_n = \alpha + \Delta\alpha.$$

$$\phi_n = \phi + \Delta\alpha + \psi_{cvc}.$$

$$\psi_{cvc} = \text{cross-vertical correction}.$$

**Problem 3.22**

Measured depth = 3725 ft, Inclination = 3.25°, and Azimuth = 165°.
Measured depth = 3900 ft, Inclination = 5.15°, and Azimuth = 166°.
$\zeta$ (Rand number) = 0.375.

Calculate the tortuosity using the random inclination azimuth method.

*Solution:*

$$\Delta\alpha = \frac{0.375 \times (3900 - 3725)}{100} \times 1 = 0.66^\circ.$$

$$\alpha_n = 5.15 + 0.66 = 5.81^\circ$$

$$\phi_n = 166 + 0.66 + 0 = 166.66^\circ.$$

### 3.11.5 Random Inclination Dependent Azimuth Method

$$\Delta\alpha = \zeta \times \delta, \tag{3.65}$$

$$\delta = \frac{\Delta MD}{P} M, \tag{3.66}$$

where $\zeta$ = random number, $\alpha_n = \alpha + \Delta\alpha$, and

$$\phi_n = \phi + \frac{\Delta\alpha}{2\sin\alpha_n} + \psi_{cvc}. \tag{3.67}$$

**Problem 3.23**

Measured depth = 3725 ft, Incinination = 3.25°, and Azimuth = 165°.
Measured depth = 3900 ft, Inclination = 5.15°, and Azimuth = 166°.
Random number = 0.375.

Calculate the tortuosity using the random inclination dependent azimuth method.

*Solution:*

$$\Delta\alpha = \frac{0.375 \times (3900 - 3725)}{100} \times 1 = 0.66^\circ.$$

$$\alpha_n = 5.15 + 0.66 = 5.81°.$$

$$\phi_n = 166 + \frac{0.66}{2\sin 5.81} + 0 \ = \ 169.26°.$$

## 3.12 Well Profile Energy

Wellpath profile energy between survey stations can be given as

$$E_{(abs)_n} = \left( \frac{\sum_{i=1}^{n} \left(\kappa_i^2 + \tau_i^2\right)\Delta D_i}{D_n + \Delta D_n} \right) \tag{3.68}$$

To quantify the change of the well trajectory after applying the artificial tortuosity, a relative energy is defined, which is the energy of the wellbore relative to the absolute energy and is given below:

$$E_{s(rel)_n} = E^{tor}_{s(abs)_n} - E^{notor}_{s(abs)_n}. \tag{3.69}$$

**Table 3.1** Data for Problem 3.24.

| ***L*, *ft*** | **$\alpha$, deg.** | **$\phi$, deg.** |
|---|---|---|
| 13200 | 59.52 | 224.36 |
| 14400 | 59.71 | 224.55 |
| 15600 | 60.29 | 225.13 |
| 16200 | 59.52 | 224.36 |
| 17400 | 59.71 | 224.55 |
| 18600 | 60.29 | 225.13 |
| 19800 | 60.48 | 225.32 |
| 20000 | 60 | 224.84 |

**Table 3.2** Survey results with absolute and relative tortusosities.

| *L*, ft | *α*, deg. | *ϕ*, deg. | *H*, ft | *DLS*, (°/100ft) | *Abs.Tort*, (°/100ft) | *Rel. Tort*, (°/100ft) |
|---|---|---|---|---|---|---|
| 13200 | 59.52 | 224.36 | 9842.2 | 0.16 | 0.47 | 0.01 |
| 14400 | 59.71 | 224.55 | 10443.5 | 0.31 | 0.45 | 0.03 |
| 15600 | 60.29 | 225.13 | 11043.5 | 0.36 | 0.44 | 0.05 |
| 16200 | 59.52 | 224.36 | 11342.2 | 0.16 | 0.43 | 0.06 |
| 17400 | 59.71 | 224.55 | 11943.5 | 0.31 | 0.42 | 0.07 |
| 18600 | 60.29 | 225.13 | 12543.5 | 0.36 | 0.41 | 0.09 |
| 19800 | 60.48 | 225.32 | 13142.1 | 0.09 | 0.4 | 0.1 |
| 20000 | 60 | 224.84 | 13241.3 | 0.41 | 0.4 | 0.1 |

**Problem 3.24**

With data given in Table 3.1, calculate the absolute and relative tortuosities:

Tortuosity magnitude = 0.25°.

*Solution:*

Table 3.2 shows the calculated results of the absolute and relative tortuosities at various survey points.

## 3.13 Magnetic Reference and Interference

Declination (Fig. 3.4) is the angular difference in azimuth readings between magnetic north and true north. Magnetic declination is positive when magnetic north lies east of true north, and it is negative when magnetic north lies west of true north. It is actually the error between the true north and magnetic north for a specific location.

Azimuth correction can be given as:

$$\text{Azimuth(true)} = \text{Azimuth (magnetic)} + \text{Magnetic declination.} \tag{3.70}$$

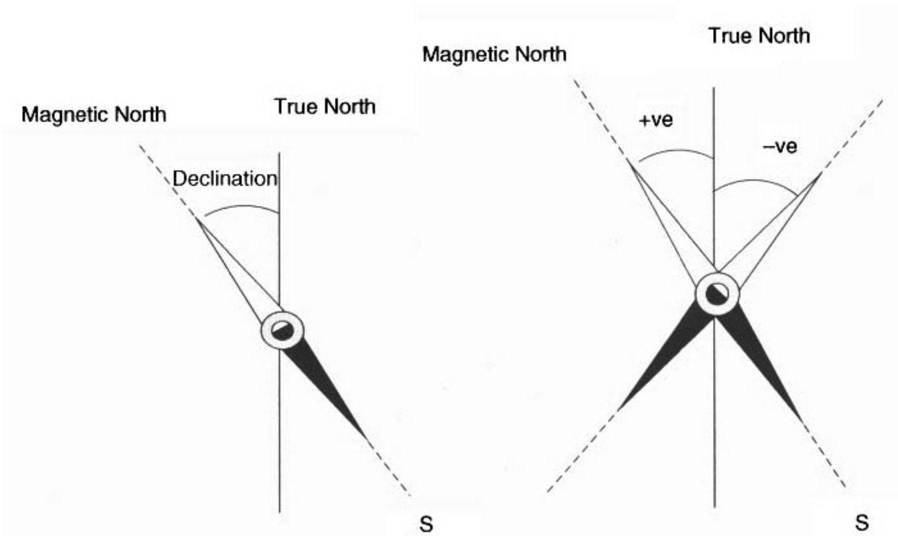



**Figure 3.4** Declination/East-West Declination.

For westerly declination, the azimuth correction can be given as

$$\text{Azimuth(true)} = \text{Azimuth (magnetic)} + (-\text{Magnetic declination}). \quad (3.71)$$

For true North and grid North, true north uses latitude and longitude coordinates of the curved earth as the reference. Longitudes converge upon the rotational pole. In the grid system, the Y-axis does not converge to single point.

$$\text{Magnetic Declination} - \text{Grid Convergence} = \text{Total Correction.} \quad (3.72)$$

$$\text{Magnetic Azimuth} + \text{Total Correction} = \text{Corrected Azimuth.} \quad (3.73)$$

**Problem 3.25**

Determine the azimuth with respect to the true north of the following wells:

- N45E, declination 5 deg west
- N45E, declination 5 deg east

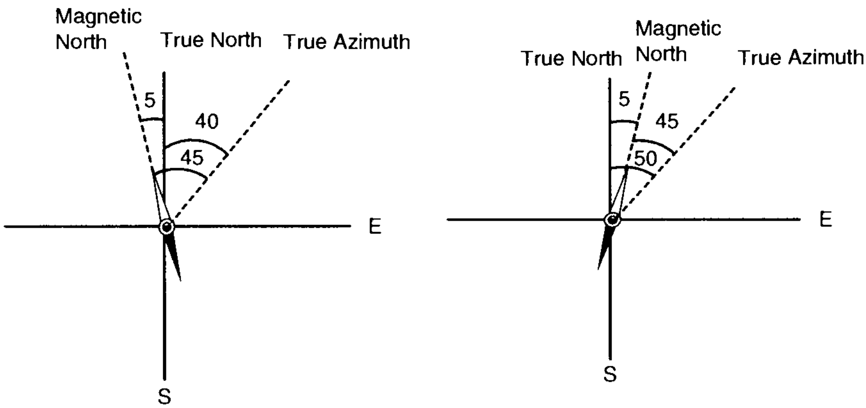



**Figure 3.5** Declinations for Problem 3.25.

*Solution:*

Azimuth = 45 – 5 = 40 deg = N40 E
Azimuth = 45 + 5 = 50 = N50 E
They are shown in Figure 3.5

**Problem 3.26**

Calculate the total correction if the magnetic declination is –4 deg and grid convergence is – 6 deg.

*Solution:*

Using equation 3.64, the total correction = magnetic declination – grid convergence:
Total correction = –4 – (–6) = 2 deg.

## 3.14 Wellbore Trajectory Uncertainty

Nominal distance between two points, i.e., from the point concerned in a reference wellbore (the reference point) to the point observed in the object wellbore (the object point), is

$$S_{or} = \sqrt{(x_o - x_r)^2 + (y_o - y_r)^2 + (z_o - z_r)^2}, \qquad (3.74)$$

where

$x_o$, $y_o$, $z_o$ are coordinates of the object point in the OXYZ system, and $x_r$, $y_r$, $z_r$ are coordinates of the object point in the OXYZ system, $S_{or}$ (Figure 3.6).

If $r_s$ represents the sum of radii, then

$$r_S = r_o + r_r, \tag{3.75}$$

where $r_o$ is the radius of an object well, and $r_r$ is the radium of a reference well.

Relative covariance matrixes are

$$\Sigma_{RR} = \Sigma_{P_oP_o} + \Sigma_{P_rP_r}, \tag{3.76}$$

where $\Sigma_{P_0P_0}$ and $\Sigma_{P_rP_r}$ are covariance matrixes of the points $P_o$ and $P_r$, respectively.

Azimuth and the dip angle of the signal point can be determined by

$$\phi = arc\sin\frac{\sqrt{U_r^2 + V_r^2}}{S_{or}}, \tag{3.77}$$

and

$$\theta = arc\tan\frac{V_r}{U_r}. \tag{3.78}$$

If $k_p$ is the amplifying factor corresponding to the ellipsoid on which the signal point lies, it is given as

$$k = \frac{S}{\sqrt{\sigma_1^2\cos^2\theta\sin^2\phi + \sigma_2^2\sin^2\theta\sin^2\phi + \sigma_3^2\cos^2\phi}} \tag{3.79}$$

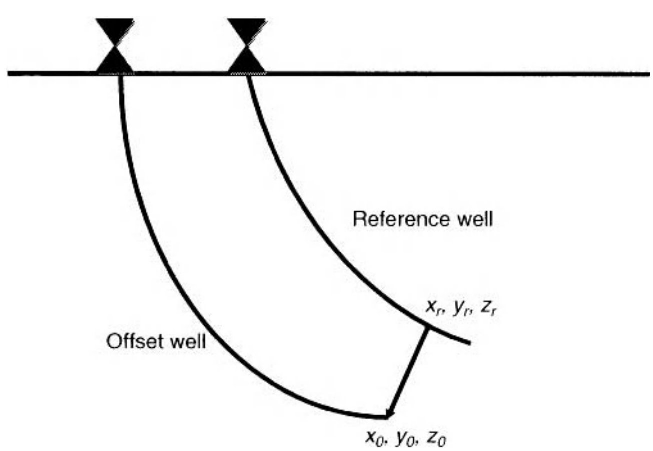



**Figure 3.6** Reference and offset well.

where $\phi$ = the azimuth angle, and $\theta$ = the dip angle.

Minimum probability of intersection is given as

$$P_p = 1 - \frac{4}{\sqrt{\pi}} \int_0^{\frac{K_P}{\sqrt{2}}} \exp[-r^2]\, r^2 dr, \tag{3.80}$$

or

$$P_p = \frac{4}{\sqrt{\pi}} \int_{\frac{K_P}{\sqrt{2}}}^{\infty} \exp[-r^2]\, r^2 dr. \tag{3.81}$$

**Problem 3.27**

A pair of parallel horizontal well is planned to be drilled. Through survey, calculation, and error analysis, a group of data is obtained in Table 3.3 for analyzing the probability of intersection of the two points given.

*Solution:*

From equation 3.75,

$$S_{or} = \sqrt{1+1+25} = 5.196 \text{ (m)}.$$

From equation 3.75,

$$r_S = (215+215)/2 = 215 \text{ (mm)}.$$

From equation 3.76,

$$\Sigma_{RR} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & \sqrt{2} \\ 0 & \sqrt{2} & 3 \end{bmatrix}.$$

(1) Determining the orthogonal matrix (T)

(a) Calculating the eigenvalue and eigenvectors of $\sum_{RR}$:

**Table 3.3** Basic parameters for the calculation.

| | Coordinate (m) | Diameter (mm) | Covariance Matrix ($m^2$) | | |
|---|---|---|---|---|---|
| Object well (point) | X=1000 | 215 | 2 | 0 | 0 |
| | Y=1000 | | 0 | 1 | $\frac{\sqrt{2}}{2}$ |
| | Z=1000 | | 0 | $\frac{\sqrt{2}}{2}$ | 1 |
| Reference well (point) | X=1001 | 215 | 1 | 0 | 0 |
| | Y=1001 | | 0 | 1 | $\frac{\sqrt{2}}{2}$ |
| | Z=1005 | | 0 | $\frac{\sqrt{2}}{2}$ | 2 |

From

$$\begin{vmatrix} \lambda-3 & 0 & 0 \\ 0 & \lambda-2 & \sqrt{2} \\ 0 & \sqrt{2} & \lambda-3 \end{vmatrix} = 0$$

we get $\lambda_1 = 4, \lambda_2 = 3, \lambda_3 = 1$.
When $\lambda = 4$, from

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & \sqrt{2} \\ 0 & \sqrt{2} & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

we get the eigenvector: $a_1 = [0 \quad 1 \quad -\sqrt{2}]^T$.
When $\lambda_1 = 3$, from

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

we get $a_2 = [1\ 0\ 0]^T$.

When $\lambda_1 = 1$, from

$$\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & \sqrt{2} \\ 0 & \sqrt{2} & -2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

we get $a_3 = [0\ \ \sqrt{2}\ \ 1]^T$.

(b) Making the unit vectors from above three eigenvectors:

$$e_1 = [0\ \ \tfrac{\sqrt{3}}{3}\ \ -\sqrt{\tfrac{2}{3}}]^T\ ; e_2 = [1\ \ 0\ \ 0]^T\ ; e_3 = [0\ \ \sqrt{\tfrac{2}{3}}\ \ \tfrac{\sqrt{3}}{3}]^T.$$

Thus, the orthogonal matrix T can be written as

$$T = \begin{bmatrix} 0 & 1 & 0 \\ \frac{\sqrt{3}}{3} & 0 & \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{2}{3}} & 0 & \frac{\sqrt{3}}{3} \end{bmatrix}.$$

(2) Make an orthogonal transformation using the matrix T. The relative uncertainty ellipsoid cluster can be obtained as

$$\frac{U^2}{4} + \frac{V^2}{3} + \frac{W^2}{1} = k^2.$$

(3) Determining amplifying factor

(a) Distance from the signal point to the center point: $S = 5.196 - 0.215 = 4.98$ m

(b) Relative coordinates of the reference point in the OUVW system:

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{\sqrt{3}}{3} & 0 & \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{2}{3}} & 0 & \frac{\sqrt{3}}{3} \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 4.695 \\ 1.255 \end{pmatrix}$$

(c) Azimuth and dip angle of the signal point:

$$\phi = arc\sin\frac{\sqrt{1+4.695^2}}{5.196} = 67.486° \; ; \theta = arc\tan\frac{4.695}{1} = 77.976°$$

(d) Amplifying factor:

$$k_p = \frac{4.98}{\sqrt{4\times\cos^2 77.976\times\sin^2 67.486+3\times\sin^2 77.976\times\sin^2 67.486+\cos^2 67.486}}$$
$$= 3.009$$

Thus, take $k_E = 3$.

(4) Probability of intersection:

$$P_p = 1-\frac{4}{\sqrt{2\pi}}\left(\frac{3^3}{6}-\frac{3^5}{20}+\frac{3^7}{112}-\frac{3^9}{864}+\cdots\right) < 5\%$$

*Formulas and Calculations for Drilling Operations*
by G. Robello Samuel


# 4

# Fluids

This chapter focuses on different basic calculations associated with drilling fluids.

## 4.1 Equivalent Mud Weight

The pressure ($P$ – psi) in the wellbore or formation can be expressed in terms of equivalent mud weight (EMW) and is given as

$$\mathrm{EMW} = \frac{P}{0.052 \times L_{tvd}} \text{ ppg.} \tag{4.1}$$

where $L_{tvd}$ = true vertical depth (ft).

If the well is deviated $\alpha$ deg from the vertical, the EMW is given as

$$\mathrm{EMW} = \frac{\mathrm{P_h}}{0.052 \times \mathrm{D_h} cos\alpha} \text{ppg.} \tag{4.2}$$

The pressures that may be encountered during drilling are classified as the following:

- Normal pressure – the pressure gradient is approximately 0.433 psi/ft of depth (4.3)
- Abnormal pressure – the pressure gradient is greater than 0.433 psi/ft of depth (4.4)
- Subnormal pressure – the pressure gradient is less than 0.433 psi/ft of depth (4.5)

**Problem 4.1**

Calculate the hydrostatic pressure at a well depth of 10,000 ft (TVD) with a mud weight of 10 ppg.

*Solution:*

The hydrostatic pressure is

$$P_h = 0.052 \times \text{Mud weight (ppg)} \times \text{True vertical depth (ft) psi},$$

$$P_h = 0.052 \times \rho_m \times L_{tvd} = 0.052 \times 10 \times 10000 = 5,200 \text{ psi}.$$

**Problem 4.2**

Calculate the equivalent mud weight at a well depth of 9,000 ft (TVD) with a bottomhole pressure of 6,555 psi.

*Solution:*

Using equation 4.1, equivalent mud weight is given as

$$\text{EMW} = \frac{P}{0.052 \times L_{tvd}}.$$

Substituting the values,

$$\text{EMW} = \frac{6555}{0.052 \times 9000} = 14 \text{ ppg}.$$

**Problem 4.3**

Casing is set on a east Texas well at 7,800 ft (TVD). It was estimated the fracture pressure was 6,815 psi. Calculate the equivalent fracture mud weight and fracture gradient.

*Solution:*

Equivalent mud weight for fracture is given as

$$\text{EMW} = \frac{P}{0.052 \times L_{tvd}} = \frac{6815}{0.052 \times 7800} = 16.8 \text{ ppg}$$

$$\text{Fracture gradient} = \frac{6815}{7800} = 0.8737 \text{ psi/ft}$$

## 4.2 Mud Weighting

The four fundamental equations used in developing mud weighting mathematical relations are 1) the material balance equation, 2) the volume balance equation, 3) the relationship between weight and volume equation, and 4) the volume balance in low specific gravity solids equation.

$$W_f = W_o + W_a, \tag{4.6}$$

where $W_f$, $W_o$, and $W_a$ = final mixture weight, original liquid weight, and added material weight, respectively.

$$V_f = V_o + V_a, \tag{4.7}$$

where $V_f$, $V_o$, and $V_a$ = final mixture volume, original liquid volume, and added material weight, respectively.

$$\rho = \frac{W}{V}, \tag{4.8}$$

where $\rho$, $W$, and $V$ = weight density, weight, and volume, respectively.

$$V_f f_{vf} = V_o f_{vo}, \tag{4.9}$$

where $f_{vf}$ and $f_{vo}$ = volume fraction of low specific gravity solids in final mixture and original liquid, respectively.

These equations may be combined and rearranged algebraically to suit the particular calculations desired. From equations 4.6 to 4.8 for example, the following can be obtained as a general relationship equation:

$$V_1\rho_1 + V_2\rho_2 + V_3\rho_3 + V_4\rho_4 + \ldots = V_f\rho_f, \tag{4.10}$$

$$V_1 + V_2 + V_3 + V_4 + \ldots = V_f,$$

where $V_1$, $V_2$, $V_3$, and $V_4$ = volumes of materials 1, 2, 3, and 4, respectively, and $\rho_1$, $\rho_2$, $\rho_3$, and $\rho_4$ = the densities of materials 1, 2, 3, and 4, respectively.

A simultaneous solution of the two equations above results in any two sought unknowns and the rest of the parameters are known.

The following are example formulations for commonly encountered field problems:

(a) The amount of weighting materials required to increase original mud density, $\rho_o$, to a final density, $\rho_f$, is

$$W_{wm} = \frac{42\left(\rho_f - \rho_o\right)}{1-\left(\frac{\rho_f}{\rho_{wm}}\right)}, \tag{4.11}$$

where $W_{wm}$ = the required amount of weighting material, lbs/bbl, or original mud.

(b) The average weight density, $\rho_{av}$, of two added materials $i$ and $j$ of weights $\rho_i$ and $\rho_j$, respectively, is

$$\rho_{av} = \frac{\rho_i\rho_j}{f_w f_i + \left(1+f_w\right)\rho_j}, \tag{4.12}$$

where $f_w = w_j/w_i + w_j$ = the weight fraction of material $j$ with respect to added weights of materials $i$ and $j$.

(c) The amount of liquid volume, $V_l$, required to make a mud of total volume $V_f$ having a weight of $\rho_f$ is

$$V_1 = V_f \left( \frac{1-\left(\frac{\rho_f}{\rho_a}\right)}{1-\left(\frac{\rho_L}{\rho_a}\right)_j} \right), \tag{4.13}$$

where $\rho_a$ = the weight density of added material or average weight density, $\rho_{av}$, of material added to the liquid.

(d) The amount of solids *i* and *j*, such as clay and barite for example, required to make-up a specified final mud volume, $V_f$, and density, $\rho_f$, is

$$w_i = \frac{42 f_w \left(\rho_f - \rho_o\right)}{1 - f_w\left(\frac{\rho_o}{\rho_j}\right) - \left(1 - f_w\right)\left(\frac{\rho_o}{\rho_i}\right)}, \tag{4.14}$$

$$w_i = \frac{\left(1 - f_w\right)\left(\rho_f - \rho_o\right)}{\left[1 - f_w\left(\frac{\rho_o}{\rho_j}\right) - \left(1 - f_w\right)\left(\frac{\rho_o}{\rho_i}\right)\right]}, \tag{4.15}$$

where $w_i$ and $w_j$ = solid materials *i* and *j* in lbs/bbl of final mud.

(e) The final mud density, $\rho_f$ , when a certain liquid volume, $V_l$, of density $\rho_l$ is added to a mud system of original density, $\rho_o$, and volume, $V_o$, is

$$\rho_f = \frac{\rho_o + a\rho_l}{1 + a}, \tag{4.16}$$

where $\alpha = V_l / V_o$.

## 4.3 Common Weighting Materials

The average weights of the commonly used weighting materials are given in Table 4.1.

**Table 4.1** Common weighting materials.

| **Weighting Materials** | **Specific Gravity ($gm/cm^3$)** | **Density (lbm/gal – ppg)** | **Density (lbm/bbl – ppb)** |
|---|---|---|---|
| Barite Pure grade | 4.5 | 37.5 | |
| Barite API – drilling grade | 4.2 | 35 | 1,470 |
| Bentonite | 2.6 | 21.7 | 910 |
| Calcium carbonate | 2.7 | 22.5 | 945 |
| Calcium chloride | 1.96 | 16.3 | 686 |
| Sodium chloride | 2.16 | 18 | 756 |
| Water | 1 | 8.33 | 1,001 |
| Diesel | 0.86 | 7.2 | 300 |
| Iron oxide | 5 | 41.73 | 5,005 |
| Galena | 6.6 | 55 | 6,007 |
| Drilled solids | 2.7 | 21.7 | 2,702 |

**Micron Sizes:**

| | |
|---|---|
| Clay | <1 |
| Bentonite | <1 |
| Barite | 2–60 |
| Silt | 2–74 |
| API Sand | >74 |

**Micron Cut Points for Solid Removal System:**

| | |
|---|---|
| Centrifuge | 3–5 Micron |
| Desilter 3″–4″ cones | 12–60 Micron |
| Desander 5″–12″ cones | 30–60 Micron |

**Problem 4.4**

Determine the percentage of clay based on a ton (2000 lb) of clay of 10 ppg clay-water drilling mud. The final mud volume is 100 barrels.

*Solution:*

If $x$ is the percentage of clay in the final mud, it can be written as

$$\frac{x}{100} \times \rho_f \times V_f = 2000,$$

$$\frac{x}{100} \times 10 \times 500 = 2000,$$

Solving for $x$,

$$x = \frac{2000}{10 \times 100 \times 42} \times 100 = 4.76\%.$$

**Problem 4.5**

What will the final density of mud be after adding 5 tons of barite to 500 barrels of 9 ppg drilling mud. Assume the barite density = 35 ppg.

*Solution:*

Weighting material added is

$$w_{wm} = \frac{5 \times 2000}{500} = 20 \text{ lb/bbl}.$$

The final density of the mud is calculated using the relationship given in equation 4.11:

$$\rho_f = \frac{9 + \frac{20}{42}}{1 + \left( \frac{20}{35 \times 42} \right)} = 9.35 \text{ ppg}.$$

**Problem 4.6**

A mud engineer is planning to calculate the volume of mud that can be prepared with 20% of clay by weight from one ton of clay. Assume the specific gravity of the clay is 2.5.

*Solution:*

1 ton of clay = 2,000 lbs.
The weight of clay = 2.5 × 8.33 = 20.82 ppg.
The weight of mud = 2000/0.20 = 10,000 lbs.
Using the mass balance, weight of water = 10,000 – 2,000 = 8,000 lbs.
Using the volume balance, volume of mud = volume of clay + volume of water:

$$\text{Volume of mud} = \frac{2000}{20.82} + \frac{8000}{8.33} = 1056.44\text{ gal},$$

$$= 1056.44\text{ gal}\left(\frac{1\text{ bbl}}{42\text{ gal}}\right) = 25.15\text{ bbl}.$$

**Problem 4.7**

What will be the final density of an emulsion mud composed of 25% (by volume) diesel oil and a 10 ppg water-base mud? Assume the specific gravity of oil = 0.75.

*Solution:*

Starting from the mass balance equation,

$$V_{mf}\rho_{mf} = V_o\rho_o + V_{mw}\rho_{mw},$$

where $V_{mf}$ = the final mud volume, $\rho_{mf}$ = the final density mud, $V_o$ = the emulsion oil volume, and $V_{mw}$ = the volume of water-based mud.
Also, it is given as

$$V_{mf}\rho_{mf} = 0.25V_{mw}\rho_o + (1-0.25)V_{mf}\rho_{mw},$$

since

$$V_o = 0.25V_{mw} \text{ and } V_{mw} = (1-0.25)V_{mf}.$$

Therefore,

$$\rho_{mf} = 0.25\rho_o + (1-0.25)\rho_{mw}.$$

Also,

$$\rho_o = \rho_w \times \rho_d,$$

$$V_f\rho_f = \rho_w \times 0.25 \times \rho_d + (1-0.25)V_f\rho_w.$$

Substituting the values, the final density can be calculated as

$$\rho_f = 8.33 \times 0.25 \times 0.75 + (1-0.25) \times 10 = 9.06 \text{ ppg}.$$

## 4.4 Diluting Mud

When diluting mud with a liquid, the resulting density of the diluted mud can be given as

$$\rho_f = \rho_o + \left(\frac{V_a}{V_o}\right)(\rho_o - \rho_a), \tag{4.17}$$

where $\rho_o$ = the original mud weight, ppg, $V_a$ = the original volume in barrels, $V_o$ = the final volume in barrels, and $\rho_a$ = the density of material added, ppg.

**Problem 4.8**

Determine the final mud density of mud if 100 barrels of diesel with a specific gravity of 0.82 are mixed with 500 barrels 9 ppg mud.

*Solution:*

The final volume of mud after adding diesel = 500 + 100 = 600 bbl.

Using the density relationship, the final density of the mud can be calculated as

$$\rho_f = 9 + \left(\frac{100}{600}\right)(9 - 0.82 \times 8.33) = 8.57 \text{ ppg}.$$

**Problem 4.9**

While drilling it was desired to increase the mud weight to 13 ppg. The volume of existing mud in the mud system was 1000 bbl and mud weight was 12.5 ppg. Calculate the amount of barite required to increase the mud weight. Also, calculate the number of sacks required. Assume the barite density = 35 ppg and 1 sack = 100 lb.

*Solution:*

The barite required in lb per bbl of original mud is

$$w_{wm} = \frac{42(13-12.5)}{1-\left(\frac{13}{35}\right)} = 33.4 \text{ lb/bbl.}$$

The amount of barite sacks required is

$$w_{wm} = \frac{1000 \times 33.4}{100} = 334 \text{ sacks.}$$

**Problem 4.10**

It is desired to increase a mud weight from 12 ppg to 0.75 psi/ft using barite having a 4.4 specific gravity. Determine the cost due to mud weight increase, assuming barite is $101/100 lbs.

*Solution:*

$$\rho_f = \frac{0.75}{0.052} = 15\text{ppg}.$$

Using equation 4.11, the weight of the barite required per barrel of original mud can be estimated as

$$w_{wm} = \frac{42\left(\rho_f - \rho_o\right)}{1-\left(\frac{\rho_f}{\rho_{wm}}\right)} = 213.34 \text{ lbs/bbl of original mud.}$$

The total cost of the barite required is calculated as Cost = 213.34 × 1.01 = $215.47/bbl of original mud.

**Problem 4.11**

How many sacks of weighting material with a specific gravity of 4.3 will be required to increase the mud weight of 100 barrels of mud from 11 ppg to 12.0 ppg, and what will be the increase in volume? Assume 1 sack = 100 lb.

*Solution:*

The barite required in lb per bbl of original mud is

$$w_{wm} = \frac{42(12-11)}{1-\left(\frac{12.0}{4.3\times 8.33}\right)} = 63.15 \text{ lb/bbl.}$$

The amount of barite sacks required is

$$w_{wm} = \frac{100\times 63.15}{100} \approx 62 \text{ sacks.}$$

The final volume is

$$V_{mf}\rho_{mf} = V_o\rho_o + V_{mw}\rho_{mw},$$

where $V_{mf}$ = final mud volume, $\rho_{mf}$ = final mud density, $V_o$ = original volume, and $V_{mw}$ = the volume of weighting material.

$$V_{mf}\times 12 = 100\times 11 + \left(V_{mf} - 100\right)\times 4.33\times 8.33$$

and solving yields 104.16 bbl.

The increase in volume is 104.16 – 100 = 4.16 bbl.

**Problem 4.12**

A particular drilling phase is planned to be drilled with a mud weight of 14 ppg. Present mud weight in the tank and hole is 13 ppg. The total volume of the hole and mud tanks including reserve mud tanks are 1200 bbl. Estimate the number of sacks of barite needed.

Assume 1 sack = 100 lbs.

*Solution:*

The weight of the barite required per barrel of mud is calculated using equation 4.11

$$w_{wm} = \frac{42(14-13)}{1-\left(\frac{13}{35}\right)} = 69.47 \text{ lb/bbl.}$$

The total amount of barite required = 69.47 × 1200 = 83,364 lbs of barite.

Sacks required is

$$\frac{83364\,(\text{sack})}{100\left(\text{lb}/\text{sack}\right)} = 834 \text{ lbs } (\sim 42 \text{ tons}).$$

**Problem 4.13**

How many sacks of weighting material of density 35 ppg will be required to make 500 barrels of mud weighing 89.76 pound per cubic foot? Also, estimate how many barrels will be needed. Assum 1 sack = 100 lb.

*Solution:*

$$\text{The density of the mud} = \frac{89.76}{\text{ft}^2} \times \frac{\text{ft}^2}{7.48 \text{ gal}} = 12 \text{ ppg.}$$

Using the mass balance and volume balance equation, it can be written as

$$V_{mf}\rho_{mf} = V_o\rho_o + V_{mw}\rho_{mw},$$

or

$$500 \times 12 = 8.33 \times (500 - V_{mw}) + V_{mw} \times 35.$$

Solving for $V_{mw}$ = 68.80 bbl.

The number of sacks of weighting material required is

$$35\frac{\text{lbs}}{\text{gal}} \times 68.80 \text{ bbl} \times \frac{42 \text{ gal}}{\text{lbs}} \times \frac{1 \text{ sack}}{100 \text{ bbl}} = 1{,}011 \text{ sacks.}$$

**Problem 4.14**

What is the volume of diesel oil that needs to be added when a final density of an emulsion mud is 9.5 ppg, composed of diesel oil and 800 barrels of a 10 ppg water-based mud? Assume the specific gravity of oil is 0.75. Also, calculate the percentage of diesel oil by volume in the total mud.

*Solution:*

Density of the oil = 0.75 × 8.33 = 6.25 ppg.

The ratio of the mud to the diesel oil added can be given as

$$\frac{V_m}{V_l} = \frac{(\rho_a - \rho_o)}{\rho_f - \rho_o} - 1.$$

Substituting the values,

$$\frac{V_m}{V_l} = \frac{(\rho_a - \rho_o)}{\rho_f - \rho_o} - 1 = \frac{(6.25 - 10)}{9.5 - 10} - 1 = 6.505.$$

Volume of diesel oil added = 800/6.505 = 123 bbl.

$$\text{Percentage of diesel in the mud} = \frac{123}{123 + 800} \times 100 = 13.3\%.$$

**Problem 4.15**

It is desired to increase the mud weight from 12 ppg to 13 ppg without increasing the volume using weighting material with a density of 35 ppg. Calculate the amount of mud discarded if the original volume is 800 bbl.

*Solution:*

The volume of additives added is

$$w_{wm} = \frac{800(12 - 13)}{(12 - 35)} = 34.8 \text{ bbl}.$$

The volume of original mud dumped = 34.8 bbl.

**Problem 4.16**

Before drilling the next phase, it was desired to increase the 900 bbl of mud with a mud density of 16 ppg mud to 17 ppg. The volume fraction of low-sp-gravity solids must be reduced from 0.055 to 0.030 by dilution with water. Compute the original mud that must be discarded while maintaining the final mud volume of 900 bbl and the amount of water and barite that should be added. Assume barite density = 35 ppg.

*Solution:*

Volumetric balance is

$$V_i = V_f \frac{f_f}{f_{1i}},$$

where

$$V_f = \frac{(11 \times 5 \times 36) \times 2 \times 0.90}{5.6} = 636 \text{ bbl}.$$

Discarded volume is

$$636 - V_i = 636 - 636\frac{0.030}{0.055} = 289 \text{ bbl}.$$

For ideal mixing and performing mass balance, the volume of water added is

$$V_w = \frac{(\rho_b - \rho_f)V_f - (\rho_b - \rho_i)V_i}{(\rho_b - \rho_w)},$$

and the mass of barite needed is $m_b = (V_f - V_i - V_w)\rho_b$.
Substituting respective values yields

$$V_w = 204 \text{ bbls}; \quad m_B = 125,486 \text{ lbs}.$$

**Problem 4.17**

Calculate the percent weight increase for the following data:

- Density of water = 8.34 ppg
- Density of solids = 22 lbs/gal
- Mud weight = 86 lb/cu. ft
- 1 bbl = 42 gal
- 7.48lb/cu.ft = ppg

*Solution:*

Percent weight increase is given as

$$P = \frac{1 - \frac{\rho_w}{\rho_m}}{1 - \frac{\rho_w}{\rho_c}} \times 100.$$

Substituting the values,

$$P = \frac{1 - \frac{8.34}{30.43}}{\frac{8.34}{13.45}} = \frac{0.274}{0.62} \times 100 = 44\%.$$

**Problem 4.18**

Calculate the mud weight increase due to cuttings in the mud for the following data:

- Cuttings generated = 2 bbl/min
- Density of cuttings = 800 lbs/bbl
- Flow rate of mud = 400 gpm
- Mud weight = 10 ppg

*Solution:*

Weight of cuttings generated = 2 × 800 = 1600 lbs/min.
Weight of mud circulated = 400 × 10 = 4000 lbs/min.
Volume of cuttings generated = 1600 × 42/800 = 84 gpm.
Volume rate of mud = 400 gpm.
Mud weight in the annulus is

$$\rho_{ann} = \frac{(4000 + 600)}{(400 + 84)} = 11.57 \text{ ppg}.$$

**Problem 4.19**

A well is being drilled with a mud density of 14 ppg, and it is desired to increase the mud density to 15 ppg to prevent any formation fluid influx. The volume fraction of low-sp-gravity solids must be reduced from 0.055 to 0.030 by dilution with water. Compute the original mud that must be discarded and the amount of water and barite that should be added. Calculate the unnecessary volume of mud discarded/added if an error of ±0.02 was made in determining the original and final volume fractions of low-sp-gravity solids in the mud.

If the barite and mixing water added are 175,000 lbm and 250 bbls, respectively, calculate the present volume fraction of low specific gravity solids. Assume the barite density = 35 ppg.

Mud tank size = 11′ width × 5′ height × 36′ length.

Number of tanks = 2 and both are 90% full.

*Solution:*

Volumetric balance is

$$V_1 = V_2 \frac{f_{s2}}{f_{s1}},$$

where

$$V_2 = \frac{(11 \times 5 \times 36) \times 2 \times 0.90}{5.6} = 636 \text{ bbl}.$$

Discarded volume is

$$636 - V_1 = 636 - 636 \frac{0.030}{0.055} = 289 \text{ bbl}.$$

For ideal mixing and performing mass balance, the volume of water added is

$$V_w = \frac{(\rho_B - \rho_2)V_2 - (\rho_B - \rho_1)V_1}{(\rho_B - \rho_w)},$$

and the mass of barite needed is $m_B = (V_2 - V_1 - V_w)\rho_B$.

Substituting respective values yields

$$V_w = 204 \text{ bb1},$$

$$m_B = 125{,}486 \text{ 1bs}.$$

For an error of +0.02, mud discarded = 636 – $V_1$ = 636 – 636 $\frac{0.05}{0.075}$ = 212 bbl.

Unnecessary mud discarded in error = 77 bbl.

For an error of –0.02, mud discarded = 636 – $V_1$ = 636 – 636 $\frac{0.01}{0.035}$ = 455 bbl.

Unnecessary mud discarded in error = 166 bbl.

If the barite and mixing water added are 175,000 lbm and 250 bbl, respectively, the present volume fraction of low specific gravity solids is

$$f_2 = f_1 \frac{V_1}{V_2} = (0.055)\frac{267}{636} = 0.0231.$$

**Problem 4.20**

Derive expressions for determining the amounts of barite and water that should be added to increase the density of 100 bbl of mud from $\rho_1$ to $\rho_2$. Also, derive an expression for the increase in mud volume expected upon adding the barite and the water. Assume a water requirement of 1 gal per sack of barite, 1 sack = 100 lb.

*Solution:*

Using the volume and mass balance, it can be written as

$$V_2 = V_1 + V_w + V_B = V_1 + \frac{m_B}{\rho_B} + m_B V_{wB},$$

where $V_{wB}$ is the barite-water requirement.

Including the mass of water,

$$\rho_2 V_2 = \rho_1 V_1 + m_B + \rho_w V_{wB} m_B.$$

Solving the above equations yields

$$V_1 = V_2 \left[ \frac{\rho_B \left( \dfrac{1+\rho_w V_{wB}}{1+\rho_B V_{wB}} \right) - \rho_1}{\rho_B \left( \dfrac{1+\rho_w V_{wB}}{1+\rho_B V_{wB}} \right) - \rho_2} \right],$$

$$100 = V_2 \left[ \frac{35\left(\frac{1+8.33\times 0.01}{1+35\times 0.01}\right) - \rho_2}{100\left(\frac{1+8.33\times 0.01}{1+35\times 0.01}\right) - \rho_1} \right].$$

Given values are $\rho_w$ = 8.33 ppg, $\rho_B$ = 35 ppg, $V_1$ = 100 bbl, $V_{wB}$ = 0.01 gal/lb.

$$m_B = \frac{\rho_B}{1+\rho_B V_{wB}}(V_2 - V_1) = \frac{35\times 42}{1+35\times 0.01}(V_2 - 100).$$

Substituting $V_2$ from previous equation,

$$m_B = 109{,}00\frac{(\rho_2 - \rho_1)}{(28.02 - \rho_2)} \text{ lbm,}$$

$$= 109\frac{(\rho_2 - \rho_1)}{(28.02 - \rho_2)} \text{ sacks.}$$

$$V_w = V_{wB} m_B = \frac{m_B}{100} gal = \frac{m_B}{4200} \text{ bbl,}$$

$$V = V_w + \frac{m_B}{\rho_s} = \frac{m_B}{4200} + \frac{m_B}{1470} = 0.0091 m_B.$$

**Problem 4.21**

Consider the mud consisting of oil, water, and solids:

- $f_o$ = volume fraction of oil
- $f_w$ = volume fraction of water
- $f_s$ = volume fraction of solid
- $f_f$ = volume fraction of fluid including oil and water

Derive an expression to calculate $f_f$ in terms of the oil-water ratio (OWR), the oil-water ratio (OWR = $\frac{f_o}{f_w}$) and the following densities:

- Mud density = $\rho_m$
- $\rho_o$ = oil density

- $\rho_w$ = water density
- $\rho_s$ = solid density

*Solution:*

Consider the mud consisting of oil, water, and solids. The density of the mud can be written from mass balance as

$$\rho_m = \rho_o f_o + \rho_w f_w + \rho_s f_s. \tag{4.18}$$

In terms of the volume fraction of oil, water, and solids, it can be written as

$$f_0 + f_w + f_s = 1, \tag{4.19}$$

and the oil-water ratio, OWR, is $\frac{f_0}{f_w}$.

Also,

$$f_f + f_s = 1, f_0 + f_w = f_f, \tag{4.20}$$

where $f_f$ = the volume fraction of fluid including oil and water.

Equation 4.20 can be re-written as

$$f_f\left(\rho_o \frac{f_o}{f_f} + \rho_w \frac{f_w}{f_f}\right) + \rho_s\left(1 - f_f\right) = \rho_m. \tag{4.21}$$

Therefore, the fluid fraction can be given as

$$f_f = \frac{\rho_m - \rho_s}{\left(\rho_o \frac{f_o}{f_f} + \rho_w \frac{f_w}{f_f} - \rho_s\right)}. \tag{4.22}$$

It is known that $f_o + f_w = f_f$ and can be written as

$$\frac{f_f}{f_w} = \frac{f_0}{f_w} + 1. \tag{4.23}$$

From the above equation, $\frac{f_w}{f_f}$ can be calculated, and $\frac{f_0}{f_f}$ can be found out from the following equation:

$$\frac{f_0}{f_f} = 1 - \frac{f_w}{f_f}.$$

In terms of OWR, the volume fraction of the fluid can be written as

$$f_f = \frac{\rho_m - \rho_s}{\rho_o - \rho_s + (\rho_w - \rho_o)\left(\frac{1}{1+OWR}\right)}. \tag{4.24}$$

**Problem 4.22**

It is desired to mix $X$ barrels of $\rho_m$ ppg of water-based mud. Bentonite (weight $= \rho_c$ ppg) and barite (weight $= \rho_b$ ppg) are to be added in the ratio of $\alpha_c$ lb of bentonite to $P_b$ lbs of barite. Determine the amount of water, $V_w$, bentonite, $W_c$, and barite, $W_b$, needed to make-up the mud system.

*Solution:*

Mass balance is

$$V_f \rho_m = \rho_w V_w + \rho_b V_b + \rho_c V_c,$$

where $V_f \rho_m = 42X\rho_m$.

Volume balance, using in 42X gal of mud, is

$$42X = V_w + V_b + V_c \quad \text{or} \quad V_f = V_w + V_b + V_c.$$

Another relation given is

$$\frac{a_c}{P_b} = \frac{\rho_c V_c}{\rho_b V_b}.$$

Solving will yield

$$V_b = \frac{V_f(\rho_m - \rho_w)}{(\rho_c - \rho_w)\frac{a_c \rho_b}{P_b \rho_c} + \rho_b - \rho_w},$$

and

$$V_c = \frac{V_f(\rho_m - \rho_w)}{(\rho_c - \rho_w)\frac{a_c \rho_b}{P_b \rho_c} + \rho_b - \rho_w} \frac{a_c \rho_b}{P_b \rho_c}.$$

$$W_c = \frac{V_f\left(\rho_m - \rho_w\right)}{\left(\rho_c - \rho_w\right)\dfrac{a_c \rho_b}{P_b \rho_c} + \rho_b - \rho_w} \frac{a_c \rho_b}{P_b}.$$

$$W_b = \frac{V_f\left(\rho_m - \rho_w\right)}{\left(\rho_c - \rho_w\right)\dfrac{a_c}{P_b \rho_c} + 1 - \left(\dfrac{\rho_w}{\rho_b}\right)}.$$

$$V_w = V_f - V_b - V_c.$$

Substituting $V_b$ and $V_c$ from above yields

$$V_w = V_f\left(1 - \frac{\left(P_b \rho_c - a_c \rho_b\right)\left(\rho_m - \rho_w\right)}{a_c \rho_b\left(\rho_c - \rho_w\right) + P_b \rho_c\left(\rho_b - \rho_w\right)}\right).$$

**Problem 4.23**

How many sacks of AQUAGEL of 2.5 specific gravity are required to prepare 300 barrels of an AQUAGEL-water mud containing 5.5% AQUAGEL by weight? Assume 1 sack = 100 lb.

*Solution:*

$$P = \frac{1 - \dfrac{\rho_w}{\rho_{mf}}}{1 - \dfrac{\rho_w}{\rho_{aq}}} \times 100.$$

Substituting the values, $\rho_{mf} = 8.61$ ppg.

It can be written as

$$V_{mf} \rho_{mf} = V_{aq} \rho_{aq} + V_w \rho_w,$$

or

$$V_{aq} \rho_{aq} = V_{mf} \rho_{mf} - V_w \rho_w.$$

Since $V_w = (300 - V_{aq})$,

$$V_{aq} \gamma_{aq} = 300 \times 8.61 - 8.33\left(300 - V_{aq}\right).$$

Solving,

$$V_{aq} = 6.825 \text{ bbl.}$$

$$\text{Number of sacks} = \frac{6.825 \times 2.5 \times 8.33 \times 42}{100} = 60.$$

**Problem 4.24**

The density of 900 bbl of 16 ppg mud must be increased to 17 ppg. The volume fraction of low-sp-gravity solids must be reduced from 0.055 to 0.030 by dilution with water. A final mud volume of 900 bbl is desired. Compute the original mud that must be discarded and the amount of water and barite that should be added. Calculate the mud discarded if an error of ±0.02 was made in determining the original volume fraction of low-sp-gravity solids in the mud. Assume the barite density = 35 ppg.

*Solution:*

Volumetric balance is

$$V_1 = V_2 \frac{f_{s2}}{f_{s1}}.$$

$$\text{The initial volume of mud needed} = 900 \times \frac{0.03}{0.055} = 490.9 \text{ bbl.}$$

Thus, (900-490.9) 409 bbl of initial 900 bbl should be discarded. New volume and volume balance can be given as follows. Volume balance is

$$V_2 = V_1 + V_w + \frac{m_B}{\rho_B}.$$

Mass balance is

$$\rho_2 V_2 = \rho_1 V_1 + m_B + \rho_w V_{wB}.$$

Solving will give

$$V_w = \frac{(\rho_B - \rho_2)V_2 - (\rho_B - \rho_1)V_1}{(\rho_B - \rho_w)}.$$

Substituting,

$$V_w = \frac{(35-17)900-(35-16)490}{(35-8.33)} = 258.34 \text{ bbl.}$$

Solving, you can get the equation for the mass of barite needed:

$$m_B = (V_2 - V_1 - V_w)\rho_b = (900-490.9-258.34)42\times 35$$
$$= 221620 \text{ lbm.}$$

For an error of +0.02, mud discarded is

$$490.9 - V_1 = 490.9 - 490.9\frac{0.05}{0.075} = 163.63 \text{ bbl.}$$

Unnecessary mud discarded in error = 245.37 bbl.
For an error of –0.02, mud discarded is

$$490.9 - V_1 = 490.9 - 490.9\times\frac{0.01}{0.035} = 350.64 \text{ bbl.}$$

Unnecessary mud discarded in error = 58.36 bbl.

**Problem 4.25**

A retort analysis of a mud with a density of 13 ppg shows the following percentage of components:

- Water = 57% (fresh water)
- Diesel = 8%
- Solids = 35%

Calculate the average density of the solids.

*Solution:*

The mass balance for the mud is

$$V_m\rho_m = \rho_w V_w + \rho_o V_o + \rho_s V_s,$$

where $V$ = volume, and $\rho$ = density. Subscripts $m$ = mud, $w$ = water, $o$ = oil, and $s$ = solids.
The density of water = 8.33 ppg.
The density of diesel = 7.16 ppg.

The density of mud = 13 ppg.
Substituting the values in the equation,

$$1 \times 13 = 0.57 \times 8.33 + 0.08 \times 7.16 + \rho_s \times 0.35.$$

Solving for the density of the solids,

$$\rho_s = \frac{1 \times 13 - 0.57 \times 8.33 - 0.08 \times 7.16}{0.35} = 21.9 \text{ ppg}.$$

**Problem 4.26**

A 1,000 bbl unweighted fresh water clay mud has a density of 9 ppg. What mud treatment would be required to reduce the solids content to 5% by volume? The total volume must be maintained at 1,000 bbl, and the minimum allowable mud density is 8.8 ppg.

*Solution:*

The volume balance is

$$1000 = V_c + V_w.$$

Assuming the weight of the clay to be 20.83, the weight balance is

$$1000 \times 9 = 20.83 \times V_c + 8.33 \times V_w.$$

Substituting the volume equation and solving for $V_c$,

$$1000 \times 9 = 20.83 \times V_c + 8.33 \times (1000 - V_c),$$

$$V_c = 9464 \text{ bbl}.$$

In order to contain 5% by volume of solid content, the volume should be 5% × 1000 = 50 bbl.

## 4.5 Base Fluid – Water-Oil Ratios

The base fluid/water ratio can be calculated as the percentage by volume of base fluid in a liquid phase:

$$P_b = \left( \frac{VR_b}{VR_b + VR_w} \right) \times 100, \tag{4.25}$$

where $VR_b$ = the volume percentage of base fluid, and $VR_w$ = the volume percentage of water.

The percentage of water is

$$P_w = 100 - P_b \tag{4.26}$$

The base fluid water ratio is

$$\frac{P_b}{P_w} = \frac{VR_b}{VR_w}. \tag{4.27}$$

**Problem 4.27**

Using the following retort analysis data, calculate the base fluid water ratio:

- Base fluid = 62%
- Water = 12%
- Solids = 26%

*Solution:*

Using equation 4.25, the percentage by volume of base fluid in the liquid phase is

$$P_b = \left(\frac{VR_b}{VR_b + VR_w}\right) \times 100 = \left(\frac{62}{62+12}\right) \times 100 = 83.78.$$

The percentage of water is given by equation 4.26:

$$P_w = 100 - P_b = 100 - 83.78 = 16.22.$$

The base fluid water ratio is given by equation 4.27:

$$\frac{P_b}{P_w} = \frac{VR_b}{VR_w} = \frac{83.78}{16.22} = \frac{62}{12} = 5.17.$$

**Problem 4.28**

It was desired to reduce the base fluid water ratio to 85/100. Determine the amount of base fluid to be added to a 1200 bbl mud system. Use the data from Problem 4.27.

*Solution:*

The percentage of water is

$$P_w = \left( \frac{VR_w}{VR_b + VR_w + V_b} \right) \times 100,$$

where $V_b$ = the volume of the base fluid.

Substituting the values,

$$P_w = \left( \frac{VR_w}{VR_b + VR_w + V_b} \right) \times 100 = \left( \frac{.12}{.62 + .12 + V_b} \right) \times 100 = 0.15.$$

Solving for $V_b$ yields 0.06 bbl/bbl of mud.

The total amount of base fluid required = 0.06 (bbl/bbl of mud) × 1200 (bbl of mud ) = 72 bbl.

**Problem 4.29**

It was desired to reduce the base fluid water ratio to 80/100. Determine the amount of water to be added to a 1200 bbl mud system. Use the data from Problem 4.27.

*Solution:*

The percentage of water is

$$P_b = \left( \frac{VR_b}{VR_b + VR_w + V_w} \right) \times 100,$$

where $V_b$ = the volume of the base fluid.

Substituting the values,

$$P_b = \left( \frac{VR_b}{VR_b + VR_w + V_w} \right) \times 100 = \left( \frac{.80}{.62 + .80 + V_w} \right) \times 100 = 0.035.$$

Solving for $V_w$ yields 0.035 bbl of water/bbl of mud.

The total amount of water required = 0.035 (bbl/bbl of mud) × 1200 (bbl of mud ) = 35 bbl.

**Problem 4.30**

Determine the amount (lb/bbl) of a 5.2 specific gravity-weighing material that must be added to a mud system to increase its pressure gradient from 0.52 psi/ft to 0.624 psi/ft.

*Solution:*

The initial pressure gradient = 0.52 psi/ft = 10 ppg.
The final pressure gradient = 0.624 psi/ft = 12 ppg.
Weighting material = 5.2 = 5.2 × 8.33 =43.316 ppg:

$$w_{wm} = \frac{42(12-10)}{1-\left(\frac{12}{43.32}\right)} = 116 \text{ lbs/bbl}.$$

## 4.6 Fluid Loss

The volume fraction of solids in mud, $f_{vm}$, can be written as

$$f_{vm} = \frac{V_s}{V_m} = f_{vc} + \frac{hA}{V_f + hA}, \tag{4.28}$$

where $V_f$ = the amount of filtrate volume, $h$ = the cake thickness, $A$ = the filtration area, $V_m$ = the total volume of mud filtered, and $V_s$ = the volume of solids deposited in mud cake .

API water loss is given as

$$V_{30} = 2V_{7.5} - V_{sp} \text{ cm}^3, \tag{4.29}$$

where $V_{7.5}$ = water loss in 7.5 min, cm³, and $V_{sp}$ = spurt loss, cm³.

Filter loss is always estimated with reference to the square root of time.

Filtrate volume is given as

$$V_f = A\left[\frac{2k\left(\frac{f_{vc}}{f_{vm}} - 1\right)\Delta Pt}{\mu}\right]^{\frac{1}{2}} + V_s, \tag{4.30}$$

where $\Delta P$ = the differential pressure, $\mu$ = filtrate viscosity, $t$ = filtration time, and $k$ = the cake permeability.

**Problem 4.31**

The spurt loss of a mud is known to be 2 cc. If a filtrate of 10 cc is collected in 15 minutes using a filter press, what is the standard API filtrate for this mud?

*Solution:*

The filter loss at 7.5 minutes is found by extrapolating the data given at time zero and fifteen minutes.

Let $x$ be the filter loss at 7.5 minutes. So, the filter loss at time 7.5 minutes is

$$2 = \frac{10 - x}{\sqrt{15} - \sqrt{7.5}} \times \sqrt{7.5}.$$

Solving for $x$ will yield 6.69 cc.

Using equation 4.29, API filter loss is

$$V_{30} = 2V_{7.5} - V_{sp} = 2 \times 6.69 - 10 = 3.38\text{cc}.$$

**Problem 4.32**

A kick was taken at 10000′ while drilling with a 14 ppg mud. Stabilized shut-in drill pipe pressure = 600 psi. Determine the amount of barite to be added to 800 bbl of original mud in order to contain formation pressure such that a differential pressure of 300 psi is achieved after killing the well.

*Solution:*

Equivalent mud weight needed to control the kick with the excess differential pressure is

$$\frac{600 + 0.052 \times 14 \times 10000(7280) + 300}{0.052 \times 10000} = 15.73.$$

The amount of barite required is

$$m_B = 42 \times 28.02 \frac{(15.73 - 14)}{(28.02 - 15.73)} = 132593\ \text{lbm}.$$

Assuming 100 lbm per sack, the number of sacks of barite required is 1325.

## 4.7 Acidity–Alkalinity

pH is given as the negative logarithm of $[H^+]$ or $[OH^-]$ and is a measurement of the acidity of a solution. It is easy to compare by expressing it as below:

$$pH = -\log([H^+]), \tag{4.31}$$

$$pH = -\log([OH^-)], \tag{4.32}$$

where $[H^+]$ or $[OH^-]$ are hydrogen and hydroxide ion concentrations, respectively, in moles/litter.

Also, at room temperature, $pH + pOH = 14$.

For other temperatures,

$$pH + pOH = pK_w, \tag{4.33}$$

where $K_w$ = the ion product constant at that particular temperature.

At room temperature, the ion product constant for water is $1.0 \times 10^{-14}$ moles/litter (mol/L or M).

A solution in which

$$\left[H^+\right] > \left[OH^-\right] \tag{4.34}$$

is acidic, and a solution in which

$$\left[H^+\right] < \left[OH^-\right] \tag{4.35}$$

is basic .

Table 4.2 provides the ranges of acidity/alkalinity.

Moles per litter of hydroxide ion concentration required to change from one $pH_1$ to another $pH_2$ of a solution can be given as

$$\Delta\left[OH^-\right] = 10^{(pH_2-14)} - 10^{(pH_1-14)}\ \text{mol/L}. \tag{4.36}$$

The amount of material required = $\Delta[OH^-] \times$ MW gm/L, where MW = molecular weight of the material.

**Table 4.2** Acidity/alakalinity ranges.

| pH | [H⁺] | Solution |
|---|---|---|
| $<7$ | $>1.0 \times 10^{-7}$ M | Acid |
| $>7$ | $<1.0 \times 10^{-7}$ M | Basic |
| 7 | $= 1.0 \times 10^{-7}$ M | Neutral |

**Problem 4.33**

An aqueous potassium hydroxide completion fluid has a pH of 9. Determine the hydrogen ion concentration of this solution?

*Solution:*

From the $pH$ equation $pH = -\log([H^+])$, hydrogen ion concentration can be given as

$$\left[H^+\right] = 10^{-pH} = 10^{-10} = 1 \times 10^{-9} \text{ mol/L (M)}.$$

**Problem 4.34**

An aqueous hydrochloric acid solution has a pH of 5.24. Estimate the mass of the hydrochloric acid present in one litter of this solution. Use a molar mass of HCL = 36.5 g/mol.

*Solution:*

From the pH equation $pH = -\log([H^+])$, the hydrogen ion concentration can be given as

$$\left[H^+\right] = 10^{-pH} = 10^{-5.24} = 5.8 \times 10^{-6} \text{ mol/L}.$$

Mass of hydrochloric acid = molar mass (g/mol) × hydrogen ion concentration (mol/L).

Mass of hydrochloric acid = $36.5 \times 5.8 \times 10^{-6} = 2.1 \times 10^{-4}$ hydrogen ion concentration (mol/L).

**Problem 4.35**

An aqueous solution has a pH of 12.3. Determine the concentration of $H^+$ and $OH^-$ in mole per litter.

*Solution:*

Hydrogen ion concentration is

$$\left[H^+\right] = 10^{-pH} = 10^{-12.3} = 2.0 \times 10^{-13} \text{ mol/L}.$$

Using the relationship,

$$pH + pOH = 14,$$

$$pOH = 14 - 12.3 = 1.3.$$

Therefore, $OH^-$ concentration is

$$\left[OH^-\right] = 10^{-pOH} = 10^{-1.3} = 0.05012 \text{ mol/L}.$$

**Problem 4.36**

Calculate the amount of caustic required to raise the pH from 8 to 10. The molecular weight of caustic is 40.

*Solution:*

The moles of hydroxide concentration for increasing the pH can be given as

$$\Delta\left[OH^-\right] = 10^{(pH_2-14)} - 10^{(pH_1-14)}.$$

Substituting the values,

$$\Delta\left[OH^-\right] = 10^{(10-14)} - 10^{(8-14)} = 9.9 \times 10^{-5} \text{ mol/L}.$$

The weight of caustic required per liter of solution = $40 \times 9.9 \times 10^{-5} = 0.00396$ g/L.

## 4.8 Marsh Funnel

The time for fresh water to drain = 26 secs ± 0.5 second per quart for API water at 70°F + 0.5°F.

## 4.9 Mud Rheology

Shear stress ($\tau$) and shear rate ($\gamma$) for Newtonian fluid are given as

$$\tau = \mu\gamma, \tag{4.37}$$

where $\mu$ is the Newtonian viscosity.

In engineering units, $\tau$ = dynes/cm$^2$ = 4.79 lb/100 ft$^2$, $\gamma$ = sec$^{-1}$, and $\mu$ = poise = dyne $\times$ sec/cm$^2$.

The field unit of viscosity is the centipoise (1 poise = 100 centipoise). The field unit of shear stress is lb/100 ft$^2$.

For the Bingham plastic model, the relationship is given by

$$\tau = \tau_y + \mu_p\gamma. \tag{4.38}$$

For the power law model, shear stress ($\tau$) and shear rate ($\gamma$) are given by

$$\tau = K\gamma^n. \tag{4.39}$$

Shear rate and shear stress relationship for various fluids are given in Figures 4.1 to 4.3.

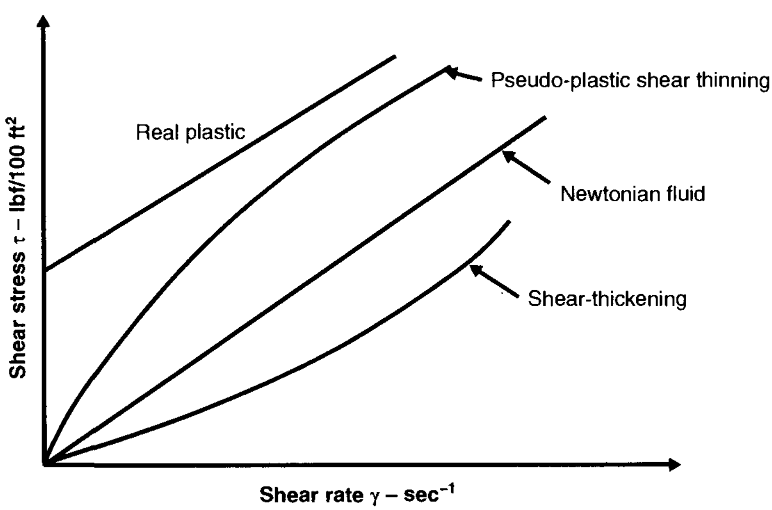



**Figure 4.1** Shear rate – shear stress relationship of time independent non-Newtonian fluids.

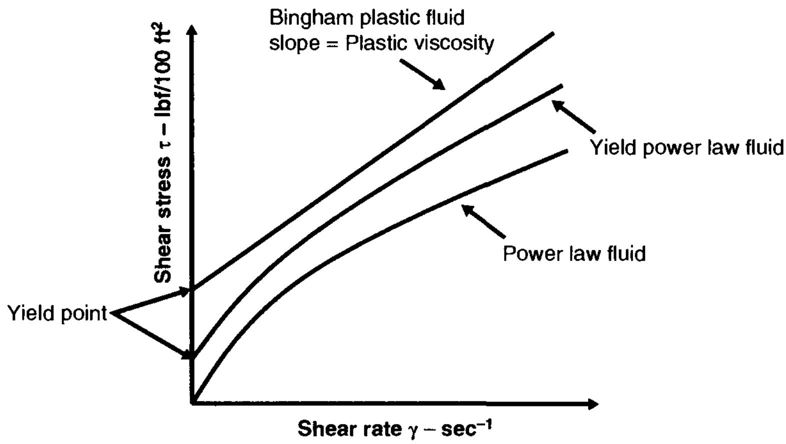



**Figure 4.2** Shear rate – shear stress relationship of non-Newtonian fluids.

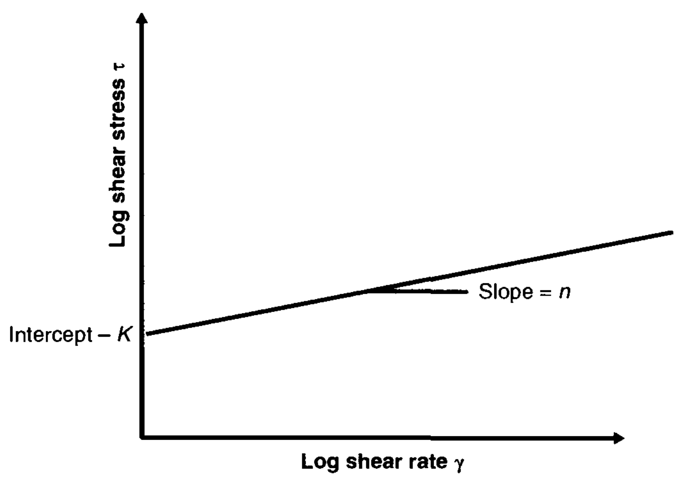



**Figure 4.3** Shear rate – shear stress relationship of a power-law fluid on logarithmic scale.

For the yield power law (Herschel Bulkley), shear stress ($\tau$) and shear rate ($\gamma$) are given by

$$\tau = \tau_y + K\gamma^n, \tag{4.40}$$

where $\tau_y$ = the yield value or yield stress, $\mu_p$ = Bingham plastic viscosity, $K$ = the consistency index, and $n$ = the power law index.

## 4.10 Plastic Viscosity, Yield Point and Zero-Sec-Gel

### 4.10.1 Bingham Plastic Model

Plastic viscosity, yield point, and zero-sec-gel can be calculated from the Fann reading using the following relationships:

$$PV = \theta_{600} - \theta_{300}, \tag{4.41}$$

$$YP = 2\theta_{300} - \theta_{600}, \tag{4.42}$$

$$\tau_0 = \theta_3. \tag{4.43}$$

Alternatively, the dial readings can be reverse calculated by using $PV$, $YP$, and zero-gel as shown below:

$$\theta_{300} = PV + YP, \tag{4.44}$$

$$\theta_{600} = 2PV + YP, \tag{4.45}$$

$$\theta_3 = \tau_0,$$

where $\theta_{600}$ = the Fann dial reading at 600 rpm, $\theta_{300}$ = the Fann dial reading at 300 rpm, and $\theta_3$ = the dial reading at 3 rpm.

If the Fann RPMs are anything other than 600 and 300, the following relationship can be used:

$$\mu_p = \frac{300}{N_2 - N_1}\left(\theta_{N_2} - \theta_{N_1}\right), \tag{4.46}$$

$$\tau_y = \theta_{N_1} - \mu_p \frac{N_1}{300}.$$

### 4.10.2 Shear Stress and Shear Rate

Shear stress and shear rate can be calculated using the following relationships:

$$\tau_1 = (0.01065)\theta\left(\frac{\text{lbf}}{\text{ft}^2}\right), \tag{4.47}$$

$$\tau_2 = (1.065)\theta\left(\frac{\text{lbf}}{100\ \text{ft}^2}\right), \tag{4.48}$$

$$\gamma = (1.703)\text{N}\left(\frac{1}{\text{sec}}\right), \tag{4.49}$$

where N = the dial speed in (1/sec).

### 4.10.3 Power Law

The rheological equation for the power law model can be given as

$$\tau = K\gamma^n, \tag{4.50}$$

where $\gamma$ = the shear rate (1/sec), and $\tau$ = the shear stress (lb/ft$^2$).

The flow behavior index can be given as

$$n = 3.322\log\left(\frac{\theta_{600}}{\theta_{300}}\right), \tag{4.51}$$

and for the modified power law it can be given as

$$n = 3.322\log\left(\frac{YP + 2PV}{YP + PV}\right), \tag{4.52}$$

where $PV$ = plastic viscosity, and $YP$ = yield point.

The consistency index, $K$, is given as

$$K = \frac{510\theta_{300}}{(511^n)}\ \text{eq.cP} \tag{4.53}$$

$K$ can be expressed in (lb × sec$^n$/ft$^2$) using the conversion factor:

$$\left(\frac{\text{lb}\times\text{sec}^n}{\text{ft}^2}\right) = 0.002088543\times\text{eq.cP}.$$

The consistency index, $K$, for the modified power law is given as

$$K = \frac{YP + 2PV}{(100)(1022^n)} \tag{4.54}$$

If the Fann RPMs are anything other than 600 and 300, the following relationship can be used:

$$n = \frac{\log\left(\dfrac{\theta_{N_2}}{\theta_{N_1}}\right)}{\log\left(\dfrac{N_2}{N_1}\right)}, \tag{4.55}$$

$$K = \frac{510\theta_N}{(1.703 \times N)^n}. \tag{4.56}$$

**Problem 4.37**

Using the following Fann data, calculate the *PV* and *YP* for the Bingham plastic model and *n* and *K* for the power law model:

- $\theta_{600} = 48$ and $\theta_{300} = 28$
- $\theta_{400} = 44$ and $\theta_{200} = 29$

*Solution:*

*Bingham plastic model:*

Plastic and yield point are calculated as

$$PV = \theta_{600} - \theta_{300} = 48 - 28 = 20 \text{ cP},$$

and

$$YP = 2\theta_{300} - \theta_{600} = 2 \times 28 - 48 = 8 \text{ lbf/100 ft}^2.$$

For the second set of readings,

$$\mu_p = \frac{300}{N_2 - N_1}\left(\theta_{N_2} - \theta_{N_1}\right) = \frac{300}{400 - 200}(44 - 29) = 22.5 \text{ cP},$$

$$\tau_y = \theta_{N_1} - \mu_p \frac{N_1}{300} = 29 - 22.5 \times \frac{200}{300} = 14 \text{ lbf/100 ft}^2.$$

*Power law model:*

Flow behavior and consistency indices are calculated as

$$n = 3.322\log\left(\frac{\theta_{600}}{\theta_{300}}\right) = 3.322\log\left(\frac{48}{28}\right) = 0.777,$$

and

$$K = \frac{510\theta_{300}}{\left(511^{n}\right)} = \frac{510\times 28}{\left(511^{0.777}\right)} = 111.84 \text{ eq.cP.}$$

Using the conversion $\left(\frac{\text{lb}\times\text{sec}^{n}}{\text{ft}^2}\right) = 0.002088543\times\text{eq.cP},$

$$K = 111.84\times 0.002088543 = 0.0020\left(\frac{\text{lb}\times\text{sec}^{0.77}}{\text{ft}^2}\right).$$

For the second set of readings,

$$n = \frac{\log\left(\frac{\theta_{N_2}}{\theta_{N_1}}\right)}{\log\left(\frac{N_2}{N_1}\right)} = \frac{\log\left(\frac{44}{29}\right)}{\log\left(\frac{400}{200}\right)} = 0.601,$$

$$K = \frac{510\times 29}{(1.703\times 300)^{0.601}} = 443.55 \text{ eq.cP.}$$

Using the conversion $\left(\frac{\text{lb}\times\text{sec}^{n}}{\text{ft}^2}\right) = 0.002088543\times\text{eq.cP},$

$$K = 443.55\times 0.002088543 = 0.00926\left(\frac{\text{lb}\times\text{sec}^{0.77}}{\text{ft}^2}\right).$$

**Problem 4.38**

The following Fann data are obtained while weighting up the mud. Estimate the plastic viscosity and yield point using the Bingham plastic model.

- $\theta_{600} = 63$ and $\theta_{300} = 41$
- $\theta_{200} = 26$ and $\theta_{100} = 21$
- $\theta_6 = 9$ and $\theta_3 = 8$

*Solution:*

The Fann data given are converted to shear stress and stress as shown in the table below.

**Table 4.3** Shear rate and shear stress – problem 4.38.

| RPM | Shear Rate (1/sec) | Dial Reading | Shear Stress lbf-s 100ft² |
|---|---|---|---|
| 600 | $1.703 \times 600 = 1021.8$ | 63 | $1.065 \times 63 = 67.095$ |
| 300 | $1.703 \times 300 = 510.9$ | 41 | $1.065 \times 41 = 43.665$ |
| 200 | $1.703 \times 200 = 340.6$ | 26 | $1.065 \times 26 = 27.69$ |
| 100 | $1.703 \times 100 = 170.3$ | 21 | $1.065 \times 21 = 22.365$ |
| 6 | $1.703 \times 6 = 10.218$ | 9 | $1.065 \times 9 = 9.585$ |
| 3 | $1.703 \times 3 = 5.109$ | 8 | $1.065 \times 8 = 8.52$ |

It is easy to use an Excel spreadsheet and, using a linear fit, the slope and intercept can be found:

- Slope = 0.0576 lbf-s/ft
- Intercept = 10.048 lbf-s 100ft$^2$

The slope and intercept are the viscosity and yield point, respectively.

Using the conversion $(\text{lb} \times \text{s/ft}^2) = 4.79 \times 10^4$ cP, equivalent viscosity is

$$0.0576 \times \frac{4.79 \times 10^4}{100} \text{ cP.}$$

Intercept = yield point = 10.048 lbf/100 ft$^2$.

**Problem 4.39**

Using the following Fann data, estimate the flow behavior index and consistency index using the power law model.

- $\theta_{600} = 52$ and $\theta_{300} = 40$
- $\theta_{200} = 33.6$ and $\theta_{100} = 26$
- $\theta_6 = 9$ and $\theta_3 = 6.5$

*Solution:*

The above Fann data are converted to shear stress and stress as shown in the table below.

**Table 4.4** Shear rate and shear stress – problem 4.39.

| RPM | Shear Rate (1/sec) | Dial Reading | Shear Stress Lbf/100 ft$^2$ |
|---|---|---|---|
| 600 | 1.703 × 600 = 1021.8 | 52 | 1.065 × 52= 55.38 |
| 300 | 1.703 × 300 = 510.9 | 40 | 1.065 × 40 = 42.6 |
| 200 | 1.703 × 200 = 340.6 | 33.6 | 1.065 × 33.6 = 35.784 |
| 100 | 1.703 × 100 = 170.3 | 26 | 1.065 × 26 = 27.69 |
| 6 | 1.703 × 6 = 10.218 | 9 | 1.065 × 9 = 9.585 |
| 3 | 1.703 × 3 = 5.109 | 6.5 | 1.065 × 6.5 = 6.922 |

Using an Excel spreadsheet and a power law fit, $n = 0.3878$ and $K = 0.0377$ lbf/ft$^2$.

*Formulas and Calculations for Drilling Operations*
by G. Robello Samuel


# 5

# Hydraulics

This chapter focuses on the different basic calculations involved in the rig hydraulics and associated operations.

## 5.1 Equivalent Mud Weight

The pressure in the wellbore or formation can be expressed in terms of equivalent mud weight (EMW). This is a convenient way to compare the pressures at any depth. EMW is calculated as

$$\text{EMW} = \frac{P_h}{0.052 \times L_{tvd}} \text{ppg}. \tag{5.1}$$

where $L_{tvd}$ = true vertical depth (TVD), ft, and $P_h$ = pressure, psi.

If the well is deviated $\alpha$ deg from the vertical, the EMW is given by

$$\text{EMW} = \frac{P_h}{0.052 \times D_h \cos\alpha} \text{ppg}. \tag{5.2}$$

where $D_h$ = is the measured depth, ft.

## 5.2 Equivalent Circulating Density

Equivalent circulating density results from the addition of the equivalent mud weight, due to the annulus pressure losses ($\Delta p_a$), to the original mud weight ($\rho_m$). This is calculated as

$$\mathrm{ECD} = \rho_m + \frac{\Delta p_a}{0.052 \times L_{tvd}} \text{ ppg.} \tag{5.3}$$

In deviated wells vertical depth should be used, and the equation for multiple sections is given by

$$\mathrm{ECD} = \rho_m + \left( \frac{\sum_{i=1}^{n} \Delta p_a}{0.052 \times \sum_{i=1}^{n} \Delta L_{tvd}} \right) \text{ppg.} \tag{5.4}$$

where $n$ is the number of wellbore sections.

**Problem 5.1**

Calculate the equivalent mud weight at a depth of 10,000 ft (TVD) with an annulus back pressure of 500 psi. The mud density in the annulus is 10 ppg.

*Solution:*

$$\mathrm{EMW} = \rho_m + \frac{500}{0.052 \times 10000} = 10.96 \text{ ppg.}$$

Alternatively, it can be calculated as follows.
Pressure exerted at the bottom is

$$P = 0.052 \times 10000 \times 10 + 500 = 5700 \text{ psi.}$$

Using equation 5.1,

$$\mathrm{EMW} = \frac{5700}{0.052 \times 10000} = 10.96 \text{ ppg.}$$

**Problem 5.2**

Calculate the ECD for the following data:

- Inclination of the well = 30 deg
- Measured depth = 5000 ft
- Calculated true vertical depth based on minimum curvature method = 4330 ft
- Annular pressure loss gradient = 0.03 psi/ft
- Mud weight = 9.2 ppg

*Solution:*

Total annular pressure loss = 0.03 × 5000 = 150 psi.

$$\Delta p_a = 0.036(\text{psi/ft}) \times 4500(\text{ft}) = 162 \text{ psi.}$$

Using equation 5.3,

$$\text{ECD} = 9.2 + \frac{150}{0.052 \times 4330} = 9.866 \,\text{ppg.}$$

## 5.3 Hydraulics: Basic Calculations

### 5.3.1 Critical Velocity

Critical velocity is the velocity at which the flow regime changes from laminar to turbulent. It can be determined for the Bingham plastic model.

$$V_c = \frac{1.08PV + 1.08\sqrt{PV^2 + 12.34\rho_m D_i^2 YP}}{\rho D_i}, \tag{5.5}$$

or

$$V_c = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 12.34\rho_m D_i^2 \tau_y}}{\rho D_i} \text{ft/sec}, \tag{5.6}$$

where $PV = \mu_p$ = plastic viscosity, cP, $YP = \tau_y$ = yield point, lbf/100 ft$^2$, and $\rho_m$ = mud density, ppg.

Critical flow rate is

$$Q_c = 2.448 \times V_c \times D_i^2 \text{ gpm,} \tag{5.7}$$

where $D_i$ = inside diameter of the pipe, in.

**Problem 5.3**

Using the following data, calculate the critical flow rate at which laminar flow changes to turbulent flow.

- Drill pipe ID = 3 ¾″
- Mud density 10 ppg, $\theta_{600} = 37$; $\theta_{300} = 25$
- Target depth = 10000 ft (TVD)

*Solution:*

Using the Fann reading, the yield and plastic viscosity are, respectively,

$$\tau_y = 2 \times 25 - 37 = 13 \text{ lbf/100ft}^2,$$

$$\mu_p = 37 - 25 = 12 \text{ cP.}$$

Using equation 5.5 to calculate the critical velocity inside the pipe, the critical velocity is

$$V_c = \frac{1.08\mu_p + 1.08\sqrt{\mu_p^2 + 12.34\rho_m D_i^2 \tau_y}}{\rho D_i},$$

$$V_c = \frac{1.08 \times 12 + 1.08\sqrt{12^2 + 12.34 \times 3.75^2 \times 12 \times 10}}{10 \times 3.75} = 4.69 \text{ ft/sec.}$$

Critical flow rate = velocity × flow area = 4.69 × 2.448 × $3.375^2$ = 161.3 gpm.

## 5.3.2 Pump Calculations

Pump pressure can be given as

$$P_p = \Delta p_b + P_d \text{ psi,} \tag{5.8}$$

where $\Delta p_b$ = bit pressure drop, psi, and $P_d$ = frictional pressure losses, psi.

Hydraulic horsepower (HHP) of the bit is

$$\frac{Q \times \Delta p_b}{1714} \text{hp}, \tag{5.9}$$

where $\Delta p_b$ = bit pressure drop, psi, and $Q$ = flow rate, gpm.

**Problem 5.4**

A pump is capable of a pressure of 2,000 psi at a circulation rate of 400 gpm. Frictional pressure loss is estimated to be 900 psi with 10 ppg mud. It is found that an increase in mud weight results in an increase of 55 bit HHP for the same flow rate. Estimate the new mud weight.

*Solution:*

Using equation 5.7, the pump pressure can be given as $P_p = \Delta p_b + P_d$, from which the bit pressure drop can be written as $\Delta p_b = P_p - P_d$.

Substituting the values, the bit pressure drop is

$$\Delta p_b = 2000 - 900 = 1100 \text{ psi.}$$

Using equation 5.9, the hydraulic horsepower of the bit is

$$\frac{400 \times 1100}{1714} = 256.7 \text{ hp.}$$

$$\text{New HP} = 256.7 + 55 = 311.7 \text{ hp.}$$

Bit pressure drop for the new hydraulic horsepower is

$$\Delta p_{b_1} = 311.7 \times \frac{1714}{400} = 1335.7 \text{ psi.}$$

Using the relationship,

$$\frac{\Delta p_{b_1}}{\Delta p_{b_2}} = \frac{\rho_1 Q_1^2}{\rho_2 Q_1^2} = 12.14 \text{ ppg,}$$

$$\frac{1100}{1335.7} = \frac{10 \times 400^2}{\rho_2 \times 400^2}.$$

Therefore,

$$\rho_2 = \frac{10 \times 400^2}{\left(\dfrac{1100}{1335.7}\right) \times 400^2} = 12.14\,\text{ppg}.$$

Bit hydraulic horsepower per square inch of bit (HSI) is

$$\text{HSI} = \frac{\text{HHP}}{\dfrac{\pi}{4} \times D_b^2}\,\text{hp/in}^2, \tag{5.10}$$

where $D_b$ = diameter of the bit, in.

Bit hydraulic horsepower per square inch of hole drilled (HSI) is

$$\text{HSI} = \frac{\text{HHP}}{\dfrac{\pi}{4} \times D_h^2}\,\text{hp/in}^2, \tag{5.11}$$

where $D_h$ = diameter of the hole, in.

**Problem 5.5**

At a certain depth while drilling a 12 ¼″ hole, the pump pressure is 3,000 psi, and parasitic pressure loss is 1,500 psi at a circulation rate of 500 gpm with the mud weight of 12.5 ppg. Calculate the flow rate to achieve a bit HSI of 1.2 with the mud weight remaining the same. $C_d = 0.95$.

*Solution:*

Using the pump pressure drop equation, $P_p = \Delta P_b + P_d$, bit pressure drop can be calculated as

$$\Delta p_b = 3000 - 1500 = 1500\text{ psi}$$

$$\text{HSI} = \frac{\text{HHP}}{\dfrac{\pi}{4} \times 12.25^2} = 1.2.$$

Substituting the equation for new HHP and other respective values, it can be written as

$$1.2 = \frac{\dfrac{\Delta p_{b_1} \times Q_1}{1714}}{\dfrac{\pi}{4} \times 12.25^2} = \frac{\dfrac{\frac{1500 \times Q_1^2}{500^2} \times Q_1}{1714}}{\dfrac{\pi}{4} \times 12.25^2} = \frac{1500 \times Q_1^3}{500^2 \times 1714 \times \dfrac{\pi}{4} \times 12.25^2}.$$

Solving for $Q$, flow rate will be 343 gpm.

**Problem 5.6**

At a certain depth during drilling, the pump pressure is 3,000 psi, and parasitic pressure loss is 1,500 psi at a circulation rate of 500 gpm. The mud weight is 12.5 ppg. Calculate the available bit hydraulic horsepower if the mud weight is increased to 14.5 ppg and the circulation rate remains same at 500 gpm.

*Solution:*

$$P_p = \Delta p_b + P_d,$$

$$\Delta p_b = 3000 - 1500 = 1500 \text{ psi}.$$

$$\Delta p_b = \frac{8.311 \times 10^{-5} \rho Q^2}{C_d^2 A_n^2}.$$

$$\frac{\Delta p_{b_1}}{\Delta p_{b_2}} = \frac{\rho_1 Q_1^2}{\rho_2 Q_1^2},$$

$$\Delta p_{b_1} = 1500 \times \frac{14.5}{12.5} = 1740 \text{ psi}.$$

$$\text{Bit HHP} = \frac{1740 \times 500}{1714} = 507 \text{ hp}.$$

## 5.4 Bit Hydraulics

### 5.4.1 Basic Calculations

Total nozzle flow area is calculated as

$$A_n = \frac{\pi \times S_n^2}{64} \text{in}^2. \tag{5.12}$$

Nozzle velocity is calculated as

$$V_n = 0.3208 \frac{Q}{A_n} \text{ ft/sec.} \tag{5.13}$$

The pressure drop across bit is calculated as

$$\Delta p_b = \frac{8.311 \times 10^{-5} \rho_m Q^2}{C_d^2 A_n^2} \text{ psi,} \tag{5.14}$$

where $C_d$ = discharge coefficient (usually .95 used).

The pressure drop across bit can also be calculated with a 0.95 discharge coefficient as

$$\Delta p_b = \frac{\rho_m \times Q^2}{10858 \times A_n^2} \text{ psi.} \tag{5.15}$$

The pressure drop across the bit is calculated with a nozzle velocity as

$$\Delta p_b = \frac{\rho_m \times V_n^2}{1120} \text{ psi.} \tag{5.16}$$

The percentage of pressure drop across bit is calculated as

$$\frac{\Delta p_b}{\Delta p} \times 100. \tag{5.17}$$

The bit hydraulic power (BHHP) is calculated as

$$HP_b = \frac{Q \times \Delta p_b}{1714} \text{ hp.} \tag{5.18}$$

Hydraulic bit (jet) impact force (IF) is calculated as

$$F_{imp} = 0.01823 \times C_d \times Q \sqrt{\rho_m \times \Delta p_b} \text{ lbf.} \tag{5.19}$$

Hydraulic bit (jet) impact force (IF) can also be written as

$$F_{imp} = \frac{Q \times \sqrt{\rho_m \times \Delta p_b}}{57.66} \text{ lbf.} \tag{5.20}$$

Hydraulic bit (jet) impact force (IF) with nozzle velocity can be written as

$$F_{imp} = \frac{\rho_m \times Q \times V_n}{1930} \text{lbf}. \tag{5.21}$$

Impact force per square inch of the bit area is calculated as

$$\frac{F_{imp}}{\left(\frac{\pi \times D_b^2}{4}\right)} \text{lbf/in}^2. \tag{5.22}$$

Impact force per square inch of the hole area is calculated as

$$\frac{F_{imp}}{\left(\frac{\pi \times D_h^2}{4}\right)} \text{lbf/in}^2. \tag{5.23}$$

## 5.4.2 Optimization Calculations

Flow index is

$$m = \left( \frac{\log\left(\frac{(\Delta p_{\mathrm{d}})_{\mathrm{j}}}{(\Delta p_{\mathrm{d}})_{\mathrm{i}}}\right)}{\log\left(\frac{Q_{\mathrm{j}}}{Q_{\mathrm{i}}}\right)} \right). \tag{5.24}$$

For maximum bit hydraulic horsepower, the optimum bit pressure drop is

$$\Delta P_{bopt} = \frac{m}{m+1} \Delta p_{pmax}, \tag{5.25}$$

and the optimum flow rate is

$$Q_{opt} = Q_a \text{alog}\left[\frac{1}{m} \log\left(\frac{\Delta p_{d_{opt}}}{\Delta p_{d_{Q_a}}}\right)\right]. \tag{5.26}$$

### *5.4.2.1 Limitation 1 – Available Pump Horsepower*

For maximum impact force, the optimum bit pressure drop is

$$\Delta p_{b_{opt}} = \frac{m+1}{m+2} \Delta p_{p_{opt}}. \tag{5.27}$$

The optimum flow rate is

$$Q_{opt} = \left( \frac{2 \times \Delta p_{p\max}}{c(m+2)} \right)^{\frac{1}{m}}. \tag{5.28}$$

### *5.4.2.2 Limitation 2 – Surface Operating Pressure*

For maximum impact force, the optimum bit pressure drop is

$$\Delta p_{b_{opt}} = \frac{m}{m+2} \Delta p_{p_{max}}. \tag{5.29}$$

**Problem 5.7**

Estimate the optimum nozzle size and optimum flow rate for the following conditions:

- $m = 1.66$
- Maximum allowed operating pressure = 5440 psi
- Frictional pressure loss = 2334 psi for a flow rate of 300 gpm
- Volumetric efficiency of the pump = 80%
- Mud weight = 15.5 ppg
- Minimum flow rate required for holecleaning = 265 gpm

Use limiting condition 2 for the calculations.

*Solution:*

Using limiting condition 2,

$$\Delta p_{d_{opt}} = \frac{2}{m+2} \Delta p_{p_{max}},$$

$$\Delta p_{d\,opt} = \frac{2}{1.66+2} \times 5440 = 2972.7 \text{ psi},$$

$$\Delta p_{b_{opt}} = \Delta p_{p_{max}} - \Delta p_{d\,opt},$$

$$\Delta p_{b_{opt}} = 5440 - 2975 = 2467.3 \text{ psi}.$$

Using the corresponding equation for the optimized flow rate,

$$Q_{opt} = Q_a a \log\left[\frac{1}{m}\log\left(\frac{\Delta p_{d_{opt}}}{\Delta p_{d_{Q_a}}}\right)\right]$$

$$Q_{opt} = 300\, a \log\left[\frac{1}{1.66}\log\left(\frac{2975}{2334}\right)\right] = 359 \text{ gpm},$$

$$\Delta p_{b\,opt} = \frac{8.3\times 10^{-5} \times 15.5 \times 347^2}{0.95^2 \times A_{nopt}^2} = 2465 \text{ psi},$$

$$A_{opt} = 0.2728 \text{ in}^2.$$

The average diameter of the nozzle can be calculated from the total area of the nozzle as three nozzles: 11-11-11.

**Problem 5.8**

The following friction pressure loss-flow rate relationship is given:

$$P_f = cQ^m,$$

where $P$ = friction pressure loss, $c$ = constant, $m$ = flow exponent, and $Q$ = flow rate.

Find an expression to determine the flow exponent, $m$, if the following measurements are known:

- $P_f = P_{fi}$ at $Q = Q_i$
- $P_f = P_{fj}$ at $Q = Q_j$

*Solution:*

Using the given relationships,

$$c = \frac{P_{fi}}{Q_i^m}$$

and

$$c = \frac{P_{fj}}{Q_j^m},$$

$$c = \frac{P_{fi}}{Q_i^m} = \frac{P_{fj}}{Q_j^m}.$$

Taking logarithm on both sides and solving for $m$,

$$m = \left( \frac{\log\left(\frac{P_{fi}}{P_{fi}}\right)}{\log\left(\frac{Q_j}{Q_i}\right)} \right),$$

or

$$m = \log\left(\frac{P_{fj}}{P_{fi}} - \frac{Q_j}{Q_i}\right).$$

**Problem 5.9**

Calculate the optimal nozzle size using the maximum bit hydraulic horsepower criterion with the following data:

- Frictional pressure loss is $P_d$ = 2400 psi for $Q$ = 350 gpm
- $m = 1.86$
- Maximum allowed pump operating pressure = 5000 psi
- Pump hydraulic horsepower = 1600 hp
- Pump volumetric efficiency = 80%

- Minimum flow rate required for holecleaning = 200 gpm
- Mud density = 10 ppg

*Solution:*

$$p_{d_{opt}} = \left(\frac{1}{m+1}\right)p_{p_{max}} = \frac{1}{1.86+1}\times 5000 = 1748\,\text{psi},$$

$$\Delta p_{b_{opt}} = \frac{1.86}{2.86}\times 5000 = 3252\,\text{psi}.$$

$$Q_{opt} = 350\times \text{alog}\left[\left(\frac{1}{1.86}\log\frac{1748}{2400}\right)\right] = 295\,\text{gpm},$$

$$Q_{\max} = \frac{1714\times 1500\times 0.85}{5000} = 437\,\text{gpm}.$$

Optimum flow rate check is

$$200\text{ gpm} < Q_{opt} < 403\text{ gpm}.$$

From the pressure drop equation,

$$\Delta p_b = \frac{8.311\times 10^{-5}\rho_m Q^2}{C_d^2 A_n^2},$$

the area of the nozzle can be estimated as

$$A_{nopt}^2 = \frac{8.3\times 10^{-5}\times 10\times 295^2}{0.95^2\times 3252} = 0.1568\,\text{in}^2.$$

**Problem 5.10**

Calculate the optimal flow rate optimal nozzle area using the bit impact force criterion (limitation 2) with the following data:

- Maximum allowed pump operating pressure = 5000 psi
- Minimum flow rate required for holecleaning = 300 gpm
- Mud density = 10 ppg

- Pump pressures of 4000 psi and 2500 psi were recorded while circulating mud at 500 gpm and 387 gpm, respectively, with 11-11-11 nozzles.

*Solution:*

$$m = \frac{\log\left(\frac{p_{d2}}{p_{d1}}\right)}{\log\left(\frac{Q_2}{Q_1}\right)} = \frac{\log\left(\frac{2970}{1779}\right)}{\log\left(\frac{500}{387}\right)} = 1.80.$$

Using the bit impact force criterion (limitation 2),

$$p_{dopt} = \left(\frac{2}{m+2}\right) p_{pmax} = \left(\frac{2}{1.8+2}\right) \times 5000 = 2630 \text{ psi}.$$

Optimum bit pressure drop can be estimated as

$$p_{b_{opt}} = p_{max} - p_{dopt} = 5000 - 2630 = 2369 \text{ psi}.$$

Using the optimum values, the optimum flow rate can be calculated as

$$Q_{opt} = Q_a \text{a}\log\left[\frac{1}{m}\log\left(\frac{\Delta p_{dopt}}{\Delta p_{da}}\right)\right] = 387 \times \text{a}\log\left[\frac{1}{1.8}\log\left(\frac{2630}{1779}\right)\right]$$
$$= 435 \text{ gpm}.$$

With the pressure drop equation and using the optimum flow rate, the corresponding optimum nozzle area can be calculated as

$$\Delta p_{b_{opt}} = \frac{8.3 \times 10^{-5} \times 10 \times 435^2}{0.95^2 \times A_n^2} = 2369 \text{ psi}.$$

$A_n^{\ 2} = 0.07337 \text{ in}^4$, and the total nozzle area is

$$A_n = \sqrt{0.07337} = 0.2708 \text{ in}^2.$$

**Problem 5.11**

Calculate the maximum flow rate at which the driller can pump if the maximum hydraulic horsepower is 1500 hp. The maximum bit hydraulic horsepower criterion yields an optimum flow rate of

350 gpm. Frictional pressure loss is $p_d = 1975$ psi for $Q = 300$ gpm. Calculate the HSI at optimum condition for a 12¼-inch hole. Use $m = 1.6$ and $C_d = 0.95$.

*Solution:*

Using the maximum bit hydraulic horsepower criterion,

$$\Delta p_{d_{opt}} = 1975 \times \text{a}\log\left[1.6\log\left(\frac{350}{300}\right)\right] = 2527 \text{ psi},$$

$$p_{d_{opt}}(m+1) = p_{p_{max}} = (1.6+1)\times 2527 = 6570 \text{ psi},$$

$$\Delta p_{d_{opt}} = 1975 \times \text{a}\log\left[1.6\log\left(\frac{350}{300}\right)\right] = 2527 \text{ psi},$$

$$Q_{max} = \frac{1714 \times 1500}{6570} = 391 \text{ gpm}.$$

The optimum bit pressure drop is

$$\Delta p_{b_{opt}} = 6570 - 2527 = 4044 \text{ psi}.$$

The optimum HSI is

$$\text{HSI}_{\text{bit}} = \frac{Q_{opt} \times \Delta P_{b_{opt}}}{1714 \times \frac{\pi}{4} d_b^2} = \frac{350 \times 4044}{1714 \times \frac{\pi}{4} 12.25^2} = 7 \text{ hp/in}^2.$$

**Problem 5.12**

A 10 ppg-liquid is being circulated at 540 gpm through the system shown in Figure 5.1. A triplex-single-acting pump, having a volumetric efficiency of 85%, is being used. What must be the pump hydraulic horsepower requirement for the above operating conditions? Assume the friction pressure loss gradient in the circulating system is 0.06 psi/ft. It is given that the hydrostatic pressure of a liquid column is given by $P = 0.052h$, where

$$P = 0.052\,h\rho,$$

where $h$ = column height in feet, and $\rho$ = liquid density in ppg.

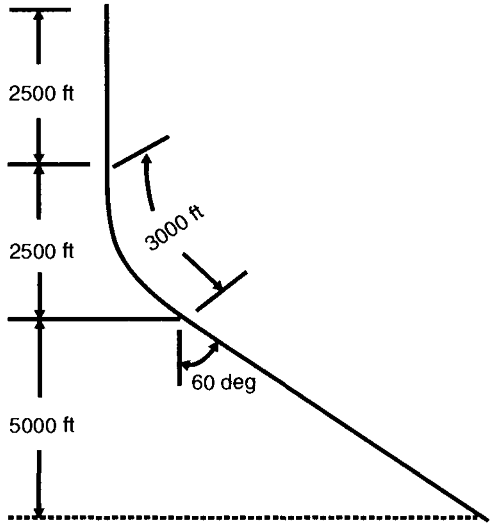



**Figure 5.1** Well profile for Problem 5.18.

*Solution:*

Total head acting (6800 – 6000) = 800 ft.
Hydrostatic pressure is $P = 0.052 \times 10 \times 800 = 416$ psi.
Frictional pressure loss in the pipe = (6000 + 1000 + 6000 + 800 + 8000) 0.06 = 1,308 psi.
Total pressure pump at the pump = 416 + 1308 = 1,724 psi.
Actual flow rate = 540 gpm.
Theoretical flow rate = 540/0.85 =635 gpm.

$$\text{Hydraulic horsepower} = \frac{Q\Delta P}{1714} = \frac{635 \times 1{,}724}{1714} = 640 \text{ hp.}$$

**Problem 5.13**

A bit currently has 3 × 12 nozzles. The driller has recorded that when 10 ppg mud is pumped at a rate of 500 gpm, a pump pressure of 3,000 psi is observed. When the pump is slowed to a rate of 250 gpm, a pump pressure of 800 psi is observed. The pump is rated at 2,000 hp and has an overall efficiency of 90%. The minimum flow rate to lift the cuttings is 240 gpm. The maximum allowable surface pressure is 5,000 psi.

A. Determine the pump operating conditions and bit nozzle sizes for maximum bit horsepower for the next bit run.
B. What bit horsepower will be obtained at the conditions selected?

*Solution:*

A. Bit pressure drop is 2097 psi for 500 gpm. Therefore, the frictional pressure losses can be calculated using the following equation:

$$P_{fl} = P_p - P_b = 3000 - 2097 = 903 \text{ psi}.$$

Using equation 5.24, the flow index $m$ can be found as

$$m = \frac{\log \frac{903}{275.5}}{\log \frac{500}{250}} = 1.71.$$

$$Q_{max} = 1714 \times 0.9 \times \frac{2000}{5000} = 617 \text{ gpm}.$$

Find the optimum friction pressure loss using the maximum hydraulic horsepower criterion:

$$P_{fopt} = \frac{1}{m+1} P_{pmax} = 1845 \text{ psi},$$

$$P_{bopt} = 5000 - 1845 = 3155 \text{ psi}.$$

$$Q_{opt} = 500\, a\log\left[\frac{1}{1.71}\log\frac{1845}{900}\right] = 761 \text{ gpm}.$$

Check the lowest limit, which is 240 gpm. Use the maximum flow rate as it cannot be exceeded, and calculate the area of the nozzle and the nozzle sizes, 12-12-12.

B. The nozzle pressure drop can be calculated using equation 5.14:

$$\Delta p_b = \frac{8.3 \times 10^{-5} \rho \times Q^2}{C_d^2 \times A_n^2} = \frac{8.3 \times 10^{-5} \times 10 \times 617^2}{0.95^2 \times 0.3313^2} = 3189 \text{ psi}.$$

The bit hydraulic horsepower is

$$\text{HHP}_b = \frac{Q \Delta P_b}{1714} = \frac{617 \times 3189}{1714} = 1148 \text{ hp}.$$

**Problem 5.14**

Using the data in Problem 5.13, calculate the optimum nozzle sizes for the maximum impact force (limitation 2) criteria for the next bit run.

*Solution:*

If the bit pressure drop is 2097 psi for 500 gpm, then the frictional pressure losses can be calculated using the following equation:

$$P_{f1} = P_p - \Delta p_b = 3000 - 2097 = 903 \text{ psi}.$$

The flow index $m$ can be obtained using equation 5.24:

$$m = \frac{\log \frac{903}{275.5}}{\log \frac{500}{250}} = 1.71.$$

$$Q_{max} = 1714 \times 0.9 \times \frac{2000}{5000} = 617 \text{ gpm}.$$

$$Q_{min} = 240 \text{ gpm}.$$

Find the optimum friction pressure loss using the maximum hydraulic horsepower criterion:

$$P_{fopt} = \frac{2}{m+2} P_{pmax} = \frac{2}{3.71} \times 5000 = 2695.4 \text{ psi},$$

$$\Delta p_{bopt} = 5000 - 2695.4 = 2304.6 \text{ psi},$$

$$Q_{opt} = 500 \text{ a} \log\left[\frac{1}{1.71} \log \frac{2695.4}{900}\right] = 950 \text{ gpm}.$$

Since the calculated 950 gpm is greater than the maximum flow rate, the optimum flow rate is 617gpm. This flow rate is greater than the required minimum flow rate.

Using this flow rate, the optimum nozzle area can be found as

$$A_n = \sqrt{\frac{8.3 \times 10^{-5} \rho \times Q^2}{C_d^2 \times \Delta P_b}} = \sqrt{\frac{8.3 \times 10^{-5} \times 10 \times 617^2}{0.95^2 \times 2304}} = 0.39 \text{ in}^2$$

Nozzle sizes can be 13-13-13.

## 5.5 Bingham Plastic Model

### 5.5.1 Reynolds Number

For the pipe side, the Reynolds number is calculated as

$$N_{\mathrm{Re}p} = \frac{928 \rho_m v_p D_i}{\mu_{ep}}, \tag{5.30}$$

where $\rho_m$ = mud weight in ppg, $v_p$ = velocity of the fluid in fps,

$$v_p = \frac{Q}{2.448 D_i^2} \text{fps}, \tag{5.31}$$

$Q$ = flow rate, gpm, $D_i$ = inside diameter of the pipe, in, $\mu_{ep}$ = equivalent viscosity of the fluid and is calculated as

$$\mu_{ep} = \mu_p + \frac{20 D_i \tau_y}{3 v_p}, \tag{5.32}$$

$\mu_p$ = plastic viscosity, cP, and $\tau_y$ = yield point, lbf/100 ft$^2$.

The pressure gradient for the unit length $dL$ and for the laminar flow is calculated as

$$\left(\frac{dp_f}{dL}\right) = \frac{\mu_p v_p}{1{,}500 D_i^2} + \frac{\tau_y}{225 D_i} \text{psi/ft}. \tag{5.33}$$

For the annulus side,

$$N_{\mathrm{Re}a} = \frac{757 \rho_m v_a \left(D_2 - D_p\right)}{\mu_{ep}}, \tag{5.34}$$

$$\mu_{ep} = \mu_p + \frac{5\left(D_2 - D_p\right)\tau_y}{v_a}, \tag{5.35}$$

where $D_2$ = annulus diameter, in, $D_p$ = outside diameter of the pipe, in, and $v_a$ = velocity of the fluid in fps,

$$v_a = \frac{Q}{2.448\left(D_2^2 - D_p^2\right)} \text{ft/sec}. \tag{5.36}$$

The pressure gradient for laminar flow is calculated as

$$\left(\frac{dp_f}{dL}\right) = \frac{\mu_p v_a}{1{,}000(D_2 - D_p)^2} + \frac{\tau_y}{200(D_2 - D_p)} \text{psi/ft.} \quad (5.37)$$

**Problem 5.15**

Calculate the change in the bottomhole pressure when the yield value is 5 lb/100 ft² and the viscosity is 30 cp. Use the following data:

- Hole size = 9 ⅞″
- Depth = 10,000 ft
- Pipe OD = 4 ½″
- Flow rate = 400 gpm
- Mud weight = 12 ppg
- Yield point = 60 lb/100 ft²
- Plastic viscosity = 40 cp

*Solution:*

The annular velocity of the fluid is

$$v_a = \frac{400}{2.448\left(9.875^2 - 4.5^2\right)} = 2.11\,\text{ft/sec.}$$

The equivalent viscosity is

$$\mu_{ep} = 40 + \frac{5 \times 60\,(9.875 - 4.5)}{2.11} = 802.5\,\text{cP.}$$

The Reynolds number is

$$N_{\text{Re}a} = \frac{757 \times 12 \times 2.11 \times (9.875 - 4.5)}{802.5} = 128.$$

Since the Reynolds's number is less than 2100, the flow is laminar. The equivalent viscosity with new YP and PV is

$$\mu_{ep} = 5 + \frac{5 \times 30\,(9.875 - 4.5)}{2.11} = 93.5\text{ cP,}$$

and the Reynolds's number is 1104. Therefore, the flow is still laminar.

Using equation 5.37,

$$\left(\frac{dp_f}{dL}\right) = \frac{(40-5)v_a}{1,000(9.875-4.5)^2} + \frac{(60-30)}{200(9.875-4.5)} = 0.051895\,\text{psi/ft.}$$

$$\text{Total pressure change} = 0.051895 \times 10000 = 519\ \text{psi.}$$

**Problem 5.16**

Two speed viscometer readings are $\theta_{600} = 52$ and $\theta_{300} = 35$. Mud density = 10 ppg. The hole size is 8 ¾″, and the OD of the drill string is 4 ½″.

A. Using the Bingham plastic model, find out whether the flow is laminar or turbulent for a flow rate of 400 gpm.
B. Find the velocity at which the flow changes the regime.

*Solution:*

Viscosity is

$$\mu_p = \theta_{600} - \theta_{300} = 52 - 35 = 17\ \text{cP.}$$

Yield point is

$$\tau_y = 35 - 17 = 18\ \text{lb/100 ft}^2.$$

Annular velocity of the fluid is

$$v_a = \frac{400}{2.448\left(8.75^2 - 4.5^2\right)} = 2.90\ \text{ft/sec.}$$

The equivalent viscosity is

$$\mu_{ep} = 17 + \frac{5(8.75-4.5)18}{2.90} = 148.9\ \text{cP.}$$

The Reynolds number is

$$N_{\text{Re}a} = \frac{757 \times 10 \times 2.9 \times (8.75-4.5)}{148.9} = 626.$$

The flow is laminar.

**Problem 5.17**

Two speed viscometer readings are $\theta_{600} = 52$ and $\theta_{300} = 34$. Mud density = 10 ppg. The hole size is 8 ½″, and the OD of the drill string is 4 ½″.

A. Using the Bingham plastic model, find out whether the flow is laminar or turbulent for a flow rate of 400 gpm.
B. Find out the percentage increase in the pressure loss gradient when the mud density is 11 ppg.

*Solution:*

$$\mu_p = \theta_{600} - \theta_{300} = 52 - 34 = 18 \text{ cP.}$$

$$\tau_y = 35 - 18 = 16 \text{ lb/100 ft}^2.$$

$$v_a = \frac{400}{2.448\left(8.5^2 - 4.5^2\right)} = 3.14 \text{ ft/sec.}$$

$$\mu_{ep} = 18 + \frac{5(8.5 - 4.5)16}{3.14} = 119.9 \text{ cP.}$$

$$N_{\text{Re}a} = \frac{757 \times 10 \times 3.14 \times (8.5 - 4.5)}{119.9} = 792.$$

Flow is laminar.

**Problem 5.18**

Determine the pump hydraulics horsepower requirement in order to drill to a target depth of 10,000 ft (TVD). Assume a pump volumetric efficiency of 85%. Calculate the bottomhole pressure and equivalent circulating density (ECD). Also, plot the measured depth versus pressure for this condition. Use the Bingham plastic model and the following data:

- Drill pipe: 4.5″ OD, 3 ¾″ ID, 18.10 ppf
- Drill collar: 1200 ft, 7″ OD, 3 ¾″ ID, 110 ppf
- Surface connections: 250′ (length) × 4″ (inside diameter).
- Last intermediate casing set: 5000 (TVD) and 9 ⅞″ outside diameter; 0.3125 wall thickness
- Next hole size: 8 ½″ tri-cone roller, 3 × 14 jets

- Mud density: 10 ppg, $\theta_{600} = 37$; $\theta_{300} = 25$
- For annular hole cleaning, 120 fpm of fluid velocity is required.

*Solution:*

Use the thickness of the pipe and calculate the flow rate required to clean the hole against the drill pipe:

$$Q = v_{min} A_{adp} = 120 \times \frac{\pi}{4}\left(\left(\frac{9.25}{12}\right)^2 - \left(\frac{4.5}{12}\right)^2\right) \times 7.48 = 320\,\text{gpm},$$

$$\tau_y = 2 \times 25 - 37 = 13 \frac{\text{lbf}}{100\ \text{ft}^2},$$

$$\mu_p = 37 - 25 = 12\ \text{cP}.$$

*Pipe side calculation: drill pipe of 14,300 ft*

$$v_{dp} = \frac{320}{2.448 \times 3.75^2} = 9.3\ \text{ft/sec},$$

$$N_{He} = \frac{37100 \times 10 \times 13 \times 3.75^2}{12^2} = 47100,$$

$$N_{\text{Re}\,cr} \cong 13000,$$

$$N_{\text{Re}p} = \frac{928 \times 10 \times 9.3 \times 3.75}{12} = 26970.$$

Since $N_{\text{Re}\,cr} < N_{\text{Re}'}$, the flow regime is turbulent and the frictional pressure drop can be found to be $\Delta p_{fdp} = 790$ psi.

Similarly, the following pressure loss can be calculated:

For a drill collar of 1200 ft, the flow regime is turbulent, and the pressure drop is $\Delta p_{fdc} = 66$ psi.

Similarly, for surface connections the flow is turbulent, and the pressure drop is $\Delta p_{fdc} = 10$ psi.

Bit pressure drop can be estimated as

$$\frac{8.31 \times 10^{-5} \times 10 \times 320^2}{0.95^2 \times \left(3 \times \frac{\pi}{4}\left(\frac{14}{32}\right)^2\right)^2} = 463\ \text{psi}.$$

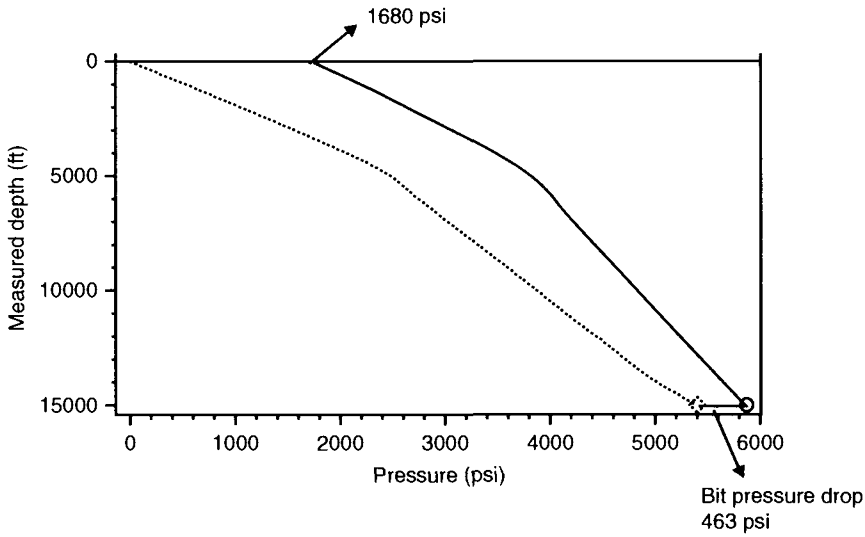



**Figure 5.2** Pressure plot.

*Annulus side calculation:*

Annulus 1 (hole/DC) length is 1200 ft.
Flow is turbulent and the pressure drop is $\Delta p_{afdc} = 110$ psi.
Annulus 2 (hole/DP) length is 8800 ft.
Flow is laminar and the pressure is $\Delta p_{afdp} = 160$ psi.
Annulus 3 (casing/DP) length is 5500 ft.
Flow is laminar and the pressure drop is $\Delta p_{afdp} = 80$ psi.

$$P_p = 790 + 66 + 10 + 463 + 110 + 160 + 80 = 1680 \text{ psi},$$

$$\text{HHP}_\text{p} = \frac{1680 \times 320}{1714} = 314 \text{ hp}.$$

The bottomhole pressure is

$$\begin{aligned} p_h + \Delta p_a &= 0.052 \times 10 \times \text{TVD} + \Delta p_a \\ &= 0.052 \times 10 \times 10000 + 350 = 5550 \text{ psi}. \end{aligned}$$

$$\begin{aligned} \text{Equivalent circulating density} &= \frac{\text{bottomhole pressure}}{0.052 \times \text{TVD}} \\ &= \frac{5550}{0.052 \times 10000} = 10.67 \text{ ppg}. \end{aligned}$$

The measured depth vs. pressure is shown in Figure 5.2.

## 5.6 Power Law Model

The power law constant for a pipe is calculated as

$$n_p = 3.32 \log\left(\frac{R_{600}}{R_{300}}\right). \tag{5.38}$$

The fluid consistency index ($K_p$) for a pipe is given by

$$K_p = \frac{5.11\,R_{300}}{1{,}022^{n_p}}\,\frac{\text{dynesec}^n}{\text{cm}^2}, \tag{5.39}$$

$$K_p \cong \frac{510\,R_{300}}{511^{n_p}}\ \text{eq. cP}. \tag{5.40}$$

The equivalent viscosity is calculated as

$$\mu_{ep} = 100K_p\left(\frac{96V_p}{D}\right)^{n_p-1}\left(\frac{3n_p+1}{4n_p}\right)^{n_p}. \tag{5.41}$$

The Reynolds number for a pipe is

$$N_{\text{Re}_p} = \frac{928\,D\,V_p\rho}{\mu_{ep}}. \tag{5.42}$$

The friction factor in a pipe can be calculated as follows.
For $N_{Re_p} < 2100$,

$$f_p = \frac{16}{N_{\text{Re}_p}}. \tag{5.43}$$

For $N_{Re_p} > 2100$,

$$f_p = \frac{a}{N_{\text{Re}_p^b}}, \tag{5.44}$$

where $a = \dfrac{\log n_p + 3.93}{50}$ and $b = \dfrac{1.75 - \log n_p}{7}$.

The friction pressure loss gradient in the pipe is calculated as

$$\left(\frac{dP}{dL}\right)_{dp} = \frac{f_p V_p^2 \rho}{25.81 d} \frac{\text{psi}}{\text{ft}}. \tag{5.45}$$

The power-law constant in the annulus is calculated as follows:

$$n_a = 0.657 \log\left(\frac{R_{100}}{R_3}\right), \tag{5.46}$$

$$K_a = \frac{5.11 R_{100}}{170.2^{n_a}} \frac{\text{dyne} \times \text{sec}^n}{\text{cm}^2}, \tag{5.47}$$

$$K_a = \frac{510 R_{100}}{511^{n_a}} \text{ eq. cP}, \tag{5.48}$$

$$\mu_{ea} = 100 K_a \left(\frac{144 V_a}{d_2 - d_1}\right)^{n_a - 1} \left(\frac{2n_a + 1}{3n_a}\right)^{n_a}. \tag{5.49}$$

The friction factor in the annulus can be calculated as follows.
For $N_{Re_p} < 2100$,

$$f_a = \frac{24}{N_{Re_a}}. \tag{5.50}$$

For $N_{Re_p} > 2100$,

$$f_a = \frac{a}{N_{Re_a}^b}, \tag{5.51}$$

where $a = \dfrac{\log n_a + 3.93}{50}$ and $b = \dfrac{1.75 - \log n_a}{7}$.

The friction pressure loss gradient in the pipe is calculated as

$$\left(\frac{dP}{dL}\right)_a = \frac{f_a V_a^2 \rho}{25.81 (d_2 - d_1)} \frac{\text{psi}}{\text{ft}}. \tag{5.52}$$

**Problem 5.19**

Using the power law, calculate the change in the bottomhole pressure when the 600 rpm reading is 60. Use the following data:

- Hole size = 9 ⅞″
- Depth = 10,000 ft
- Pipe OD = 4 ½″
- Flow rate = 400 gpm
- Mud weight = 12 ppg
- Viscometer readings are $\theta_{600} = 55$, $\theta_{300} = 37$, and $\theta_3 = 3$.

*Solution:*

$$\theta_{100} = \theta_{300} - \frac{2(55-37)}{3} = 25,$$

$$n_a = 0.61,$$

$$K = 5.7,$$

$$\mu_a = 100 \times 5.7\left(\frac{144 \times 2.1}{5.375}\right)^{.61-1}\left(\frac{2 \times 0.61 + 1}{3 \times 0.61}\right)^{0.61} = 130 \text{ cP},$$

$$R_e = \frac{928 \times 5.375 \times 2.1 \times 12}{130.6} = 968.$$

Since the Reynolds number is less than 2100, the flow is laminar:

$$f_a = \frac{24}{968} = 0.025,$$

$$\Delta p_a = 95 \text{ psi}.$$

Use the same calculations for the second case:

$$R_e = \frac{928 \times 5.375 \times 2.1 \times 12}{121} = 1040.$$

The flow is laminar:

$$f_a = \frac{24}{1040} = 0.023,$$

$$\Delta P_a = 88 \text{ psi}.$$

**Problem 5.20**

Given the following data for the directional S-shaped well, determine the pump hydraulics horsepower requirement in order to drill to a target depth as shown in Figure 5.3. Assume a pump volumetric efficiency of 85% and a maximum flow rate of 400 gpm. Calculate the bottomhole pressure and ECD. Also, plot the measured depth versus pressure and the measured depth versus annular velocity for this condition. Use power law models for the calculations

- Mud density = 10.2 ppg, $\theta_{600} = 65$; $\theta_{300} = 40$; $\theta_3 = 15$.
- Bit used is tri-cone roller cone type with 12-12-12 nozzles
- Drill pipe: 4.5″ OD, 4″ ID, 14 ppf in air
- Drill collar: 1000 ft, 7.5″ OD, 4″ ID, 110 ppf in air.
- Surface connections: 500 ft × 4″ ID.
- Hole: Last intermediate casing set is 8.5″ ID set at 16000′ (TVD) openhole washed out to 8.5″ using 7.5″ bit.

*Solution:*

Calculating $n$ and $K$ as before using the respective equation will be 0.7 and 2.6, respectively. The velocity in the pipe is the same as calculated before.

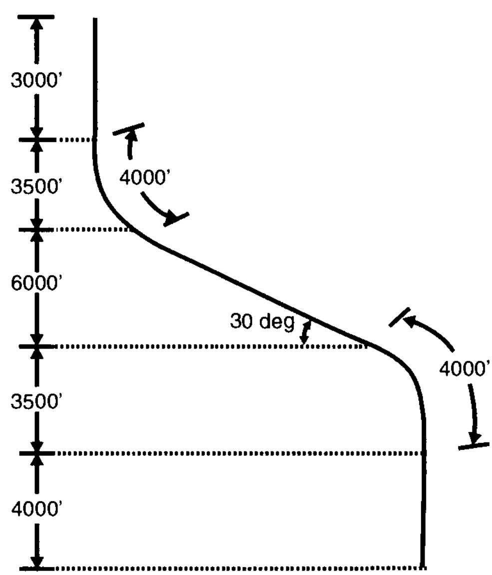



**Figure 5.3** Well profile for problem 5.20.

Calculate the flow regime.
Calculate the Reynolds number:

$$\frac{928 \times 10.2 \times 10.2 \times 4}{53.6} = 7208 = \text{turbulent.}$$

Calculate the friction factor as follows:

$$a = \frac{\log 0.7 + 3.93}{50} = 0.0755,$$

$$b = \frac{1.75 - \log 0.7}{7} = 0.2721,$$

$$f_p = \frac{0.0755}{7208^{0.2721}} = 0.0067.$$

Calculate the friction pressure loss inside the pipe, collar, and surface connection as follows:

$$\frac{0.0067 \times 10.2^2 \times 10.2}{25.81 \times 4} \times 27500 = 1894\,\text{psi}.$$

The bit pressure drop is

$$\frac{8.31 \times 10^{-5} \times 10.2 \times 400^2}{0.95^2\left(3 \times \frac{\pi}{4}\left(\frac{12}{32}\right)^2\right)^2} = 1369\,\text{psi}.$$

In a similar way, the Reynolds number around the drill collar in the annulus is

$$R_{100} = 40 - \frac{2(65-40)}{3} = 23.33.$$

$$n_a = 0.657 \log\left(\frac{23.33}{15}\right) = 0.126.$$

$$K_a = \frac{5.11 \times 23.33}{170.2^{0.126}} = 62.5.$$

The effective viscosity will be

$$= 100 \times 62.5\left(\frac{144 \times 10.2}{8.5 - 7.5}\right)^{0.126-1}\left(\frac{2 \times 0.126 + 1}{3 \times 0.126}\right) = 12.17\,\text{cP},$$

$$N_{\text{Re}_a} = \frac{928\,(8.5 - 7.5) \times 10.2 \times 10.2}{12.17} = 7934.$$

Therefore, the flow regime is turbulent.

The friction pressure loss against the collar in the annulus is calculated as

$$\frac{0.002 \times 10.2^2 \times 10.2}{25.81 \times (8.5 - 7.5)} \times 1000 = 82\ \text{psi}.$$

The Reynolds number against the drill pipe in the annulus = 1018. Therefore, the flow is laminar.

The friction pressure loss against the pipe in the annulus is calculated as

$$\frac{0.0236 \times 3.138^2 \times 10.2}{25.81 \times (8.5 - 4.5)} \times 26000 = 597\,\text{psi}.$$

Total pump pressure = 1894 + 1369 + 82 + 597 = 3942 psi.

$$HHP_p = \frac{3942 \times 400}{1714 \times 0.85} = 1082\ \text{hp}.$$

The bottomhole pressure is calculated as

$$\begin{aligned} p_h + \Delta p_a &= 0.052 \times 10.2 \times \text{TVD} + \Delta p_a \\ &= 0.052 \times 10.2 \times 20000 + 82 + 597 = 11287\ \text{psi}. \end{aligned}$$

$$\begin{aligned} \text{The equivalent circulating density} &= \frac{\text{bottomhole pressure}}{0.052 \times \text{TVD}} \\ &= \frac{11287}{0.052 \times 20000} \\ &= 10.85\,\text{ppg}. \end{aligned}$$

**Problem 5.21**

Use the following well data:

- Drill pipe: 4.5" OD; 4" ID; 12.75 lb/ft
- Drill collar: 7.5" OD; 4" ID; 107.3 lb/ft; 1000′ length
- Intermediate casing: 10 ¾" OD; 0.4" wall thickness
- Current depth = 10000′ (TVD)
- TD = 15000′
- Setting depth = 6500′ (TVD)
- Drill bit = 8 ½" with 9-9-9 nozzles
- Mud related data Rheology = Bingham plastic; density = 10 ppg; Fann data: $\theta_{600} = 95$; $\theta_{300} = 50$
- Minimum annular velocity required for cuttings removal = 120 fpm
- Pump: 2000 hp
- Maximum allowed surface pressure = 4800 psi
- Volumetric efficiency = 90%
- Pump pressure = 2333 psi at 250 gpm at current depth; 4377 psi at 350 pgm at current depth

A. Find the bottom hole equivalent circulating mud density at a flow rate of 450 gpm.
B. Plot the depth versus pressure and depth versus flow velocity.
C. Determine if formation fracturing will occur during circulation. Assume the formation fracture gradient at the casing shoe is 0.545 psi/ft.
D. Check whether the nozzle size is optimum based on maximum HHP criterion.

*Solution:*

*1. Calculating the pressure losses into the drill pipe:*
Calculating the viscosity and yield point,

$$\mu_p = \theta_{600} - \theta_{300},$$

$$\mu_p = 95 - 50 = 45,$$

and

$$\tau_y = 2 \cdot \theta_{300} - \theta_{600},$$

$$\tau_y = 2 \cdot 50 - 95 = 5.$$

Since the minimum velocity for holecleaning is 120 fpm, the flow rate has to be checked against the drill pipe in the bigger clearance depth. The minimum flow rate against the drill pipe (maximum annular clearance) will satisfy other annular areas because it will be less. Hence, the velocity is higher.

The minimum flow rate for holecleaning is

$$Q_{min} = v_a .2.448 \cdot \left(d_2^2 - d_1^2\right),$$

$$Q_{min} = \frac{120 \times 2.448 \times \left(9.95^2 - 4.5^2\right)}{60} = 385 \text{ gpm}.$$

$$Q_{op}(450) > Q_{min}.$$

$$v_p = \frac{q}{2.448 \cdot d^2},$$

$$v_p = \frac{450}{2.448 \cdot 4^2} = 11.49 \text{ ft/sec}.$$

$$\mu_{ep} = \mu_p + \frac{20 \cdot d \cdot \tau_y}{3 \cdot v_p},$$

$$\mu_{ep} = 45 + \frac{20 \cdot 4 \cdot 5}{3 \cdot 11.49} = 56.60 \text{ cP}.$$

$$N_{\text{Re}} = \frac{928 \cdot \rho \cdot v_p \cdot d}{\mu_{ep}},$$

$$N_{\text{Re}} = \frac{928 \times 10 \times 11.49 \times 4}{56.60} = 7535.$$

$N_{\text{Re}} = 7535$. Therefore, the flow regime is turbulent.

$$\left(\frac{dp_f}{dL}\right)_p = \frac{\rho^{0.75} v_p^{1.75} \mu_p^{0.25}}{1{,}800 \cdot d^{1.25}},$$

$$\left(\frac{dp_f}{dL}\right)_p = \frac{10^{0.75} 11.49^{1.75} 45^{0.25}}{1{,}800 \cdot 4^{1.25}} = 0.102568 \text{ psi/ft},$$

$$\Delta p_{dp} = 0.102568 \text{ psi/ft},$$

$$\Delta p_{dp} = \Delta P_{dp} \times L_{dp} = 0.102568 \times 14{,}000 \text{ psi},$$

$$\Delta p_{dp} = 1{,}436 \text{ psi}.$$

Since the drill collar inside diameter is also the same, the pressure drop inside the drill collar is

$$\Delta P_{dc} = 103 \text{ psi}.$$

Therefore, the total frictional pressure loss in the drill string is 1,540 psi.

*2. Calculating the pressure losses into the annulus between the drill pipe and borehole:*
There are three sections in the annulus.

For the annulus between the drill pipe and casing,

$$v_a = \frac{q}{2.448 \cdot \left(d_2^2 - d_1^2\right)},$$

$$v_a = \frac{450}{2.448 \cdot \left(9.95^2 - 4.5^2\right)} = 2.33 \text{ ft/sec}.$$

$$\mu_{ep} = \mu_p + \frac{5 \cdot \left(d_2 - d_1\right) \cdot \tau_y}{v_a},$$

$$\mu_{ep} = 45 + \frac{5 \cdot (9.95 - 4.5) \cdot 5}{2.33} = 103.5.$$

$$N_{\text{Re}} = \frac{757 \cdot \rho \cdot v_a \cdot \left(d_2 - d_1\right)}{\mu_{ep}},$$

$$N_{\text{Re}} = \frac{757 \times 10 \times 2.33 \times (9.95 - 4.5)}{103.5},$$

$$N_{\text{Re}} = 929.$$

Therefore, the flow regime is laminar.

$$\left(\frac{dp_{f_{adp}}}{dL_{adp}}\right)_a^1 = \frac{\mu_p v_a}{1{,}000 \cdot (d_2 - d_1)^2} + \frac{\tau_y}{200 \cdot (d_2 - d_1)}$$

$$= \frac{45 \times 2.33}{1{,}000 \cdot (9.95 - 4.5)^2} + \frac{5}{200 \cdot (9.95 - 4.5)}$$

$$= 0.008117 \text{ psi/ft.}$$

$$\Delta P_{adp} = 0.008117 \text{ psi/ft,}$$

$$\Delta P_{adp} = \Delta P_{adp} \times L_{adp} = 0.008117 \times 6{,}500 = 53 \text{ psi.}$$

Similarly, the pressure gradient for the annulus between the drill pipe/openhole and drill collar/openhole can be calculated as

$$\Delta P_{adp} = 0.0162 \text{ psi/ft,}$$

$$\Delta P_{adp} = \Delta P_{adp} \times L_{adp} = 0.0162 \times 7{,}500 = 121.5 \text{ psi.}$$

$$\Delta P_{adc} = 0.5416 \text{ psi/ft,}$$

$$\Delta P_{adc} = \Delta P_{adc} \times L_{adc} = 0.5416 \times 1{,}000 = 542 \text{ psi.}$$

Therefore, the total frictional pressure loss in the annulus is 717 psi.

3. *Calculating the pressure losses across the bit:*
Assuming a coefficient of discharge, $C_d = 0.95$,

$$\Delta p_b = \frac{8.311 \times 10^{-5} \times \rho \times Q^2}{C_d^2 \cdot A_n^2} \text{ psi.}$$

$$A_n = \frac{\pi}{4}\left(d_1^2 + d_2^2 + d_3^2\right) \text{in}^2,$$

$$A_n = \frac{3.1416}{4}\left(0.28125^2 + 0.28125^2 + 0.28125^2\right) \text{in}^2,$$

$$A_n = 0.18637 \text{ in}^2.$$

$$\Delta p_b = \frac{8.311 \times 10^{-5} \times 10.0 \times 450^2}{0.95^2 \times 0.18637^2} = 5368 \text{ psi,}$$

$$\Delta p_b = 5{,}368 \text{ psi.}$$

4. *Calculating the ECD at TD:*

$$\mathrm{ECD} = \rho_m + \frac{\Delta p_{fa}}{0.052 \times D},$$

$$\mathrm{ECD} = 10.0 + \frac{717}{0.052 \times 10{,}000},$$

$$\mathrm{ECD} = 11.379 \text{ ppg}.$$

5. *Formation fracture check at the shoe:*
The formation fracture at the shoe is 0.545 psi/ft. So, the formation pressure at the shoe is 3543 psi.

$$\text{ECD at the shoe during circulation} = 10.0 + \frac{53}{0.052 \times 6500}$$
$$= 10.1568 \text{ ppg}.$$

Therefore, the pressure = 0.052 × 10.1568 × 6500 = 3433 psi. This is equivalent to 0.5281 psi/ft. So there will not be any fracture at the shoe.

Similarly, a formation fracture check can be done at the TD using the uniform fracture gradient at TD.

6. *Optimum nozzle size calculation using the bit HHP criterion:*
Calculating the pressure drop across the bit with the test flow rates gives the following.

For 250 gpm,

$$\frac{8.3 \times 10^{-5} \times 10 \times 250^2}{0.95^2 \times 0.18637^2} = 1{,}657 \text{ psi},$$

and for 350 gpm,

$$\frac{8.3 \times 10^{-5} \times 10 \times 350^2}{0.95^2 \times 0.18637^2} = 3247 \text{ psi}.$$

From calculating $m$, which is calculated as

$$m = \left( \frac{\log\left( \dfrac{(\Delta P_d)_j}{(\Delta P_d)_i} \right)}{\log\left( \dfrac{Q_j}{Q_i} \right)} \right)$$

$$m = \left( \frac{\log\left( \frac{(2333 - 1657)_j}{(4377 - 3247)_i} \right)}{\log\left( \frac{250}{350} \right)} \right) = 1.53,$$

$$\Delta p_{d\,opt} = \frac{1}{m+1} \Delta p_{p_{max}},$$

$$\Delta p_{d\,opt} = \frac{1}{1.53+1} \times 4800 = 1900 \text{ psi},$$

and

$$\Delta p_{b_{opt}} = \Delta p_{p_{max}} - \Delta p_{d\,opt},$$

$$\Delta p_{b_{opt}} = 4800 - 1900 = 2900 \text{ psi}.$$

Or the following equation can be used:

$$\Delta p_{b\,opt} = \frac{m}{m+1} \Delta p_{p_{max}}.$$

Using the corresponding equation for the optimized flow rate,

$$Q_{opt} = Q_a \mathrm{a}\log\left[ \frac{1}{m} \log\left( \frac{\Delta p_{d_{opt}}}{\Delta p_{d_{Q_a}}} \right) \right]$$

$$Q_{opt} = 250 \text{ a}\log\left[ \frac{1}{1.53} \log\left( \frac{1900}{2333 - 1657} \right) \right] = 492 \text{ gpm},$$

$Q_{max}$ can be calculated as

$$P_{HP} = \frac{P_p \cdot q_{\max}}{1{,}714},$$

$$Q_{max} = \frac{2000 \times 1714 \times 0.8}{4800} = 643 \text{ gpm}.$$

Checking for the limits, $Q_{max} > Q_{opt} > Q_{min}$. Therefore, $Q_{opt}$ is within the limits.

Using the nozzle equation, the optimum nozzle area can be obtained:

$$\Delta p_{bopt} = \frac{8.3\times10^{-5}\times10\times520^2}{0.95^2\times A_n^2} = 2928\,\text{psi},$$

$$A_{opt} = 0.2766\ \text{in}^2.$$

Select the closest nozzle size. Nozzle sizes can be 11-11-11.

*7. Depth vs. pressure calculation:*

$$\Delta P_p = \Delta P_f + \Delta P_b,$$

$$P_f = \Delta P_{fdp}L_{fdp} + \Delta P_{fdc}L_{fdc} + \Delta P_{fadp}L_{fadp} + \Delta P_{fadc}L_{fadc} + \Delta P_{PDM} + \Delta P_{MWD} + \Delta P_{fsurf}L_{fsurf}.$$

Since there is no mud motor or MWD tool in the string, pressure losses are neglected.

$$P_f = 1540 + 717 = 2257\ \text{psi},$$

$$\Delta P_p = 2257 + 5368 = 7625\ \text{psi},$$

$$\Delta P_p = 7{,}625\ \text{psi}.$$

Since the allowed pump pressure is 4800 psi and the optimum nozzle is 11-11-11, when used it will result in a pressure bit pressure drop of

$$\Delta p_b = \frac{8.3\times10^{-5}\times10\times450^2}{0.95^2\times.2928^2} = 2410\ \text{psi}.$$

Note that in the above equation the nozzle size is using the actual area of 3 × 11 nozzles.

The surface pressure will be 4300 psi, which is within the limits.

*8. Bottomhole pressure calculation:*

$$P_{bhp} = P_h + \Delta P_a,$$

$$P_h = 0.052 \times \rho \times D,$$

$$P_h = 0.052 \times 10 \times 10{,}000 = 5200 \text{ psi},$$

$$P_{bhp} = 5200 + 717 = 5917 \text{ psi}.$$

You can also use this bottomhole pressure to calculate the ECD:

$$\text{ECD} = \frac{5917}{0.052 \times 1000} = 11.379 \text{ ppg}$$

as obtained earlier in (4).

Note that by using fluid statics, the bottomhole pressure can be calculated as

$$P_{bh} = P_p - P_{fpdp} - P_{fpdc} - \Delta P_{mm} - \Delta P_{m/pwd} - \Delta P_b + 0.052 \times \rho_m \times D_v,$$

$$P_{bh} = 4302 - 1540 - 0 - 0 - 2045 + 0.052 \times 10 \times 10000 = 5917 \text{ psi},$$

$$P_{bhp} = 5{,}917 \text{ psi}.$$

## 5.7 Gel Breaking Pressure

In the pipe side, the gel breaking pressure is calculated as

$$P = \frac{\tau_g L}{300 D_i} \text{ psi}, \quad (5.53)$$

where $\tau_g$ = gel strength, $\tau_g \rightarrow \frac{lbf}{100 ft^2}$, $D_i$ = inside diameter of the pipe, in, and $L$ = length of the pipe, ft.

In the annular side,

$$P = \frac{\tau_g L}{300\left(D_h - D_p\right)} \text{psi}, \quad (5.54)$$

where $D_h$ = diameter of the hole, in, and $D_p$ = outside diameter of the pipe, in.

**Problem 5.22**

Mud circulation has been stopped for sufficient time to develop a gel strength of 15 lbf/100ft$^2$. If the pipe is not moved, calculate the pressure surge required to break the circulation.

- Depth = 10,000 ft
- Drill pipe = 5″ × 4.27″
- Drill collar = 450 ft, 6.5″ × 3″
- Hole diameter = 8 ½″

*Solution:*

The pressure required to break circulation is

$$P=\left(\frac{15\times(9550)}{300\times 4.27}\right)+\left(\frac{15\times(450)}{300\times 3}\right)+\left(\frac{15\times(9550)}{300\times(8.5-5)}\right)+\left(\frac{15\times(450)}{300\times(8.5-6.5)}\right)\text{psi},$$

$$P=111.8+7.5+136.43+11.25=267\,\text{psi}.$$

## 5.8 Hole Cleaning – Cuttings Transport

The cuttings concentration for 45° with one tool is given by

$$C_c=3.22\left(1+N_{Ta}\right)^{-0.472}+5703.6N_{\text{Re}}^{-0.776}+69.3N_{Rop}^{-0.051}-63.3, \quad (5.55)$$

$$(1450<N_{\text{Re}}<3700,\ 0<N_{Ta}<5800,\ 19.7<N_{ROP}<23).$$

The cuttings concentration for 90° with one tool is given by

$$C_c=-5.22\times10^{-5}N_{Ta}^{1.36}+605.714\,N_{\text{Re}}^{-0.0124}+8.86e^{230.43}N_{Rop}^{-77.58}-529.4, \quad (5.56)$$

$$(1450<N_{\text{Re}}<3700,\ 0<N_{\text{Ta}}<5800,\ 19.7<N_{ROP}<23).$$

For the above case with no tools and with a 45° inclination,

$$C_c=1.8\left(1+N_{Ta}\right)^{-8.13}+21.25\times e^{177.57}N_{\text{Re}}^{-24.63}+15.7\times e^{3.54}N_{Rop}^{-0.03}-487.67, \quad (5.57)$$

$$(1450<N_{\text{Re}}<3700,\ 0<N_{Ta}<5800,\ 19.7<N_{ROP}<23).$$

The cuttings concentration for 90° with no tool is given by

$$C_c = -14.04 \times e^{-316}\left(1 + N_{Ta}\right)^{36.33} + 284.96 N_{\mathrm{Re}}^{-0.073} + 58.2 \times e^{30.9} N_{Rop}^{-12.46} - 140.88, \quad (5.58)$$

$$(1450 < N_{Re} < 3700,\ 0 < N_{Ta} < 5800,\ 19.7 < N_{ROP} < 23).$$

In the above equations, $N_{Ta}$ = Taylor's number, or

$$N_{Ta} = \frac{\omega \rho D^2}{\mu_{eff}}, \quad (5.59)$$

where ω = angular velocity of the tool, rad/sec, $\rho_m$ = mud density, ppg, $\mu_{eff}$ = effective viscosity, cp, and $D_h$ = diameter of the hole, in.

$N_{\mathrm{Re}}$ = Reynolds number, or

$$N_{\mathrm{Re}} = \frac{\rho V^{2-n} D^n}{K 8^{n-1}}, \quad (5.60)$$

where $K$ = consistency index, $V$ = velocity of the fluid, and $n$ = power law index.

$N_{Rop}$ = ROP number, or

$$N_{ROP} = \frac{\rho D \times \mathrm{ROP}}{\mu_{eff}}, \quad (5.61)$$

where ROP = rate of penetration.

## 5.9 Transport Velocity

Critical transport fluid velocity is calculated as

$$\overline{V}_{ca} = \overline{V}_{cr} + \overline{V}_{cs}, \quad (5.62)$$

where $\overline{V}_{ca}$ = critical transport average annular fluid velocity, $\overline{V}_{cr}$ = cuttings rise velocity, and $\overline{V}_{cs}$ = cuttings average slip velocity.

$$\overline{V}_{cr} = \frac{1}{\left[1 - \left(\frac{D_{op}}{D_h}\right)^2\left[0.64 + \frac{18.16}{\mathrm{ROP}}\right]\right]}. \quad (5.63)$$

$$\bar{V}_{cs} = V_{cs} C_{ang} C_{size} C_{mwt}$$

where

$$V_{cs} = 0.00516\mu_a + 3.006 \quad \mu_a \le 53,$$

$$V_{cs} = 0.02554\mu_a + 3.280 \quad \mu_a > 53,$$

$$\mu_a = PV + 1.12YP\left(D_h - D_{op}\right) \quad PV < 20\,\text{cP} \quad YP < 20, \frac{\text{lb}}{100\text{ft}^2}$$

$$\mu_a = PV + 0.9YP\left(D_h - D_{op}\right) \quad PV > 20\,\text{cP} \quad YP > 20\frac{\text{lb}}{100\text{ft}^2},$$

and where $C_{ang}$, $C_{size}$, $C_{mwt}$ are correction factors for change in hole angle, cuttings size, and mud weight, respectively.

$$C_{ang} = 0.0342\alpha - 0.000233\alpha^2 - 0.213,$$

$$C_{size} = -1.04D_{50} + 1.286,$$

and

$$C_{ang} = 1 - 0.0333\left(\rho_m - 8.7\right),$$

where $\alpha$ = hole inclination from vertical (degrees), and $\rho_m$ = mud weight, ppg.

For the subcritical flow, the amount of non-moving cuttings concentration in the annulus is found from the following equations:

$$C_{corr} = C_{cal} C_{bed}, \tag{5.64}$$

$$C_{cal} = \left[1 - \frac{\bar{V}_{opr}}{\bar{V}_{crit}}\right](1 - \phi)100$$

$$C_{cal} = \left[1 - \frac{Q_{opr}}{Q_{crit}}\right](1 - \phi)100$$

where $\bar{V}_{opr}$, $Q_{opr}$ = operating annular fluid velocity and operating flow rate, respectively, and $\phi$ = bed porosity (30 to 40%).

$C_{bed}$ = correction factor. Thus,

$$C_{bed} = 0.97 - 0.00231\mu_a. \tag{5.65}$$

**Problem 5.23**

The well was in the rotary mode with a string rotation of 50 rpm. One mechanical holecleaning device is being in the well. Calculate and plot the volumetric concentration in a 100 ft section on the well for various inclination angles with and without tool. Draw your conclusions. Use the given data:

- Hole Diameter = 12 ¼″
- Drill pipe outside diameter = 5″
- Flow rate = 500 gpm
- Mud: polymer mud with a density of 10 ppg
- Viscosity: 45 cp, $n = 0.65$, $K = 0.25$ lb-s$^n$/100 ft$^2$
- Rate of penetration = 50 ft/hr

*Solution:*

$$\rho_m = 10 \text{ ppg } = 10 \times 7.48 = 74.8 \text{ lbm/ft}^3.$$

$$\mu = 45 \text{ cp} = 0.000672 \times 45 = 0.0302 \text{ lbm/ft} \times \text{sec}.$$

$$\omega = 50 \text{ rpm} = 50 \times 2 \times \pi / 60 = 5.24 \text{ rad/sec}.$$

$$\text{ROP} = 50\text{ft/hr} = \frac{50}{3600} = 0.0139 \text{ ft/sec}.$$

$$V = \frac{Q}{A} = \frac{500}{2.448 \times (8.5^2 - 5^2)} = 4.32 \text{ ft/sec}.$$

$$D_h = D_{hole} - D_{pipe} = 12.25 - 5 = 7.25'' = 0.6 \text{ ft}.$$

$$k = 0.25 \text{ lb-sn/100 ft}^2 = 0.0025 \text{ lb-sn/ft}^2.$$

$$N_{re} = \frac{\rho_m V^{2-n} D_h^2}{K 8^{n-1} g} = \frac{74.8 \times 4.32^{2-0.65} \times 0.292^2}{0.0025 \times 8^{2-0.65} \times 32.2} = 2689.$$

$$N_{rop} = \frac{\rho_m D_h \; ROP}{\mu} = \frac{74.8 \times 0.292 \times 0.0139}{0.0302} = 20.76.$$

$$N_{Ta} = \frac{\omega \rho_m D_h^2}{\mu} = \frac{5.24 \times 74.8 \times 0.6^2}{0.0302} = 4728.$$

Using the range,

$$1450 < N_{\text{Re}} < 3700, \; 0 < N_{Ta} < 5800, \; 19.7 < N_{Rop} < 23.$$

The cutting concentration for 90° (horizontal) with one tool is

$$\begin{aligned} C^0_{c(90^o)} &= -5.22 \times 10^{-5} \times N_{Ta}^{1.36} + 605.714 \times N_{\text{Re}}^{-0.0124} + 8.86 \times e^{230.43} N_{Rop}^{-77.58} - 529.4 \\ &= -5.22 \times 10^{-5} \times 4728^{1.36} + 605.714 \times 2689^{-0.0124} + 8.86 \times e^{230.43} \times 20.76^{-77.58} - 529.4 \\ &= 18.04. \end{aligned}$$

The cutting concentration for 90° (horizontal) without a tool is

$$\begin{aligned} C^1_{c(90^o)} &= -14.04 \times e^{-316} \times (1 + N_{Ta})^{36.33} + 284.96 \times N_{\text{Re}}^{-0.073} + 58.2 \times e^{30.9} N_{Rop}^{-12.46} - 140.88 \\ &= -14.04 \times e^{-316} \times (1 + 4728)^{36.33} + 284.96 \times 2689^{-0.073} + 58.2 \times e^{30.9} \times 20.76^{-12.46} - 140.88 \\ &= 25.6. \end{aligned}$$

The cutting concentration for 45° with one tool is

$$\begin{aligned} C^0_{c(45^o)} &= 3.22 \times (1 + N_{Ta})^{-0.472} + 5703.6 \times N_{\text{Re}}^{-0.776} + 69.3 \times e^{230.43} N_{Rop}^{-0.051} - 63.3 \\ &= 3.22 \times (1 + 4728)^{-0.472} + 5703.6 \times 2689^{-0.776} + 69.3 \times 20.76^{-0.051} - 63.3 \\ &= 14.3. \end{aligned}$$

The cutting concentration for 45° without a tool is

$$\begin{aligned} C^1_{c(45^o)} &= 1.8 \times (1 + N_{Ta})^{-8.13} + 21.25 \times e^{177.57} \times N_{\text{Re}}^{-24.63} + 15.7 \times e^{3.54} N_{Rop}^{-0.03} - 487.67 \\ &= 1.8 \times (1 + 4728)^{-8.13} + 21.25 \times e^{177.57} \times 2689^{-24.63} + 15.7 \times e^{3.54} \times 20.76^{-0.03} - 487.67 \\ &= 6.6. \end{aligned}$$

The cutting concentration for 60° with one tool is

$$C^0_{c(45^o)} = \frac{1}{3} C^0_{c(90^o)} + \frac{2}{3} C^0_{c(45^o)} = 10.6.$$

The cutting concentration for 60° without tool is

$$C^1_{c(60^\circ)} = \frac{1}{3}C^1_{c(90^\circ)} + \frac{2}{3}C^1_{c(45^\circ)} = 10.7.$$

**Problem 5.24**

Use the following well data:

- Mud: YP = 12; PV =12; mud density = 9 ppg
- Hole details: 9″ diameter, 70° inclination
- Cuttings average size = 0.174″
- Drill pipe: 3.5″ OD
- Bed porosity = 40%
- ROP = 60 ft/hr

A. Determine the critical flow rate.
B. Determine the amount of cuttings (% per volume of annulus) if the operating flow rate is 70% of the critical.

*Solution:*

$$D_{op} = 9 - 3.5 = 5.5.$$

$$\text{ROP} = 60/\text{hr}.$$

$$\bar{V}_{cr} = \frac{1}{\left[1-\left(\frac{D_{op}}{D_h}\right)^2\right]\left(0.64+\frac{18.16}{\text{ROP}}\right)} = \frac{1}{\left[1-\left(\frac{3.5}{9}\right)^2\right]\left(0.64+\frac{18.16}{60}\right)}$$
$$= 1.25\,\text{ft/sec}.$$

$$C_{mud} = 1 - 0.03333 \times (\rho_m - 8.7) = 1 - 0.03333 \times (9 - 8.7) = 0.99.$$

$$C_{ang} = 0.0324\alpha - 0.000233\alpha^2 - 0.213 = 0.0324 \times 70 - 0.000233 \times 70^2 - 0.213 = 1.04$$

$$C_{size} = -1.04 D_{50} + 1.286 = -1.04 \times 0.174 + 1.286 = 1.105.$$

$$\mu_s = PV + 1.12 \times YP \times (D_h - D_{op}) = 12 + 1.12 \times 12 \times (9 - 3.5) = 86\,\text{cP}.$$

The critical transportation fluid velocity is

$$\bar{V}_{ca} = \bar{V}_{cr} + \bar{V}_{cs} = 1.25 + 3.92 = 5.17\ \text{ft/sec}.$$

The critical flow rate is

$$\bar{Q}_{ca} = \bar{V}_{ca}\frac{\pi}{4}(D_h^2 - D_p^2) = 5.17 \times 2.448 \times (9^2 - 3.5^2) = 870\ \text{gpm}.$$

The operating flow rate is

$$Q_{opt} = \bar{Q}_{ca} \times 0.7 = 870 \times 0.7 = 609\ \text{gpm}.$$

The operating flow velocity is

$$V_{opt} = \bar{V}_{ca} \times 0.7 = 5.17 \times 0.7 = 3.62\ \text{ft/sec}.$$

For the sub critical flow,

$$C_{corr} = C_{cal}C_{bed} = \left(1 - \frac{V_{opt}}{\bar{V}_{cr}}\right)(1-\phi) \times (0.97 - 0.002331\mu_a)$$
$$= (1-0.7)(1-0.4) \times (0.97 - 0.002331 \times 86) = 0.139 = 13.9\%.$$

# 6

# Tubular Mechanics

This chapter focuses on the different basic calculations related to tubulars.

## 6.1 Drill Collar Length

The drill collar size required is calculated by

$$D_{dc} = (2D_{csg} - D_b), \tag{6.1}$$

where $D_{dc}$ = diameter of the drill collar, $D_{csg}$ = diameter of the casing coupling, and $D_b$ = diameter of the bit.

The length of the drill collar required is calculated by

$$L_{dc} = \frac{\text{WOB} \times \text{DF}}{w_{dc} \times \text{BF} \times \cos a}, \tag{6.2}$$

where WOB = weight on bit in lbs, DF = design factor, $w_{dc}$ = unit weight of the collar in lbf/ft, BF = buoyancy factor, and $a$ = wellbore inclination in degrees.

**Problem 6.1**

An 8½", 22° hole is planned to be drilled and cased with 7", 38 ppf, P-110 – BTC casing. The mud weight to be used is 12 ppg. The weight on bit desired is 25 kips. Calculate the size and length of drill collars required for a design factor of 1 and 1.2.

*Solution:*

Coupling OD of the 7" 38 ppf BTC is 7.656 in.

Using equation 3.1, the diameter of the drill collar is

$$D_{dc} = 2 \times 7.656 - 8.5 = 6.812".$$

The closest available drill collar sizes are 6¾" and 6½". Typically, 6½" is selected so that annular pressure losses against the drill collars, and thereby the equivalent circulating density, (ECD) are reduced. From Table 3.3, the drill collar size that can be selected is 6½" 99 ppf.

*For design factor 1:*

The buoyancy factor is

$$\text{BF} = 1 - \frac{12}{65.39} = 0.816.$$

The length of the drill collars is

$$L_{dc} = \frac{25000 \times 1}{99 \times 0.82 \times \cos 22} = 333 \text{ ft}.$$

To pull out the hole in full stands, 4 stands of 6½" drill collars of a total length of 360 ft can be used, which will give a design factor of 1.08.

*For design factor 2:*

The length of the drill collars is

$$L_{dc} = \frac{25000 \times 1.2}{99 \times 0.82 \times \cos 22} = 400 \text{ ft}.$$

To pull out the hole in full stands, 4 stands of 6½" drill collars of a total length of 450 ft can be used, which will give a design factor of 1.35.

## 6.2 Bending Stress Ratio (BSR)

Bending strength ratio (BSR) is defined as the ratio of the box section modulus to the pin section modulus, and the dimensions used are illustrated in Figure 6.1.

$$\mathrm{BSR} = \frac{\dfrac{D^4 - b^4}{D}}{\dfrac{R_t^4 - d^4}{R_t}} \tag{6.3}$$

where $D$ = the connection or tool outside diameter in inches, $b$ = the thread root diameter of box threads at the end of the pin in inches, $R_t$ = the thread root diameter of pin threads ¾ inch from the shoulder of the pin in inches, and $d$ = the pin inside diameter in inches.

Typical accepted ranges of BSR are 2.25 – 2.75 for critical service application, 2.0 – 3.0 for normal service application, and 1.9 – 3.2 for limited service application.

## 6.3 Pipe Wall Thickness

The corrected outside diameter is calculated as

$$D_{cp} = c \times D_p + D_i(1-c), \tag{6.4}$$

where $c$ = the class multiplier, $D_{cp}$ = the corrected pipe diameter, $D_p$ = the original pipe diameter, and $D_i$ = the inside pipe diameter.

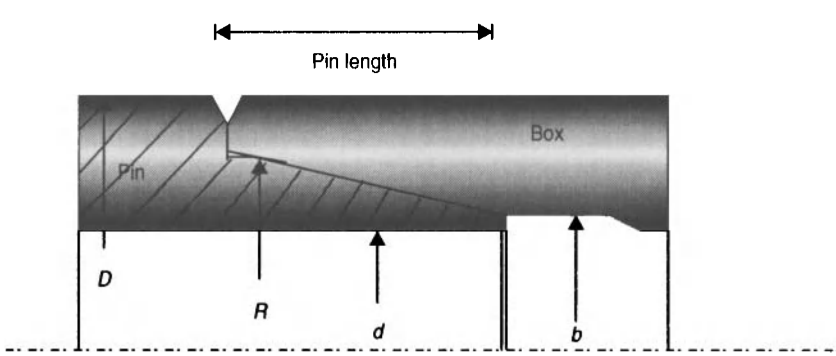



**Figure 6.1** Pin-box dimensions.

The pipe class multipliers for various classes of pipes are given below:

- N = new, $c = 1.000$
- C = critical, $c = 0.875$
- P = premium, $c = 0.800$
- 2 = class 2, $c = 0.700$
- 3 = class 3, $c = 0.650$

**Problem 6.2**

Calculate the new outside diameter and wall thickness for the following pipe with the given pipe condition:

- Outside diameter = 5.0″
- Inside diameter = 4.276″
- Wall thickness = 0.362″
- Class = premium

*Solution:*

Using equation 6.4, the new outside diameter = 0.800 × 5.0 + 4.276 (1.0 – 0.800) = 4.8552″.

New wall thickness = 0.2896″.

**Problem 6.3**

A drilling engineer checks the diameter of 4.5″ OD, 3.5″ ID drill pipes in the rack that are found to average 4.25″ OD. What should be the API classification of the pipe with this condition? Calculate the new outside diameter and wall thickness based on the API classification that he should use for his engineering calculation.

*Solution:*

From equation 6.4, the class multiplier can be expressed as

$$c = \frac{D_{cp} - D_i}{D_p - D_i} = \frac{4.25 - 3.5}{4.5 - 3.5} = 0.75.$$

The calculated classification falls between the premium and class 2 classifications. Derating downwards in order to be conservative for the engineering design, the pipe can be classified as class 2.

The new outside diameter = 0.700 × 4.5 + 3.5 (1.0 – 0.70) = 4.2″.

The new wall thickness = 0.35″.

## 6.4 Resonant Frequency

The natural frequency of a single span beam is given by

$$f_i = \frac{\lambda_i}{2\pi L^2}\sqrt{\frac{EI}{m}}, \tag{6.4a}$$

where $\lambda_i$ = a non-dimensional parameter, $i$ = the modal index, $L$ = the length of the span, $E$ = the modulus of elasticity, $I$ = the moment of inertia, and $m$ = mass.

**Problem 6.4**

Assuming the geometrical parameters to be the same for the three materials (aluminium, titanium, and steel) and comparing only the ratio, $R_m = \sqrt{E/\rho}$ gives results as follows.

For steel with Young's modulus, $E = 29.9 \times 10^6$ psi and density $\rho = 0.288\ \text{lb/in}^3$,

$$R_s = \sqrt{\frac{29.9\times 10^6}{0.288}} = 10189.$$

For aluminum with $E = 10 \times 10^6$ psi and $\rho_s = 0.0975\ \text{lb/in}^3$,

$$R_a = \sqrt{\frac{10\times 10^6}{0.0975}} = 10127.$$

Similarly for titanium,

$$R_t = \sqrt{\frac{16.8\times 10^6}{0.164}} = 10121.$$

## 6.5 Tensions

Effective tension is given by

$$F_e = F_t + F_{bs}, \tag{6.5}$$

which can also be written as

$$F_e = F_t + P_o A_o - P_i A_i, \tag{6.6}$$

where $F_{bs}$, the buckling stability force, is

$$F_{bs} = P_o A_o - P_i A_i. \tag{6.7}$$

True tension is given by

$$F_t = \sum\left[W_s \cos a + F_D + \Delta F_{area}\right] - F_{bottom} - \mathrm{WOB}, \tag{6.8}$$

$$F_e = \sum\left[W_s \cos\alpha + F_D + \Delta F_{area}\right] - F_{bottom} - \mathrm{WOB} + F_{bs}, \tag{6.9}$$

where $W_s$ = air weight of the segment = $Lw_{air}$, $L$ = length of drill string hanging below point in feet, $w_{air}$ = weight per foot of the drill string in air in lb/ft, $\alpha$ = inclination in degrees, $F_{bottom}$ = bottom pressure force, and $F_{bs}$ = buckling stability force.

The bottom pressure force is a compressive force due to fluid pressure applied over the cross sectional area of the bottom component.

$\Delta F_{area}$ = the change in force due to a change in area .

## 6.6 Drag Force

Drag force is given as

$$F_D = F_s \times \mu \times \frac{|V_{ts}|}{|V_{rs}|}. \tag{6.10}$$

Here, $V_{ts}$ = trip speed in inches per second, $|V_{rs}|$ = resultant speed = $\sqrt{(V_{ts}^2 + \omega^2)}$, $|\omega|$ = angular speed = diameter $\times \pi \times$ RPM/60 in inches per second, $F_s$ = side or normal force in lbs, and $\mu$ = the coefficient of friction.

$$F_D = F_s \times \mu_v \times \frac{|V_{ts}|}{|V_{rs}|}, \tag{6.11}$$

where the variable friction coefficient is given as

$$\mu_v = \mu_s \times e^{-k|V_{rs}|}, \tag{6.12}$$

where $\mu_s$ = static friction and k is a constant.

## 6.7 Side Force Calculation

In the soft string model, the side force or normal force is calculated using the following equation for a non-buckled condition:

$$F_s = \sqrt{\left(F_e\ \Delta\phi\ Sin\alpha_{avg}\right)^2 + \left(F_e\ \Delta\alpha + W_b\ Sin\alpha_{avg}\right)^2} \qquad (6.13)$$

where $F_e$ = the axial force at the bottom of the section, which is calculated using the buoyancy method, i.e., the effective tension calculation. $W_b$ = buoyed weight of the section = $w_b S_L$, where $S_L$ = the section length, and $w_b$ = buoyed weight per unit length of the section. $\Delta\phi$ = the change in azimuth over the section length. $\alpha_{avg}$ = the average inclination over the section, and $\Delta\alpha$ = the change in inclination over the section length.

The sign in front of the term $W_b$ is based on whether the wellbore is building or dropping.

**Problem 6.5**

Using the data given below, calculate the side force.

- Top point measured depth = 961 ft; $\alpha = 6.25$ deg; $\phi = 163.0$ deg
- Bottom point measured depth = 990.9 ft; $\alpha = 6.87$ deg; $\phi = 165.07$ deg
- $S_L = 990.9 - 961 = 29.9$ ft
- Effective tension at the point of calculation is 153.3 kips.
- Mud weight = 10.5 ppg
- Linear weight of section = 22.08 lb/ft.

*Solution:*

$\alpha_{avg} = (6.25 + 6.87)/2 = 6.56$ deg = 0.11449 rad.
$\Delta\phi = 163.0 - 165.07 = -2.07$ deg = $-0.036128$ rad.
$\Delta\alpha = 6.87 - 6.25 = 0.62$ deg = 0.010821 rad.

$$\text{Buoyancy factor} = 1 - \frac{10.5 \times 7.48}{489.024} = 0.8394.$$

Density of steel = 489.024 lb/ft$^3$.
$W = 22.08 \times 0.8394 = 18.53$ lb/ft.

Using the side force equation, equation 6.13,

$$F_s = \sqrt{\left(F_e\ \Delta\phi\ Sin\alpha_{avg}\right)^2 + \left(F_e\ \Delta\alpha + W_b\ Sin\alpha_{avg}\right)^2},$$

$$F_s = \sqrt{(153300 \times (-0.036128)\sin(0.11449)^2 + (15300 \times 0.010821 - 18.53 \times \sin(0.11449))^2}$$

Note the negative sign in the inclination term as it is a build section.

$$F_s = 654 \text{ lbf}.$$

Adjusting for the section length gives

$$F_s = 654 \times 31/29.9 = 677 \text{ lbf}.$$

**Problem 6.6**

A well is being drilled at a depth of 7500 ft with a mud density of 9.2 ppg. The drill string consists of a drill pipe, a 5″ – 19.5 ppf, E grade, class 1 pipe. Tool joint effects may be neglected. Assume a wellbore inclination of 20°. Azimuth = 0, and the friction factor = 0.25. Compute the true and effective tensions when the driller is rotating off bottom without circulation.

*Solution:*

*True tension calculation:*

Average weight = 19.5 ppf since the tool joint effect is neglected. Otherwise, the average weight will be 20.89 ppf.

$$\text{Weight component} = 7500 \times 19.5 \times \cos(20) = 137.43 \text{ kips}.$$

Since it is rotating off bottom, WOB = 0 kips.
Buoyancy factor = (1 – 9.2/64.5) = 0.8573.
Side force = 0.8573 × 19.5 × sin(20) = 5.71(lb/ft).
Linear velocity ratio = 0.0.
DF = 0 kips.
$\Delta F_{area} = 0$ as the cross sectional area is the same.
$F_{bot}$ is the compressive force due to fluid pressure applied at the bottom of the pipe. The bottom force should be equal to the stability force at the bottom of the pipe. This bottom force is applied

throughout the pipe uniformly, whereas the buckling stability force is calculated using the same equation but with different depths.

$$F_{bot} = P_e\left(A_e - A_i\right) = 0.052 \times 7500 \times 9.2\left(\frac{\pi}{4}5^2 - \frac{\pi}{4}4.276^2\right)$$
$$= -18.92 \text{ kips},$$

where force is compressive.

*Effective tension at the surface:*

$F_{bs} = 0$ due to the fact that $P_i = P_e = 0$, pertaining to this case.

So, the true tension and effective tension at the surface for the tripping out operation is 137.43 + 0 – 18.92 = 118.5 kips.

## 6.8 Torque and Makeup Torque

Torque is given by

$$T = F_a \times \frac{d}{2} = \mu r F_s. \tag{6.14}$$

When the pipe is rotated and reciprocated, the torque is

$$T = \mu \times F_s \times r \times \frac{|\omega|}{|V_{rs}|}, \tag{6.15}$$

where $|V_{ts}|$ = trip speed,

$$|V_{rs}| = \text{resultant speed} = \sqrt{\left(V_{ts}^2 + \omega^2\right)}, \tag{6.16}$$

$$|\omega| = \text{angular speed} = \text{diameter} \times \pi \times \frac{\text{RPM}}{60}, \tag{6.17}$$

$F_s$ = side or normal force, $\mu$ = the coefficient of friction, and $r$ = the radius of the component.

With equilibrium, the angle torque can be given as

$$T = \frac{\mu}{\sqrt{1+\mu^2}} \times F_s \times r \times \frac{|\omega|}{|V_{rs}|}, \tag{6.18}$$

$$T = \frac{\mu_v}{\sqrt{1+\mu_v^2}} \times F_s \times r \times \frac{|\omega|}{|V_{rs}|}, \tag{6.19}$$

where the variable friction coefficient is given as

$$\mu_v = \mu_s \times e^{-k|V_{rs}|}, \tag{6.20}$$

where $k$ = the speed constant.

With equilibrium, the angle due to pipe rotation with perfect contact is given by

$$\psi = \tan^{-1}\left(\frac{F_e\ \Delta\phi\ \sin\alpha_{avg}}{F_e\ \Delta\alpha + W_b\ \sin\alpha_{avg}}\right) + \tan^{-1}(\mu) \tag{6.21}$$

**Problem 6.7**

A drill string is rotated and pulled out simultaneously. The rotational speed is 10 RPM, and tripping speed is 60 feet/min. The normal or side force acting on a 6″ OD downhole tool present in the drill string is estimated to be 2000 lbf. Estimate the torque and drag in the tool. Assume a friction factor of 0.3.

*Solution:*

Since the pipe is simultaneously rotated and tripped out, the resultant tool speed is

$$|V_{rs}| = \sqrt{\left(V_{ts}^2 + \omega^2\right)} = \sqrt{60^2 + \left(\frac{\pi \times 6 \times 10}{12}\right)^2} = 62 \text{ ft/min.}$$

Drag at the tool is

$$F_D = F_s \times \mu \times \frac{|V_{ts}|}{|V_{rs}|} = 2000 \times 0.3 \times \frac{60}{62} = 580 \text{ lbf.}$$

Similarly, torque can be determined as

$$T = \mu \times F_s \times r \times \frac{|\omega|}{|V_{rs}|} = 0.3 \times 2000 \times \frac{6}{2} \times \frac{\left(\frac{\pi \times 6 \times 10}{12}\right)}{62} = 2735 \text{ ft-lbf.}$$

## 6.9 Buckling

The buckling force $F_b$ is defined by

$$F_b = -F_a + P_i A_i - P_o A_o, \tag{6.22}$$

where $F_b$ = buckling force, $F_a$ = axial force (tension positive), $P_i$ = internal pressure, $A_i = \pi r_i^2$ where $r_i$ is the inside radius of the tubing, $P_o$ = external pressure, and $A_o = \pi r_o^2$ where $r_o$ is the outside radius of the tubing.

The Paslay buckling force, $F_p$, is defined by

$$F_p = \sqrt{\frac{4EIw \sin \alpha}{r}} \tag{6.23}$$

where $F_p$ = Paslay buckling force, $w$ = the distributed buoyed weight of the casing, $\alpha$ = wellbore angle with the vertical, $EI$ = the pipe bending stiffness, and $r$ = radial annular clearance.

### 6.9.1 Buckling Criteria

For curved wellbores, the pipe contact force is given as

$$F_p = \sqrt{\frac{4EIw_c}{r}}, \tag{6.24}$$

in which the contact force, $w_c$, between the pipe and wellbore is given as

$$w_c = \sqrt{\left(w_{bp} \sin \alpha + F_b \alpha'\right)^2 + \left(F_b \sin \theta\phi'\right)^2}, \tag{6.25}$$

where $\alpha$ = inclination angle, $w_{bp}$= buoyed weight of casing, and $\phi$ = azimuth angle.

For the constant curvature of the wellbore, equation 6.25 can be expressed as

$$w_c = \sqrt{\left(w_{bp} n_z - F_b \kappa\right)^2 + \left(w_{bp} b_z\right)^2} \tag{6.26}$$

where $n_z$ = the vertical component of the normal to the curve, and $b_z$ = the vertical component of the binormal to the curve.

**Problem 6.8**

The sample calculation given here is for determining the buckling of 2 – 7/8″, 6.5 lbm/ft tubing inside a 7″, 32 lbm/ft casing. The tubing is submerged in 10 lbm/gal packer fluid with no other pressures applied. Assume that an applied buckling force of 30,000 lbf is applied at the end of the string in a well with a 60° deviation from the vertical. The effect of the packer fluid is to reduce the tubing weight per unit length through buoyancy:

$$w_e = w + A_i\rho_i - A_o\rho_o,$$

where $w_e$ is the effective weight per unit length of the tubing, $A_i$ is the inside area of the tubing, $\rho_i$ is the density of the fluid inside the tubing, $A_o$ is the outside area of the tubing, and $\rho_o$ is the density of the fluid outside the tubing.

The calculation gives

$$\begin{aligned} w_e &= 6.5\ \text{lbm/ft} + (4.68\ \text{in}^2)\ (.052\ \text{psi/ft/lbm/gal}),\\ &= (10.0\ \text{lbm/gal}) - (6.49\ \text{in}^2)\ (.052\ \text{psi/ft/lbm/gal})\\ &\quad (10.0\ \text{lbm/gal}),\\ &= 5.56\ \text{lbm/ft} = 0.463\ \text{lbm/in}. \end{aligned}$$

**Problem 6.9**

Calculate the buckling force of a tubing for a horizontal portion of a well.

Assume E = $30 \times 10^6$ psi, and use other data from Problem 6.8.

*Solution:*

From equation 6.23, the Paslay force can be calculated as

$$F_p = \sqrt{\frac{4EIw\sin\alpha}{r}}.$$

First, the value for a horizontal well is calculated as follows:

$$\begin{aligned} F_p &= \sqrt{4\ (0.463\ \text{lbm/in})(30\times10^6\ \text{psi})(1.611\ \text{in}^4)/(1.61\ \text{in})}\\ &= 7456\ \text{lbf}. \end{aligned}$$

This means that the axial buckling force must exceed 7456 lbf before the tubing will buckle.

## 6.10 Maximum Permissible Dogleg

Maximum permissible dogleg and the axial stress are given by

$$\sigma_t = \frac{F_{dls}}{A} \text{ psi,} \tag{6.27}$$

where $A$ = the cross sectional area of the drill pipe body in sq. inches, and $F_{dls}$ = the buoyed weight supported below the dogleg in lbs.

The maximum permissible bending stress for a drill pipe of grade E when the tensile stress is less than or equal to 67,000 psi is given by

$$\sigma_b = 19500 - \frac{10}{67}\sigma_t - \frac{0.6}{670^2}(\sigma_t - 33500)^2. \tag{6.28}$$

The maximum permissible bending stress for grade S drill pipe when the tensile stress is less than or equal to 133,400 psi is given by

$$\sigma_b = 20000\left(1 - \frac{\sigma_t}{145000}\right)\text{psi.} \tag{6.29}$$

The maximum permissible dogleg severity is

$$c = \frac{432000\sigma_b}{\pi E D_p}\frac{\tanh(KL)}{KL} \text{ deg/100 ft,} \tag{6.30}$$

where

$$K = \sqrt{\frac{T}{EI}}. \tag{6.31}$$

$E$ = Young's modulus in psi:

$$E = 30 \times 10^6 \text{ psi for steel,}$$
$$E = 10.5 \times 10^6 \text{ psi for aluminum.}$$

$D_p$ = drill pipe OD in inches, and $L$ = half distance between the tool joints in inches = 180 inches for Range 2 drill pipe.

Note that the above calculation does not hold true for Range 3 drill pipe.

$I$ = the drill pipe moment of inertia with respect to its diameter:

$$I = \frac{\pi}{64}\left(D^4 - ID^4\right) \text{ in}^4.$$

The expected value of lateral force or the force at the tool joint is

$$F_s = \frac{\pi \times c \times L \times T}{108000} \text{lb.} \tag{6.32}$$

## 6.11 Length Change Calculations

The total stretch or total elongation of the drill string is given as

$$\Delta L_{stretch} = \Delta L_a + \Delta L_p + \Delta L_b + \Delta L_t \tag{6.33}$$

### 6.11.1 Stretch Due to Axial Load

Stretch due to the linear change in the axial load and is given by:

$$\Delta L_a = \frac{F_T \times L}{A \times E} + \frac{\Delta F_T \times L}{2 \times A \times E} \tag{6.34}$$

where $F_T$ = true tension, axial force acting at the point of reference and is determined by the pressure area method.

$\Delta F$ = the change in pressure area axial force over the component length, $A$ = the cross sectional area of the component, and $E$ = Young's modulus of the component material.

### 6.11.2 Stretch Due to the Pressure Effect (Ballooning)

This elongation of the string is due to the differential pressure inside and outside of the workstring and is given by the following equation:

$$\Delta L_p = \frac{-v \times L_p}{E \times (R^2 - 1)} \times \left[ (\rho_s - R^2 \times \rho_a) \times L + 2 \times (P_s - R^2 \times P_a) \right], \tag{6.35}$$

where, $\Delta L_p$ = the change in length due to the ballooning mechanism, $L_p$ = the length of the workstring component element, $R$ = the ratio of the component outside diameter to the inside diameter, $E$ = Young's modulus of the component material, $v$ = Poisson's ratio of the component material, $\rho_s$ = mud density inside the workstring component, $\rho_a$ = mud density in the annulus at the depth of the workstring component, $P_s$ = the surface pressure on the drill string side, and $P_a$ = the surface pressure on the annulus side.

### 6.11.3 Stretch Due to Buckling

The length change for buckling is

$$\Delta L_b = \frac{-r^2}{4EIw}\left(F_2 - F_p\right)\left[0.3771 F_2 - 0.3668 F_p\right] \tag{6.36}$$
$$\text{for } 2.8F_p > F_2 > F_p$$

and

$$\Delta L_b = -\frac{r^2}{8EIw}\left[F_2^2 - F_1^2\right] \quad \text{for } F > 2.8 F_p. \tag{6.37}$$

**Problem 6.10**

Assume that an applied buckling force of 30,000 lbf is applied at the end of the string in a well with a 60° deviation from the vertical. The string will buckle for any force between 6,939 lbf and 30,000 lbf. The axial force will vary as $w_e \cos\theta$.

*Solution:*

$$w_a = w_e \cos(60) = 5.56 \text{ lbf/ft } (0.50) = 2.78 \text{ lbf/ft}.$$

Total buckled length is

$$L_{bkl} = \frac{(30{,}000 - 6939) \text{ lbf}}{2.78 \text{ lbf/ft}} = 8{,}295 \text{ ft}.$$

Maximum helically buckled length is

$$L_{helmax} = \frac{(30{,}000 - 9813) \text{ lbf}}{2.78 \text{ lbf/ft}} = 7{,}262 \text{ ft}.$$

Minimum helically buckled length is

$$L_{helmin} = \frac{(30{,}000 - 19{,}626) \text{ lbf}}{2.78 \text{ lbf/ft}} = 3{,}732 \text{ ft}.$$

String length change calculations involve two calculations.

The lateral buckling tubing movement is given by:

$$\Delta L_h = \frac{-(1.61 \text{ in})^2(19626 - 6939 \text{ lbf}) \times [0.3771(19626) - 0.3668(6939) \text{ lbf}]}{\ldots [(4)(30 \times 106 \text{ psi })(1.611 \text{ in}^4)(2.78 \text{ lbf/ft})]}$$

$$= 0.297 \text{ ft.}$$

The helical buckling tubing movement is given by

$$\Delta L_h = \frac{-(1.61 \text{ in})^2(30{,}000^2 - 19626^2 \text{ lbf}^2)}{(8)(30 \times 10^6 \text{ psi})\ (1.611 \text{ in}^4)(2.78 \text{ lbf/ft})}$$

$$= 1.242 \text{ ft.}$$

The total string movement is 0.297 ft. plus 1.242 ft. which equals 1.539 ft.

The length change due to pure helical buckling is

$$\Delta L_h = \frac{-(1.61 \text{ in})^2(30{,}000^2 \text{ lbf}^2)}{(8)(30 \times 10^6 \text{ psi})\ (1.611 \text{ in}^4)(2.78 \text{ lbf/ft})}$$

$$= 2.170 \text{ ft.}$$

## 6.11.4 Stretch Due to Temperature

The thermal induced stretch can be given by the following equation:

$$\Delta L_t = L a_t \Delta t, \tag{6.38}$$

where $a_t$ = the coefficient of thermal expansion defined as the fractional increase in length per unit rise in temperature, with units of in/in/F (with values of $6.9 \times 10^{-6}$ for steel, $10.3 \times 10^{-6}$ for aluminum, and $4.9 \times 10^{-6}$ for titanium). $\Delta t$ = the average temperature change in degrees F.

Assuming a linear variation of temperature along the wellbore as $\Delta t(z) = \Delta t_0 + \frac{\Delta t}{\Delta z} zs$, equation 6.38 can be written as

$$\Delta L_t = a\left[\Delta t_0 + \frac{\Delta t}{\Delta z}\frac{L^2}{2}\right] \tag{6.39}$$

where $z$ = measured depth, and $\Delta L$ = measured calculation interval.

**Problem 6.11**

Compute the elongation of 10,000 ft of 7-in casing due to temperature if the bottomhole temperature (BHT) is 290°F and the surface temperature is 70°F.

*Solution:*

$$\Delta t = \frac{\text{final BHT} + \text{final surface temp}}{2} - \frac{\text{initial BHT} + \text{initial surface temp}}{2},$$

$$\Delta t = 110°\text{F}.$$

$$\Delta L_t = 10000 \times 6.9 \times 10^{-6} \times 110 = 7.59 \text{ ft}.$$

## 6.12 Stresses

### 6.12.1 Radial Stress

The normal stress in the radial direction is

$$\sigma_r(r) = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} + \left(\frac{r_i^2 r_o^2}{r^2}\right)\frac{(P_o - P_i)}{(r_o^2 - r_i^2)}, \quad r_i \le r \le r_o. \tag{6.40}$$

The above equation may be written in simplified form as

$$\sigma_r(r) = M + CY, \tag{6.41}$$

where

$$M = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2},$$

$$C = \left(\frac{r_i^2 r_o^2}{r^2}\right),$$

and

$$Y = \frac{(P_o - P_i)}{(r_o^2 - r_i^2)} = \frac{\Delta P}{(r_o^2 - r_i^2)},$$

where $r$ = the radius of the pipe in inches, $r_i$ = the inside radius of the pipe in inches, $r_o$ = the outside pipe radius in inches, $P_i$ = the pipe's internal pressure in psi, $P_o$ = the pipe's external pressure in psi, and $\Delta p = (P_o - P_i)$.

The outside pipe radius should be modified based on the pipe class. When the internal and external pressures are the same,

$$\sigma_{ri} = \sigma_{ro} = -P_i = -P_o. \tag{6.42}$$

## 6.12.2 Hoop Stress (Tangential or Circumferential Stress)

The normal stress in the circumferential direction is

$$\sigma_h(r) = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} - \left(\frac{r_i^2 r_o^2}{r^2}\right)\frac{(P_o - P_i)}{(r_o^2 - r_i^2)}, \quad r_i \le r \le r_o. \tag{6.43}$$

If this is expressed in terms of $M$, $C$, and $Y$ defined above, then

$$\sigma_r(r) = M - CY.$$

Equation 6.43 is called Lame's equation. For the outer and inner walls, the above equation can be written as given below.

For the inside surface at $r = r_i$,

$$\sigma_{hi} = \frac{2r_i^2 P_i - (r_o^2 + r_i^2)P_o}{r_o^2 - r_i^2}. \tag{6.44}$$

For the outside surface at $r = r_o$,

$$\sigma_{h2} = \frac{-2r_o^2 P_o + (r_o^2 + r_i^2)P_i}{r_o^2 - r_i^2}. \tag{6.45}$$

And when the internal and external pressures are same,

$$\sigma_{h1} = \sigma_{h2} = -P_i = -P_o. \tag{6.46}$$

**Problem 6.12**

Calculate hoop stress and radial stress inside and outside of the pipe with the following data:

- Inside pressure = outside pressure = 4019 psi
- Pipe body diameter = 5″
- Pipe inside diameter = 4″

*Solution:*

*Hoop stress:*

When the internal and external pressures are same,

$$\sigma_{h1} = \sigma_{h2} = -P_i = -P_o,$$
$$\sigma_{h1} = \sigma_{h2} = -4019 \text{ psi}.$$

*Radial stress:*

When the internal and external pressures are the same,

$$\sigma_{ri} = \sigma_{ro} = -P_i = -P_o,$$
$$\sigma_{ri} = \sigma_{ro} = -4019 \text{ psi}.$$

## 6.12.3 Axial Stress

The axial stress is given by

$$\sigma_a = \frac{F_T}{\pi\left(r_o^2 - r_i^2\right)} + \sigma_{buck} \pm \sigma_b. \tag{6.47}$$

**Problem 6.13**

Calculate the axial stress of the pipe with the following data:

- True axial force = 64.6 kips
- Effective force = 97.9 kips
- Pipe body diameter = 5″
- Pipe inside diameter = 4″
- Pipe data: premium class; assume no buckling and bending occur
- $E$ = modulus of elasticity, psi; for steel = $30 \times 10^6$ psi

*Solution:*

Derating the pipe based on the premium class,

$$D_{cp} = c \times D_p + D_i\left(1 - c\right).$$

New OD = 0.800 × 5.0 + 4.00 (1.0 – 0.800) = 4.80 in.

Axial stress inside of the pipe is

$$\sigma_a = \frac{F_T}{\pi\left(r_o^2 - r_i^2\right)} = \frac{646000}{\pi\left[\left(\frac{4.8}{2}\right)^2 - \left(\frac{4}{2}\right)^2\right]} = 11{,}683 \text{ psi}.$$

Note that for stress calculations true force should be used.

## 6.12.4 Bending Stress with Hole Curvature

Bending stress can be calculated from the following equation:

$$\sigma_b = \frac{M_b \times OD}{2I}, \tag{6.48}$$

where $M_b$ = the bending moment = $EI\kappa$, $I$ = the axial moment of inertia, $OD$ = outside diameter of the pipe, and $\kappa$ = the pipe curvature.

It can also be expressed using the bending moment relationship:

$$\sigma_b = \frac{E \times OD}{2R}, \tag{6.49}$$

where $R$ is the radius of curvature ($R = 1/\kappa$).

The bending stress can conveniently be expressed using the wellbore curvature as the pipe curvature as

$$\sigma_b = \frac{rE\kappa M}{68754.9} \text{ psi}, \tag{6.50}$$

where $E$ = the modulus of elasticity in psi, $r$ = the radius of the pipe in inches, $\kappa$ = the wellbore curvature as dogleg severity in deg/100ft, and $M$ = the bending stress magnification factor.

68754.9 is the conversion factor $\left(= 12 \times 100 \times 180/\pi\right)$.

The bending stresses at the external and internal radii are given below.

$$\sigma_{bi} = \frac{r_i E\kappa M}{68754.9}. \tag{6.51}$$

$$\sigma_{bo} = \frac{r_o E\kappa M}{68754.9}. \tag{6.52}$$

When buckling occurs,

$$\sigma_{buck} = \frac{rR_c F_{real}}{2I}. \tag{6.53}$$

where $F_{real}$ is the real force, lbf.

The buckling stresses at the external and internal radii are given below.

At the inside surface,

$$\sigma_{bucki} = \frac{r_i R_c F_{real}}{2I}. \tag{6.54}$$

At the outside surface,

$$\sigma_{bucko} = \frac{r_o R_c F_{real}}{2I}. \tag{6.55}$$

**Problem 6.14**

Calculate the bending stress of the pipe where the dogleg is 3.05 deg/100 ft.

- Pipe body diameter = 5″
- Pipe inside diameter = 4″
- Pipe data: premium class; assume no buckling occurs
- $E = 30 \times 10^6$ psi

*Solution:*

Derating the pipe based on the premium class, using equation 6.4

$$D_{cp} = c \times D_p + D_i\,(1 - c).$$

New OD = 0.800 × 5.0 + 4.00 (1.0 – 0.800) = 4.80 in.
New wall thickness = 0.4 in.

The bending stress inside the pipe is

$$\sigma_{bi} = \frac{r_i \kappa k}{68754.9} = \frac{\left(\frac{4}{2}\right) 30{,}000{,}000 \times 3.05}{68754.9} = 2{,}662 \text{ psi}.$$

The bending stress outside the pipe is

$$\sigma_{bo} = \frac{r_0 \kappa k}{68754.9} = \frac{\left(\frac{4.8}{2}\right) 30{,}000{,}000 \times 3.05}{68754.9} = 3{,}194 \text{ psi}.$$

Therefore, the maximum bending stress occurs outside the body of the pipe.

**Problem 6.15**

Calculate the bending stress of a 40-ft. section of a class 1, 7.625 inch, OD drill pipe where the survey points are as follows:

- 839.2 – 799.2 ft: Incl = 15.18°, Azm = 342.50° – Incl = 14.57°, Azm = 342.381°
- 799.2 – 759.3 ft: Incl = 14.57°, Azm = 342.381° – Incl = 13.87°, Azm = 342.386°
- 759.3 – 719.3 ft: Incl = 13.87°, Azm = 342.386° – Incl = 12.64°, Azm = 342.41°

Assume no buckling occurs. $E$ = the modulus of elasticity, psi, and for steel = $30 \times 10^6$ psi.

*Solution:*

*For depth of 799.2 ft:*

The borehole curvature over the section of the pipe is

$$\kappa = \frac{\beta}{\Delta L},$$

where $\kappa$ = the average borehole curvature, and $\beta = a\cos(\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\Delta\phi)$.

$$\kappa = \frac{a\cos\left(\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\Delta\phi\right)}{\Delta L}$$

$$= a\cos\frac{(\cos 15.18 \cos 14.57) + \sin 15.18 \sin 14.57 \cos(342.51 - 342.39)}{(839.2 - 799.2)}$$

$$= 0.00027 \text{ rad/ft.}$$

$$\text{Converting to deg/100 ft} = 0.00027 \times 100 \times \frac{180}{\pi}$$

$$= 1.544 \text{ deg/100 ft.}$$

The bending stress outside the pipe is

$$\sigma_{bo} = \frac{r_0 E \kappa}{12} = \frac{\left(\frac{7.625}{2}\right) 30{,}000{,}000 \times 0.00273}{12} = 2568 \text{ psi.}$$

*For depth of 759.3 ft:*

$$\begin{aligned}\kappa &= \frac{a\cos\left(\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\Delta\phi\right)}{\Delta L}\\ &= a\cos\frac{(\cos 13.87 \cos 14.57) + \sin 13.87 \sin 14.57 \cos(342.386 - 342.381)}{(799.2 - 759.2)}\\ &= 0.0003025 \text{ rad/ft.}\end{aligned}$$

$$\begin{aligned}\text{Converting to deg/100 ft} &= 0.0003025 \times 100 \times \frac{180}{\pi}\\ &= 1.733 \text{ deg/100 ft.}\end{aligned}$$

The bending stress outside the pipe is

$$\sigma_{bo} = \frac{r_0 E\kappa}{12} = \frac{\left(\frac{7.625}{2}\right) 30{,}000{,}000 \times 0.0003025}{12} = 2883 \text{ psi.}$$

Alternatively, the bending stress can be calculated as

$$\sigma_{bo} = \frac{r_0 E\kappa}{68754.9} = \frac{\left(\frac{7.625}{2}\right) 30{,}000{,}000 \times 1.733}{68754.9} = 2883 \text{ psi.}$$

*For depth of 719.3:*

$$\begin{aligned}\kappa &= \frac{a\cos\left(\cos\alpha_1 \cos\alpha_2 + \sin\alpha_1 \sin\alpha_2 \cos\Delta\phi\right)}{\Delta L}\\ &= a\cos\frac{(\cos 13.88 \cos 12.64) + \sin 13.87 \sin 14.57 \cos(342.41 - 342.36)}{(759.2 - 719.2)}\\ &= 0.00055 \text{ rad/ft.}\end{aligned}$$

$$\begin{aligned}\text{Converting to deg/100 ft} &= 0.00055 \times 100 \times \frac{180}{\pi}\\ &= 3.17 \text{ deg/100 ft.}\end{aligned}$$

The bending stress outside the pipe is

$$\sigma_{bo} = \frac{r_0 E \kappa}{12} = \frac{\left(\frac{7.625}{2}\right) 30{,}000{,}000 \times 0.00055}{12} = 5269 \text{ psi.}$$

**Problem 6.16**

This problem provides a sample buckling bending stress calculation. Use the data from Problem 6.8.

*Solution:*

$$d_o = 2.875''.$$

Radial clearance = $r$ = 1.61 in.
The moment of inertia = $I$ = 1.611 in$^4$.

$$F_b = 30{,}000 \text{ lbf.}$$

The maximum bending stress due to buckling can be evaluated using equation 6.49.

$$\sigma_b = \frac{.25 \times 2.875 \times 1.61 \times 30{,}000}{1.611},$$
$$= 21{,}550 \text{ psi.}$$

### 6.12.5 Bending Stress with Hole Curvature, Pipe Curvature, and Tensile Force

If there is no pipe-to-wall contact between the tool joints, and if

$$\kappa < \frac{\frac{r}{L}}{\frac{1}{2} - \frac{\cosh(KL) - 1}{KL\sinh(KL)}}, \tag{6.56}$$

then

$$\kappa_p = \kappa \frac{KL}{\tanh(KL)}. \tag{6.57}$$

If there is no pipe-to-wall contact between the tool joints, then

$$\kappa_p = \kappa(KL)\frac{\sinh(KL) - KL - \left(\frac{1}{2} + \frac{r}{L_c^2}\right)KL[\cosh(KL) - 1]}{2[\cosh(KL) - 1] - KL\sinh\sinh(KL)}, \quad (6.58)$$

where $\kappa_p$ = the pipe curvature at the tool joint, rad/in, $\kappa$ = the wellbore curvature on the distance between tool joints, rad/in, $L$ = half the distance between tool joints (for example, $L$ = 180" for range 2 drill pipes, and L = 270" for range 3 drill pipes), in, K = $\sqrt{F/EI}$, $F$ = the tensile force applied to the pipe, lbf, and $EI$ = the product of the modulus of elasticity and the moment of inertia, lb-in$^2$.

$$r = \frac{D_{otj} - D_p}{2},$$

where $D_{otj}$ = the tool joint outside diameter, in.

**Problem 6.17**

Calculate the bending stress of the pipe where the dogleg is 3.05 deg/100 ft. Use the following data:

- Pipe body diameter = 5″
- Inside diameter = 4″
- Tool joint diameter = 7.25″
- Range 2, class 1
- Buoyed weight of the drill string below the dogleg is 40,000 lb.
- $E = 30 \times 10^6$ psi

*Solution:*

Checking the condition using equation 6.56

$$KL = \sqrt{\frac{40000}{30000000 \times \frac{\pi}{64}(5^4 - 4^4)}} \times 180 = 1.544,$$

$$\frac{\frac{r}{L}}{\frac{1}{2} - \frac{\cosh(KL) - 1}{KL\sinh(KL)}} = \frac{\frac{\left(\frac{7.25-5}{2}\right)}{180}}{\frac{1}{2} - \frac{\cosh(1.54) - 1}{(1.54)\sinh(1.54)}} = 0.03756.$$

The above calculated value is greater than the curvature of the wellbore, $\kappa = 6.17$ deg/100 ft $= 7.7 \times 10^{-5}$ 1/in.

Therefore, the drill pipe curvature is

$$\kappa_p = \kappa \frac{KL}{\tanh(KL)} = 7.7 \times 10^{-5} \times 12 \times \frac{1.54}{\tanh(1.54)} = 0.00157 \ 1/\text{ft}.$$

Converting to deg/100 ft,

$$0.00157 \times 100 \times \frac{180}{\pi} = 9 \text{ deg/100 ft}.$$

Calculating the bending stress,

$$\sigma_{bo} = \frac{r_0 E \kappa}{68754.9} = \frac{\left(\frac{5}{2}\right)30{,}000{,}000 \times 9}{68754.9} = 9819 \text{ psi}.$$

## 6.12.6 Torsional or Twisting Shear Stress

If the pipe is subjected to torsion, the torsional shear stress is given by

$$\tau_{tor} = \frac{12rT}{J}, \tag{6.59}$$

where $J$ = the polar moment of inertia in in$^4$, and $T$ = torque in ft-lbf.

The torsional shear stress at the inner and outer walls is given by

$$\tau_{tori} = \frac{12r_iT}{J} \tag{6.60}$$

and

$$\tau_{toro} = \frac{12r_0T}{J}, \tag{6.61}$$

respectively.

## 6.12.7 Transverse Shear Stress

The transverse shear acting normal to the longitudinal axis of the drill string is given by

$$\tau_s = \frac{2F_s}{A}, \tag{6.62}$$

where $F_s$ = the normal or side force in lb, and $A$ = the cross sectional area of the component in in$^2$.

This can also be written as

$$\tau_{si} = \tau_{so} = \frac{2\left(\sqrt{F_1^2 + F_2^2}\right)}{A}, \tag{6.63}$$

where $F_1$ = the radial force in the vertical plane in lb, and $F_2$ = the radial force in the horizontal plane in lb.

**Problem 6.18**

There is a plan to design a measurement steel tool of a length of 30 ft with an outer diameter of 5 in. The maximum torque subjected should not exceed 8,000 ft-lbf, and the angle of twist is 8°. Determine the largest inner diameter of the tool that can be designed. Assume that the twisting is totally elastic and the modulus of rigidity = 12 × 10$^6$ psi. The maximum allowable shear stress is 10,000 psi.

*Solution:*

Based on the twist angle requirement, the twist angle can be given as

$$\theta = \frac{TL}{GJ} \text{ rad},$$

where

$$J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right).$$

The inside diameter can be given by algebraically manipulating the above equations:

$$\left(D_o^4 - \frac{32TL}{\pi G\theta}\right) = D_i^4,$$

$$D_i = \sqrt[\frac{1}{4}]{\left(D_o^4 - \frac{32TL}{\pi G\theta}\right)}.$$

Substituting with appropriate units and values given in the problem,

$$D_i = \sqrt[\frac{1}{4}]{\left(5^4 - \frac{32 \times 8000 \times 12 \times 30 \times 12}{\pi \times 12 \times 10^6 \times 8 \times \left(\frac{\pi}{180}\right)}\right)} = 4.513 \text{ in.}$$

Based on the stress requirement, the torsional shear stress at the inner and outer walls is given by

$$\tau_{toro} = \frac{12 r_0 T}{J}.$$

By substituting $J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right)$ and rearranging,

$$D_i = \sqrt[\frac{1}{4}]{\left(D_o^4 - \frac{12 \times 32 T \times r_0}{\pi \tau_{toro}}\right)}$$

$$= \sqrt[\frac{1}{4}]{\left(5^4 - \frac{12 \times 32 \times 8000 \times \left(\frac{5}{2}\right)}{\pi \times 10000}\right)} = 4.416 \text{ in.}$$

Using the diameter based on the stress, it can be checked at the inner radius as follows:

$$\text{New } J = \frac{\pi}{32}\left(5^4 - 4.416^2\right) = 24 \text{ in}^4.$$

Shear stress is

$$\tau_{tori} = \frac{12 r_i T}{J} = \frac{12 \times \frac{4.416}{2} \times 8000}{24} = 8833 \text{ psi}$$

and is within the allowed limit of 10,000 psi.

Alternatively, it can be checked as follows.

Using the torsion equation, the inside diameter can be given as

$$D_i^4 + \frac{6 D_i T}{\left(\frac{\pi}{32}\right)\tau_{tor}} - D_o^4 = 0 = D_i^4 + \frac{6 D_i \times 8000}{\left(\frac{\pi}{32}\right) 10000} - 5^4.$$

Solving iteratively, the maximum inside diameter allowed will be

$$D_i = 4.49 \text{ in.}$$

Take the lowest value of the inner diameter: 4.42 in.

**Problem 6.19**

Calculate the transverse shear stress of the pipe with a side force of 1400 lbf.

- Pipe body diameter = 5″
- Pipe inside diameter = 4″
- Pipe data: premium class

*Solution:*

Derating the pipe based on the premium class,

$$D_{cp} = c \times D_p + D_i\,(1-c).$$

$$\text{New OD} = 0.800 \times 5.0 + 4.00\ (1.0 - 0.800) = 4.80 \text{ in.}$$

The pipe body cross-sectional area is

$$A_p = \pi\left[\left(\frac{4.8}{2}\right)^2 - \left(\frac{4}{2}\right)^2\right] = 5.53 \text{ in}^2.$$

The bending stress inside of the pipe is

$$\tau_s = \frac{2F_s}{A} = \frac{2 \times 1400}{5.553} = 506 \text{ psi}.$$

## 6.12.8 von Mises Stress

The von Mises failure criteria, known as the maximum energy of distortion theory, is given by

$$\sigma_{vm} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r - \sigma_h)^2 + (\sigma_h - \sigma_a)^2 + (\sigma_a - \sigma_r)^2}, \tag{6.64}$$

where $\sigma_r$ is the radial stress, $\sigma_h$ is the hoop stress, and $\sigma_a$ is the axial stress, which has already been defined in the earlier sections.

The equivalent stress is given by

$$\sigma_{eff} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_r - \sigma_h)^2 + (\sigma_h - \sigma_a)^2 + (\sigma_a - \sigma_r)^2 + 6(\tau_t^2 + \tau_s^2 + \tau_r^2)}, \tag{6.65}$$

where $\tau_r$ is the radial shear stress, which is usually zero for pipes, $\tau_t$ is the torsion shear stress, and $\tau_r$ is the transverse shear stress.

Expressing in the form that uses the terms $C$ and $Y$,

$$\sigma_{vm}^2 = 3C^2Y^2 + \left( \frac{F_e}{\pi(r_o^2 - r_i^2)} \pm \sigma_b \right)^2. \tag{6.66}$$

## 6.12.9 Stress Ratio

The stress ratio is calculated as

$$X = \frac{\sigma_y \times \%\text{Yield}}{\sigma_{vm}}, \tag{6.67}$$

where $\sigma_y$ = the yield strength of the pipe in psi, and %Yield = a percentage used to reduce the yield strength of the pipe as a factor of additional safety.

When $X$ is equal to or less than 1, there is concern of failure.

**Problem 6.20**

Calculate the stress ratio using the following pipe and wellbore data for a rotating off bottom condition.

- Wellbore measure depth = 4974.7 ft
- Hole inclination = 0°
- Hole azimuth = 0°
- Mud density = 9.5 ppg
- Pipe data: premium class
- Pipe diameter = 5″ × 4.276″
- Joint diameter = 6.375″ × 3.75″
- Minimum yield strength = 75000 psi
- Dogleg severity = 0 deg/100 ft
- Rotating off bottom torque = 7254 ft-lbf
- Rotating off bottom hook load = 100 kips

*Solution:*

Derating the pipe based on the premium class,

$$\text{New OD} = 0.800 \times 5.0 + 4.276\ (1.0 - 0.800) = 4.8552 \text{ in.}$$

New wall thickness = 0.2896 in.
True vertical depth = 4974.7 ft.
Inside and outside pressure = 0.052 × 9.5 × 4974.7 = 2455 psi.
The polar moment of inertia of the pipe is

$$J_{pipe} = \frac{\pi}{32}\left(4.855^4 - 4.276^4\right) = 21.73 \text{ in}^4.$$

$$J_{joint} = 142.73 \text{ in}^4.$$

The combined polar moment of inertia is

$$J = 22.7 \text{ in}^4.$$

$$\text{Pipe area} = \frac{\pi}{4}\left(4.855^2 - 4.276^2\right) = 4.153 \text{ in}^2.$$

Stresses are calculated both inside and outside the pipe as follows.
Plane radial stress is calculated using equation 6.40:

$$\begin{aligned}\sigma_{ri} &= \frac{2455 \times 2.138^2 - 2455 \times 2.4276^2}{2.4276^2 - 2.138^2} + \left(\frac{2.138^2 \times 2.4276^2}{2.138^2}\right)\frac{(2455 - 2455)}{(2.4276^2 - 2.138^2)} \\ &= -2455 \text{ psi} \\ \sigma_{ro} &= -2455 \text{ psi.}\end{aligned}$$

Transverse shear stress is zero for this condition.

When the internal and external pressures are the same, the hoop stress is given by equation 6.43:

$$\sigma_{hi} = \sigma_{ho} = -P_i = -P_o = -2455 \text{ psi.}$$

Torsional stress is calculated from equation 6.59 as follows:

$$\tau_{torii} = \frac{12 \times 2.138 \times 7254}{22.70} = 8200 \text{ psi,}$$

$$\tau_{torio} = \frac{12 \times 2.4276 \times 7254}{22.70} = 9311 \text{ psi.}$$

Axial stress in the absence of bending and buckling stress is given by

$$\sigma_a = \frac{100 \times 1000}{\pi\left(2.4276^2 - 2.138^2\right)} = 24074 \text{ psi.}$$

The von Mises stress inside and outside the pipe walls are calculated using equation 6.64:

$$\sigma_{effo} = \frac{1}{\sqrt{2}}\sqrt{(-2455+2455)^2 + (-2455-24704)^2 + (24704+2455)^2 + 6\left(9311^2\right)} = 31046 \text{ psi}$$

$$\sigma_{eff} = 30092 \text{ psi.}$$

The greater value of the von Mises stress is 31,046 psi, and this is used to calculate the stress ratio using equation 6.67:

$$\text{Stress ratio} = X = \frac{75000 \times .90}{31046} = 2.17.$$

Since $X$ is greater than 1, there is no concern of failure.

**Problem 6.21**

A well having a maximum dogleg of 12 deg/100 ft is being drilled with 4½″, 16.6 ppf drill pipe. The total weight below the maximum dogleg is 120 kips. Determine whether the pipe is strong enough to withstand a torque of 14,000 ft-lbf. Assume the following:

- Maximum percentage of yield = 90%
- Pipe curvature is the same as the wellbore curvature
- No buckling occurs
- Pipe data: class 1

*Solution:*

Based on the weight of the pipe, the inside diameter of the pipe = 3.826″.

Effective stress is

$$\sigma_{eff} = \frac{1}{\sqrt{2}}\sqrt{2\sigma_a + 6\tau_t^2},$$

where

$$\sigma_a = \frac{F_T}{\pi\left(r_o^2 - r_i^2\right)} + \sigma_{buck} \pm \sigma_b.$$

Since there is no buckling,

$$\sigma_a = \frac{F_T}{\pi\left(r_o^2 - r_i^2\right)} + \sigma_b$$

and

$$\tau_t = \frac{12 r_0 T}{J}.$$

Substituting for $J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right) = \frac{\pi}{32}\left(4.5^4 - 3.826^4\right) = 19.22$ and other given values,

$$\sigma_{eff} = \frac{1}{\sqrt{2}}\sqrt{2\left(\frac{F_T}{\pi\left(r_o^2 - r_i^2\right)} + \sigma_b\right)^2 + 6\left(\frac{12 r_0 T}{\frac{\pi}{32}\left(D_o^4 - D_i^4\right)}\right)^2},$$

$$= \frac{1}{\sqrt{2}}\sqrt{2\left[\left(\frac{80000}{\pi\left(\left(\frac{4.5}{2}\right)^2 - \left(\frac{3.826}{2}\right)^2\right)}\right) + \left(\frac{30\times 10^6\left(\frac{4.5}{2}\right)\times 12}{68754.9}\right)\right]^2 + 6\left(\frac{12\left(\frac{4.5}{2}\right)\times 14000}{19.22}\right)^2},$$

$$= \frac{1}{\sqrt{2}}\sqrt{2\times(27226.7 + 11780.97) + 6\times 19666} = 51787 \text{ psi}.$$

The stress ratio is

$$X = \frac{75000\times .90}{51787} = 2.24.$$

Since $X$ is greater than 1, the pipe will not fail.

## 6.13 Fatigue Ratio

The fatigue ratio is

$$FR_F = \frac{|\sigma_b| + |\sigma_{buck}|}{\sigma_{fl}}, \tag{6.68}$$

where $\sigma_b$ = the bending stress in psi, $\sigma_{fl}$ = fatigue limit, and $\sigma_{buck}$ = the buckling stress in psi.

For tension, the fatigue limit can be written as

$$\sigma_{fl} = \sigma_{el}\left(1 - \frac{F_e}{F_y}\right), \tag{6.69}$$

where $\sigma_{el}$ = the fatigue endurance limit of the pipe in psi, $F_y$ = yield tension, lbf, and $F_e$ = effective tension in lbf.

For compression, the fatigue limit is

$$\sigma_{fl} = \sigma_{el}.$$

The axial force on the pipe has a remarkable impact on the fatigue failure. The force required to generate the yield stress can be written as

$$F_y = \sigma_{y_{min}} A_e, \tag{6.70}$$

where $\sigma_{y_{min}}$ = the minimum yield of the pipe in psi, and $A_e$ = the effective cross sectional area in in$^2$.

**Problem 6.22**

Using the data from Problem 6.20, calculate the fatigue ratio. The effective tension is 100 kips. Assume the fatigue endurance limit for the pipe is 20,000 psi, and bending stress is 2100 psi.

*Solution:*

The axial force required to generate the yield stress is calculated as follows:

$$F_y = 75000 \times \frac{\pi}{4}\left[\left(0.95 \times 4.8552^2 + 0.05 \times 6.375^2\right) - \left(0.95 \times 4.276^2 + 0.05 \times 3.75^2\right)\right]$$
$$= 374237 \text{ lbf}.$$

For tension, the fatigue limit can be calculated using equation 6.69:

$$\sigma_{fl} = 2100\left(1 - \frac{100000}{374237}\right) = 14656 \text{ psi.}$$

Hence, the fatigue ratio is

$$FR_F = \frac{2100}{14656} = 0.14.$$

## 6.14 Bending Stress Magnification Factor (BSMF)

### 6.14.1 BSMF for Tensile Force

1. Compute

$$\Delta D = D_{tj} - D_p, \tag{6.71}$$

where $D_{tj}$ = the tool joint outside diameter , in, and $D_p$ = the pipe body outside diameter, in.

2. If the variables $a$, $\eta$, $\Delta D_1$, and $\Delta D_2$ are defined as given below, the BSMF can be calculated depending on the type of contact as follows:

A. Compute $a$ using

$$a = \sqrt{\frac{F_a}{EI}}, \tag{6.72}$$

where $F_a$ = axial force, lbf.

B. Compute $\eta$ using

$$\eta = \frac{aL}{2}, \tag{6.73}$$

where $L$ = the distance between tool joints, ft.

C. Compute $\Delta D_1$ as

$$\Delta D_1 = \frac{\left(\frac{\eta}{2} + \frac{(1 - \cosh\eta)}{\sinh\eta}\right)L}{aR}, \tag{6.74}$$

where $R$ = the radius of curvature of the wellbore, in.

D. Compute $\Delta D_2$ as

$$\Delta D_2 = \frac{\left(\frac{2}{\eta} + \frac{(\eta \cosh \eta + 1) - 2\sinh \eta}{(\cosh \eta - 1)}\right) L}{aR}. \tag{6.75}$$

E. If $\Delta D \leq \Delta D_2$, then the contact is wrap contact and the BSMF calculation continues.

i. Compute $\xi$ using

$$a^2 \Delta D \frac{R}{2} = 2 + \frac{\left[-2\xi \sinh \xi + \xi^2 \frac{(\cosh \xi + 1)}{2}\right]}{\cosh \xi - 1}. \tag{6.76}$$

ii. Compute $\gamma$ using

$$\gamma = 2(\cosh \xi - 1) - \xi \sinh \xi. \tag{6.77}$$

iii. Hence, BSMF is given by

$$\mathrm{BSMF} = -\frac{\left\{\left(\frac{\xi^2}{2} + a^2 \Delta D \frac{R}{2}\right)(\cosh \xi - 1) - \xi(\sinh \xi - \xi)\right\}}{\gamma}, \tag{6.78}$$

where $\xi$ and $\gamma$ are calculated using equations 6.76 and 6.77, respectively.

iv. If $\Delta D \leq \Delta D_1$, then the contact is point contact and the BSMF calculation continues.

a. Compute $\Delta$ using

$$\Delta = 2(\cosh \eta - 1) - \eta \sinh \eta. \tag{6.79}$$

b. Based on the definition of BSMF,

$$\mathrm{BSMF} = \frac{\left[-\eta\left(\frac{\eta}{2} + a\Delta D \frac{R}{L}\right)(\cosh \eta - 1) + \eta(\sinh \eta - \eta)\right]}{\Delta}. \tag{6.80}$$

c. If the case does not fall into either of the above conditions, then there is no contact and the BSMF is computed using

$$\mathrm{BSMF} = \frac{\eta \cosh \eta}{\sinh \eta}. \tag{6.81}$$

## 6.14.2 BSMF for Compressive Force

1. Compute

$$v = \frac{L^2}{R \times \Delta D} \quad \text{and} \quad u = \frac{F_c L^2}{4EI}, \tag{6.82}$$

where $F_c$ = the compressive force.

The calculations are divided into three cases: $v$ between 1 and 5, $v$ between 5 and 9, and $v$ greater than 9. For the values of $v$ greater than 5, $v$ can be used to calculate the BSMF.

2. If $v$ is between 1 and 5, the BSMF is calculated as follows:

A. Compute $a$ using

$$a = \sqrt{\frac{F_c}{EI}}. \tag{6.83}$$

B. Compute $\eta$ using

$$\eta = \frac{aL}{2}. \tag{6.84}$$

C. Compute $\Delta D_1$ using

$$\Delta D_1 = \frac{\left(-\frac{\eta}{2} + \frac{(1-\cos\eta)}{\sin\eta}\right)L}{aR}. \tag{6.85}$$

D. Compute $\Delta D_2$ using

$$\Delta D_2 = \frac{\left(\frac{2}{\eta}\frac{(\cos\eta + 1) - 2\sin\eta}{(1-\cos\eta)}\right)L}{aR}. \tag{6.86}$$

E. If $\Delta D \leq \Delta D_2$, then the contact is wrap contact and the BSMF calculation is continued as follows:

i. Compute $\xi$ using

$$a^2 \Delta D \frac{R}{2} = 2 + \frac{\left[\frac{\xi^2 (1+\cos\xi)}{2} - 2\xi \sin\xi\right]}{1-\cos\xi}. \tag{6.87}$$

ii. Compute $\gamma$ using

$$\gamma = 2(1-\cos\xi) + \xi \sin\xi. \tag{6.88}$$

iii. Compute moment $M_L$ using

$$M_L = \frac{EI}{R\gamma}\left\{-\left(-\frac{\xi^2}{2} + a^2 \Delta D_2 \frac{R}{2}\right)(1-\cos\xi) - \xi(\xi - \sin\xi)\right\}. \tag{6.89}$$

iv. Compute shearing load $V_L$ using equation 6.90:

$$V_L = \frac{-EIa\xi}{R\Delta}\left\{\left(-\frac{\xi}{2} + a\Delta D_2 \frac{R}{L}\right)(\sin\xi) + 1 - \cos\xi\right\}. \tag{6.90}$$

v. Compute $x_s$ using

$$x_s = \frac{1}{a}\tan^{-1}\left(-\frac{V_L}{aM_L}\right). \tag{6.91}$$

vi. If the condition $0 < ax_s < \xi$ is not satisfied, then compute the stationary value of moment $M_s$ as

$$M_S = \frac{EI}{R}. \tag{6.92}$$

If the condition $0 < ax_s < \xi$ is satisfied, then compute $M_s$ using

$$M_s = M_L \cos(ax_s) - V_L \sin(ax_s). \tag{6.93}$$

vii. Compute BSMF as the absolute maximum of

$$\frac{M_L R}{EI} \quad \text{and} \quad \frac{M_S R}{EI}. \tag{6.94}$$

If $\Delta D \leq \Delta D_1$ then the contact is point contact and the BSMF calculation is continued as follows:

a. Compute $\Delta$ using

$$\Delta = -2(1-\cos\eta)+\eta\sin\eta. \tag{6.95}$$

b. Compute moment $M_L$ using

$$M_L = \frac{-\eta EI}{R\Delta}\left\{\left(-\frac{\eta}{2}+a\Delta D_2\frac{R}{L}\right)(1-\cos\eta)+\eta-\sin\eta\right\}. \tag{6.96}$$

c. Compute shearing load $V_L$ using

$$V_L = \frac{-EIa\eta}{R\Delta}\left\{\left(-\frac{\eta}{2}+a\Delta D_2\frac{R}{L}\right)(\sin\eta)+1-\cos\eta\right\}. \tag{6.97}$$

d. Compute $M_T$ using

$$M_T = \frac{-EI\eta}{R\Delta}\left\{-\left(-\frac{\eta}{2}+a\Delta D_2\frac{R}{L}\right)(1-\cos\eta)+\eta\cos\eta-\sin\eta\right\}. \tag{6.98}$$

e. Compute $x_s$ using

$$x_s = \frac{1}{a}\tan^{-1}\left(-\frac{V_L}{aM_L}\right). \tag{6.99}$$

f. If the condition $0 < ax_s < \eta$ is not satisfied, then compute the stationary value of moment $M_s$ as

$$M_S = \frac{EI}{R}. \tag{6.100}$$

g. If the condition $0 < ax_s < \eta$ is satisfied, then compute $M_s$ as

$$M_s = M_L\cos(ax_s) - V_L\sin(ax_s). \tag{6.101}$$

h. Compute BSMF as the absolute maximum of $M_L R/EI$, $M_T R/EI$, and $M_S R/EI$. If the case does not fall into either of the above conditions then there is no contact and the BSMF is computed using

$$\text{BSMF} = \frac{\eta}{\sin\eta}. \tag{6.102}$$

The main observation to be made here is that equations 6.76 and 6.87 are transcendental equations and require a root-finding subroutine.

**Problem 6.23**

Calculate the bending stress magnification factor for the following conditions:

- Axial force at 13002.46 ft is 73091 lbf (compression).
- Operation: rotating on bottom mode
- Drill pipe details: pipe OD = 5.0 in, and tool joint OD = 7.0 in
- Length between tool joints = 30 ft
- $EI = 4199511$ psi.ft$^2$
- Dogleg severity = 1.28 deg/100ft

*Solution:*

Compute $\Delta D = D_{ij} - D_p = 0.166$ ft.

$$\text{Compute } a = \sqrt{\frac{F_a}{EI}} = \sqrt{\frac{80000}{4199511}} = 0.1380.$$

$$\text{Compute } \eta = \frac{aL}{2} = 0.1380 \times \frac{30}{2} = 2.070.$$

Calculate $v$ and $u$ to find the region:

$$v = \frac{L^2}{R \times \Delta D} = \frac{30^2}{4475 \times 0.166} = 1.065.$$

$$u = \frac{F_c L^2}{4 \times EI} = \frac{-80000 \times 30^2}{4 \times 4199511} = 4.28.$$

The condition of $v$ is between 1 and 5.
Compute $\Delta D_1$ using equation 6.85:

$$\Delta D_1 = \frac{\left(-\frac{\eta}{2}\right) + \left(\frac{(1-\cos\eta)}{\sin\eta}\right)}{a \times \frac{R}{L}}$$

$$= \frac{\left(-\frac{2.070}{2}\right) + \left(\frac{(1-\cos(2.070))}{\sin(2.070)}\right)}{.1380 \times \frac{4475.55}{30}} = 0.0315.$$

Compute $\Delta D_2$ using equation 6.86:

$$\Delta D_2 = \frac{\left(\frac{2}{\eta}\frac{(\cos\eta + 1) - 2\sin\eta}{(1-\cos\eta)}\right)L}{aR}.$$

$$\Delta D_2 = \frac{(2/2.07) + \left(\left(0.5 \times 2.07\left(\cos(2.07)+1\right)\right)/2 - 2\sin(2.07)\right)\left(1/\left(1-\cos(2.07)\right)\right)}{0.1380 \times 4475/30}$$

$= 0.007.$

It is a no-contact condition, and using equation 6.102,

$$\text{BSMF} = \frac{\eta}{\sin\eta} = \frac{2.07}{\sin 2.07} = 2.36.$$

## 6.15 Slip Crushing

Maximum allowable static axial load that can be supported by the slip can be given as

$$F_{max} = \frac{F_y}{\sqrt{1 + \frac{2D_p^2 f A_p}{\left(D_p^2 - D_p^2\right)A_s} + \left[\frac{2D_p^2 f A_p}{\left(D_p^2 - D_i^2\right)A_s}\right]^2}},$$

where $F_y$ = the tensile strength of the pipe, lbf, $A_s$ = the contact area between slips and pipe ($A_c = \pi D_p L_s$), in$^2$, $A_p$ = the cross sectional area of the pipe, in$^2$, $D_p$ = the outside diameter of pipe, in, $D_i$ = the inside diameter of pipe, in, $L_s$ = the length of slips, in, and $f$ = the lateral load factor of slips = $(1-\mu\tan a)/(\mu + \tan a)$, where $\mu$ = the coefficient of friction between slips and bushings, and $a$ = the slip taper angle, deg.

**Problem 6.24**

Calculate the maximum axial load capacity of the slip with the following details:

- Length of the slip = 12"
- Taper angle = 9°27'45"

- Pipe data: OD = 4.5", ID = 3.64", S-135
- Class: premium
- Coefficient of friction = 0.2
- Maximum percentage yield = 90%

*Solution:*

Derating the pipe based on the premium class,

$$D_{cp} = c \times D_p + D_i\left(1 - c\right).$$

New OD = 0.800 × 4.5 + 3.64 (1.0 – 0.800) = 4.328 in.

The cross sectional area of the pipe is

$$A_p = \frac{\pi}{4}\left(4.328^2 - 3.64^2\right) = 4.30 \text{ in}^2.$$

Since it is S-135, the minimum yield strength is 135 ksi. Using 90% yield, maximum tensile strength of the pipe is

Tensile strength (lbf) = pipe cross sectional area (in$^2$) × yield strength (psi).

$$F_y = 135{,}000(4.30) \times 0.9 = 523{,}123 \text{ lbf}.$$

Calculating the contact area of the slip and $f$,

$$A_s = \pi(4.328)(12) = 163.16 \text{ in}^2,$$

$$f = \frac{1 - (0.1)\tan(9°)}{(0.1) + \tan(9°)} = 3.81.$$

Using the equation to calculate the maximum force,

$$F_{max} = \frac{F_y}{\sqrt{1 + \frac{2D_p^2 f A_p}{\left(D_p^2 - D_p^2\right)A_s} + \left[\frac{2D_p^2 f A_p}{\left(D_p^2 - D_i^2\right)A_s}\right]^2}},$$

$$= \frac{523123}{\sqrt{1+\frac{2\times 4.328^2\times 3.81\times 4.30}{\left(4.328^2-3.64^2\right)163.16}+\left[\frac{2\times 4.328^2\times 3.81\times 4.30}{\left(4.328^2-3.64^2\right)163.16}\right]^2}}$$

$= 356050$ lbf.

## 6.16 Cumulative Fatigue Calculation

Drill pipe dog-leg is given by

$$KL=\sqrt{\frac{T}{EI}}L, \tag{6.103}$$

where, $T$ = the tension below the dogleg, lbf, $E$ = modulus of elasticity, psi, and $I$ = the moment of inertia, or

$$I=\frac{\pi(D_o^4-D_i^4)}{64}\ \text{in}^4.$$

$$c_o=\frac{c(KL)}{\tanh(KL)}. \tag{6.104}$$

Cyclic (bending) stress is given by

$$\sigma_{x,c}=\frac{Ec_oD_o}{2}\ \text{psi}, \tag{6.105}$$

where $D_o$ = outside diameter of the pipe.

Medium stress is given by

$$\sigma_{x,m}=\frac{T}{A_c}\text{psi}, \tag{6.106}$$

where

$$A_c=\frac{\pi(D_o^2-D_i^2)}{4}. \tag{6.107}$$

The Soderberg correction factor is given by

$$FC_S = \frac{Y_p}{Y_p - \sigma_{x,m}}, \tag{6.108}$$

where $Y_p$ = yield strength of the pipe, psi, and $\sigma_{x,m}$ = medium stress of pipe, psi.

Cyclic stress corrected by with Soderberg factor is given as

$$S = FC\left(\sigma_{x,c}\right) \text{psi.} \tag{6.109}$$

The number of revolutions of the drill pipe along the drilled interval is given by

$$n = \frac{N \times \Delta D}{\text{ROP}} \text{Rev,} \tag{6.110}$$

where $N$ = RPM, $\Delta D$ = depth drilled, ft, and ROP = the rate of penetration, ft/hr.

The $S$-$N$ (stress $\sigma$ – number of cycles $N$) curve of drill string components generally follows the equation

$$\log S = -m \log N + c, \tag{6.111}$$

where $c$ = constant, and $m$ = the exponent ranging from 0.0333 to 0.33.

Cumulative fatigue is calculated as

$$FC = \frac{n}{N}. \tag{6.112}$$

**Problem 6.25**

Calculate the cumulative of a drill pipe, 4 ½" OD × 3.826" ID, 16.6 lb/ft, grade E-75, range 2, connection NC50, after drilling a 90 feet interval with a dog-leg of 15°/100ft, drill string revolutions of 80 rpm, and a rate penetration of 50ft/hr. Assume the drill pipe is submitted to 70,000 lbf along the interval (considering the buoyancy effect).

*Solution:*

$$A_c = \frac{\pi(D_o^2 - D_i^2)}{4} = \frac{\pi(4.5^2 - 3.826^2)}{4} = 4.41 \text{ in}^2.$$

*Drill pipe cross-section area:*

The moment of inertia is calculated as

$$I = \frac{\pi(D_o^4 - D_i^4)}{64} = \frac{\pi(4.5^4 - 3.826^4)}{64} = 9.61 \text{ in}^4.$$

The drill pipe dog-leg is calculated using equation 6.103:

$$KL = \sqrt{\frac{T}{EI}}L = \sqrt{\frac{70000}{30E6(9.61)}}180 = 2.8048,$$

$$c_o = \frac{c(KL)}{\tanh(KL)} = \frac{\frac{15\,\text{deg}}{100\,\text{ft}} \times 2.8048 \times \frac{\pi}{180} \times \frac{1}{12}}{\tanh(2.8048)} = 6.1641\text{E}-4\text{in}^{-1}.$$

Cyclic (bending) stress is calculated using equation 6.105:

$$\sigma_{x,c} = \frac{Ec_o \times D_o}{2} = \frac{30E6(6.1641E-4)(4.5)}{2} = 41608 \text{ psi}.$$

Medium stress is calculated using equation 6.106:

$$\sigma_{x,m} = \frac{T}{A} = \frac{70000}{4.41} = 15884 \text{ psi}.$$

Using the Soderberg correction factor, it is calculated using equation 6.108:

$$FC_S = \frac{Y_p}{Y_p - \sigma_{x,m}} = \frac{75000}{75000 - 15884} = 1.27.$$

Cyclic stress corrected by with Soderberg factor is

$$S = FC\left(\sigma_{x,c}\right) = 1.27(41608) = 52842 \text{ psi}.$$

Using equation 6.110, the number of revolutions of the drill pipe along the drilled interval is

$$n = \frac{N \times \Delta D}{ROP} = \frac{80x90}{50/60} = 8640 \text{ Rev}.$$

From the *S-N* graph (Figure 6.2) for drill pipe E-75, the equation of the curve *S-N* is $\log S = -0.20422 \log N + 5.718294$.

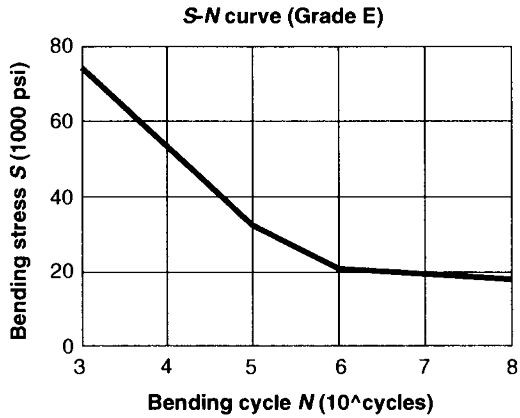



**Figure 6.2** *S-N* graph for Problem 6.25.

Then, for $S = 52842$ psi, the number of revolutions to fail is $N=10^4$ cycles.

Then, the cumulative fatigue is calculated using equation 6.112:

$$FC = \frac{n}{N} = \frac{8640}{10^4} = 0.86 = 86\%.$$

## Problem 6.26

Calculate the cumulative of a drill pipe that is 5″ OD × 4.276″ ID, 19.5 lb/ft, grade S-135, range 2, connection NC50, after drilling an 300 m interval with a dog-leg of 5°/100ft, drill string revolutions of 150 rpm, and a rate penetration of 5 m/hr. Assume the drill pipe is submitted to 200,000 lbf along the interval (considering the buoyancy effect).

*Solution:*

The drill pipe cross-section area is calculated as

$$A = \frac{\pi(D_o^2 - D_i^2)}{4} = \frac{\pi(5^2 - 4.276^2)}{4} = 5.275 \text{ in}^2.$$

The moment of inertia is

$$I = \frac{\pi(D_o^4 - D_i^4)}{64} = \frac{\pi(5^4 - 4.276^4)}{64} = 14.269 \text{in}^4.$$

The drill pipe dog-leg is calculated using equation 6.103:

$$KL = \sqrt{\frac{T}{EI}}L = \sqrt{\frac{200000}{30E6(14.269)}}180 = 3.8907,$$

$$c_o = \frac{c(KL)}{\tanh(KL)} = \frac{\frac{5\,deg}{100ft} \times \frac{\pi}{180} \times \frac{1}{12}}{\tanh(3.8907)} = 2.8769E-4in^{-1}$$

Cyclic (bending) stress is

$$\sigma_{x,c} = \frac{Ec_oD_o}{2} = \frac{30E6(2.8769E-4)(5)}{2} = 21577 \text{ psi.}$$

Medium stress is calculated using equation 6.106:

$$\sigma_{x,m} = \frac{T}{A} = \frac{200000}{5,275} = 37918 \text{ psi.}$$

Using the Soderberg correction factor, it is calculated using equation 6.108:

$$FC_S = \frac{Y_p}{Y_p - \sigma_{x,m}} = \frac{135000}{135000 - 37918} = 1.39.$$

Cyclic stress corrected by with Soderberg factor is

$$S = FC\left(\sigma_{x,c}\right) = 1.39(37918) = 30004 \text{ psi.}$$

Using equation 6.110, the number of revolutions of the drill pipe along the drilled interval is

$$n = \frac{N \times \Delta D}{ROP} = \frac{150(300)}{5/60} = 540000 \text{ Rev.}$$

From the $S$-$N$ graph (Figure 6.3) for the drill pipe S-135, the equation of the curve $S$-$N$ is: $\log S = -0.17\log N + 5.58$.

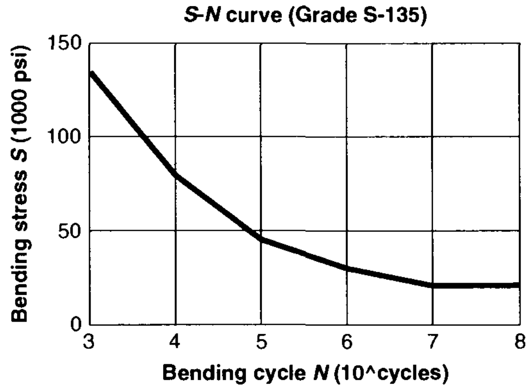



**Figure 6.3** $S$-$N$ graph for Problem 6.26.

Then, for $S = 30004$ psi, the number of revolutions to fail is $N = 3070127$ cycles.

Then, the cumulative fatigue is obtained using equation 6.112:

$$FC = \frac{n}{N} = \frac{540000}{3070127} = 0.1759 = 17.59\%.$$

# 7

# Drilling Tools

This chapter focuses on the different basic calculations using various downhole drilling tools.

## 7.1 Stretch Calculations

Based on the pipe stretch under the applied tension, the approximate depth of the stuck point can be given as

$$L = \frac{E \times e \times W}{144 \times \Delta T \times \rho_s}, \tag{7.1}$$

where $\rho_s$ = the density of steel, lb/in$^3$, $E$ = Young's modulus, psi, $\Delta T$ = the differential pull or hookload, in, $e$ = the measured elongation corresponding to the differential tension (pull), in, and $W$ = air weight of the pipe, lb/ft (ppf).

For steel it can be written as

$$L = \frac{735294 \times e \times W}{\Delta T} \text{ ft}. \tag{7.2}$$

**Problem 7.1**

A driller is planning to estimate the approximate stuck point in a vertical well by conducting the stretch test. The following is the observed data:

- Initial pull = 210 kips
- Final pull = 240 kips
- Hookloads are greater than the weight of the string.
- Stretch of the pipe observed between these hook loads is found to be 19 inches.
- Drill pipe: 5″, 19.5 ppf, IEU

Calculate the estimated depth of the stuck point.

*Solution:*

Using the above equation, the depth of the stuck point is given as

$$L = \frac{735294 \times 19 \times 19.5}{30 \times 1000} = 9{,}080 \text{ ft.}$$

## 7.2 Backoff Calculations

Force at backoff depth = axial force down to well depth – axial force down to backoff depth.

Surface axial force when the workstring is at a measured depth (MD) is

$$\sum \text{MD} \times (\text{Weight Gradient} + \text{Drag Force Gradient}) \quad (7.3)$$

The rotary table torque at surface = torque at backoff depth + backoff torque.

**Problem 7.2**

Estimate the surface action to backoff the string at a depth 9,000 ft with a backoff force and torque of 5 kips and 7,000 ft-lbf, respectively, applied at the backoff point. The string got stuck at 9,000 ft while tripping out. Other well and string details are given below:

- Well depth = 10,000 ft
- Wellbore details: casing with an inside diameter of 10″ set at 6,000 ft and an openhole of 10″ diameter
- Drill string details:

| Section Type | Length (ft) | OD (in) | ID (in) | Weight (ppf) |
|---|---|---|---|---|
| Drill Pipe | 7989 | 5 | 4 | 29 |
| Drill Collar | 540 | 8 | 2.5 | 154.3 |
| Jar | 30 | 8 | 3 | 147 |
| Drill Collar | 1440 | 8 | 2.5 | 154.3 |
| Bit | 1 | 10 | | 95.37 |

- Drill pipe tool joint details: 7.25″ OD × 3.5″ ID
- Mud weight = 10.5 ppg
- Assume a friction factor of 0.3.
- The well is inclined 5° from the surface without any azimuth change.

*Solution:*

*Buoyed weight calculation:*

For the drill pipe, assume the tool joint length is 5% of the string length.

$$A_o = \frac{\pi}{4}\left[0.95\times\left(D_p\right)^2 + 0.05\times\left(D_{jt}\right)^2\right]$$

$$= \frac{\pi}{4}[0.95\times(5)^2 + 0.05\times(7.25)^2]/144 = 0.14387 \text{ ft}^2$$

$$A_i = \frac{\pi}{4}\left[0.95\times\left(ID_p\right)^2 + 0.05\times\left(ID_{jt}\right)^2\right]$$

$$= \frac{\pi}{4}[0.95\times(4)^2 + 0.05\times(3.5)^2]/144 = 0.0862 \text{ ft}^2,$$

$$w_B = w_s + \rho_i A_i - \rho_o A_o$$

$$= 29 + 7.48052\times 10.5(0.0862 - 0.14387) = 24.4737 \text{ ppf}.$$

Similarly, the buoyed weight for the drill collar and jar are 129.56 ppf and 123.4381 ppf, respectively.

*Side force calculation:*

For the drill pipe, the side force of DP against the wellbore = the component of the buoyed weight normal to the wellbore.

$$F_s = w_b \times \sin(\alpha) = 24.4737 \times \sin 5^\circ = 2.133 \text{ ppf.}$$

Similarly, the side force against the drill collar and jar are 11.29 ppf and 10.76 ppf, respectively.

*Drag force calculation:*

For the drill pipe, the drag force gradient = side force × the friction factor (coefficient of friction).

The drag force gradient of DP against the wellbore is

$$F_d = F_s \times \mu = 2.133 \times 0.3 = 0.64 \text{ ppf.}$$

Similarly, the drag force gradient against the drill collar and jar are 3.39 ppf and 3.23 ppf, respectively.

*Axial force at the backoff depth based on the trip out condition:*

Force at the backoff depth = axial force down to the well depth – the axial force down to the backoff depth.

Surface axial force down to the well depth = $\Sigma$MD (Weight Gradient + Drag Force Gradient)

$$= 7989(24.47 + 0.64) + 540(11.29 + 3.39) + 30(10.76 + 3.23) + 1440(11.29 + 3.39) = 230089.9 \text{ lbf.}$$

Measured weight at well depth = 230,0090 + 50,000 = 280,090 lbf = 280 kips.

Axial force down to backoff depth = 7989(24.47 + 0.64) + 540(11.29 + 3.39) + 30(10.76 + 3.23) + 441(11.29 + 3.39) = 215425 lbf.

*Final surface axial force for backoff operation:*

Measured weight at the surface for backing off = axial force at backoff depth + backoff force + hoisting weight:

$$215425 + 5{,}000 + 50{,}000 = 270{,}424/1000 \text{ lbf} = 270 \text{ kips.}$$

Rotary table torque at the surface = torque at backoff depth + backoff torque.

Torque = component side force × friction factor × radius of component.

String torque at backoff depth

$$= 7989\left(2.133 \times 0.3 \times \frac{7.25}{2 \times 12}\right) + 540\left(11.29 \times 0.3 \times \frac{8}{2 \times 12}\right)$$
$$+ 30\left(10.76 \times 0.3 \times \frac{7.25}{2 \times 12}\right) + 441\left(11.29 \times 0.3 \times \frac{8}{2 \times 12}\right) = 2681$$

The surface torque for backoff = 2,681 + 7,000 = 9,681 ft-lbf.

## 7.3 Overpull/Slack-Off Calculations

When the pipe is stuck, if the surface measured weight at the stuck condition is known, then

$$F_{sp} = F_{td} - F_{sd}, \tag{7.4}$$

where $F_{sp}$ = force at stuck point depth, $F_{td}$ = axial force down to well depth, and $F_{sd}$ = axial force down to stuck depth .

**Problem 7.3**

Estimate the surface action to load the stuck point depth of 9,100 ft. Use the data from Problem 7.2.

*Solution:*

*Axial force at the stuck point depth based on the trip out condition:*

Force at stuck point depth = axial force down to well depth – axial force down to backoff depth

The axial force down to well depth is

$$F_{td} = 7989(24.47 + 0.64) + 540(11.29 + 3.39) + 30(10.76 + 3.23)$$
$$+ 1440(11.29 + 3.39) = 230{,}089 \text{ lbf.}$$

The axial force down to the stuck point = 7989(24.47 + 0.64) + 540(11.29 + 3.39) + 30(10.76 + 3.23) + 540(11.29 + 3.39) = 216,878 lbf.

The force at the stuck point depth

$$= 230{,}089 - 216{,}878 = 13{,}212 \text{ lbf} = 13.2 \text{ kips.}$$

Measured weight

$$= 338{,}790 + 50{,}000 = 388{,}790 \text{ lbf} = 389 \text{ kips.}$$

The axial force at the stuck point depth based on the trip in condition is calculated as

$$F_{id} = 7989(24.47 - 0.64) + 540(11.29 - 3.39) + 30\,(10.76 - 3.23) + 1540(11.29 - 3.39) = 207{,}035 \text{ lbf.}$$

The measured weight is

$$207036 + 50{,}000 = 257{,}035 \text{ lbf} = 257 \text{ kips.}$$

For the minimum overpull to load the string at the stuck point depth, the minimum measured weight is calculated as

$$346{,}858 + 119{,}077 + 50{,}000 = 515{,}934 \text{ lbf} = 516 \text{ kips.}$$

For the minimum slackoff to load the string at the stuck point depth, the minimum measured weight is calculated as

$$329{,}115 + 119{,}077 + 50{,}000 = 498{,}192 \text{ lbf} = 498 \text{ kips.}$$

**Problem 7.4**

Estimate the surface action to load the drill string. The stuck point depth is estimated to be 9,100 ft. At the time of the stuck condition, the weight indicator reading was 533 kips. Use other relevant data from Problem 7.2.

*Solution:*

*Axial force at the stuck point depth based on trip out condition:*

Force at stuck point depth = axial force down to well depth – axial force down to stuck depth.

The axial force down to the well depth is calculated as

$$F_{td} = 7989(24.47 + 0.64) + 540(11.29 + 3.39) + 30(10.76 + 3.23) + 1440(11.29 + 3.39) = 230{,}089 \text{ lbf.}$$

The axial force down to the stuck point is calculated as

$$7989(24.47 + 0.64) + 540(11.29 + 3.39) + 30(10.76 + 3.23) + 540(11.29 + 3.39) = 216{,}878 \text{ lbf.}$$

The force at the stuck point depth

$$= 230{,}089 - 216{,}878 = 13{,}212 \text{ lbf} = 13.2 \text{ kips.}$$

Measured weight = 338,790 + 50,000 = 388,790 lbf = 389 kips.

The axial force at the stuck point depth based on trip in condition is calculated as

$$F_{td} = 7989(24.47 - 0.64) + 540(11.29 - 3.39) + 30(10.76 - 3.23) + 1540(11.29 - 3.39) = 207{,}035 \text{ lbf.}$$

The measured weight is

$$207036 + 50{,}000 = 257{,}035 \text{ lbf} = 257 \text{ kips.}$$

Since the measured weight at the time of pipe sticking is the given minimum, overpull to load the string at the stuck point depth has to be calculated using the data provide.

The measured weight of 533 kips when stuck is greater than the calculated measured weight. So, the corrected overpull axial force will be 483 kips (533 kips – 50 kips).

The axial force at stuck point is calculated from the following relationship:

Axial force at stuck point = overpull hookload – axial force at stuck point (trip out condition). Therefore,

$$\text{Axial force at stuck point} = 483{,}000 - 346{,}858 = 136{,}142 \text{ lbf.}$$

The minimum measured weight to overpull for loading the string at the stuck point is

$$346{,}858 + 136{,}142 + 50{,}000 = 533000 \text{ lbf} = 533 \text{ kips.}$$

*Minimum slackoff to load the string at the stuck point:*

The minimum measured weight is

$$= 329{,}115 + 136{,}142 + 50{,}000 = 515{,}257 \text{ lbf} = 515 \text{ kips.}$$

## 7.4 Motor Calculations

### Problem 7.5

A bit is located at point 1, and two stabilizers are placed at points 2 and 3 with a distance of $L_{12}$-ft from the bit and $L_{23}$-ft from the first stabilizer, respectively. There is a bent sub assembly with a $\beta$-deg bend angle. What is the build rate angle for this condition?

*Solution:*

The build rate angle δ is

$$\delta = \frac{200}{L_{12} + L_{23}} \times \beta \text{ deg/100 ft.} \tag{7.5}$$

**Problem 7.6**

Calculate the build up rate angle for the following data:

- Motor length = 25 ft with a bent sub
- Bent angle = 2°
- Bent distance from bit = 6 ft

*Solution:*

The build rate angle is calculated as

$$BRA = \frac{200 \times 2}{25} = 16 \text{ deg}/100 ft.$$

### 7.4.1 Type I Motor

The equivalent motor angle is given as

$$a_{em} = a_{bh} - a_1 + a_2, \tag{7.6}$$

where $a_{bh}$ = the bent housing angle, deg, $a_1$ = the angle adjustment for the first under gauge stabilizer, deg, and $a_2$ = the angle adjustment for the second under gauge stabilizer, deg.

The angle adjustments at the respective stabilizers are given by:

$$\alpha_1 = \frac{360\, r_{c1}}{24\, \pi}\left(\frac{1}{L_{12}} + \frac{1}{L_{23}}\right), \tag{7.7}$$

$$a_2 = \frac{360\, r_{c2}}{24\, \pi}\left(\frac{1}{L_{23}}\right), \tag{7.8}$$

where $r_{c1}$ and $r_{c2}$ are radial clearances between the wellbore and the respective stabilizer blade diameters in inches.

**Problem 7.7**

Calculate the build up rate angle for the following data:

- Motor length = 25 ft
- Motor diameter = 6″
- Bent angle = 2 deg
- Bent housing diameter = 6.5
- Bent distance from bit = 6 ft
- Bit diameter = 8.5″

*Solution:*

Calculating the angle adjustments for points 1 and 2,

$$\alpha_1 = \frac{360 \times 1}{24\,\pi}\left(\frac{1}{6}+\frac{1}{19}\right) = 1.046 \text{ deg},$$

and

$$\alpha_2 = \frac{360 \times 1.25}{24\,\pi}\left(\frac{1}{19}\right) = 0.314 \text{ deg}.$$

The equivalent motor angle is calculated as

$$a_{em} = 2 - 1.046 + 0.314 = 1.267 \text{ deg}.$$

Therefore, the build rate angle is

$$\text{BRA} = \frac{200 \times 1.267}{25} = 10.14 \text{ deg/100 ft}.$$

## 7.4.2 Type II Motor

The equivalent motor angle is given by

$$\alpha_{esb} = \alpha_{bh} \frac{L_{34}}{L_{23} + L_{34}}, \tag{7.9}$$

where $L_{12}$ = the distance from the bit to the first stabilizer, $L_{23}$ = the distance from the first stabilizer to the bent housing, and $L_{34}$ = the distance from the bent housing to the second stabilizer.

The angle adjustments at the respective stabilizers are given by

$$\alpha_1 = \frac{360\, r_{c1}}{24\,\pi}\left(\frac{1}{L_{12}} + \frac{1}{L_{23} + L_{34}}\right), \tag{7.10}$$

and

$$\alpha_2 = \frac{360\, r_{c2}}{24\,\pi}\left(\frac{1}{L_{23} + L_{34}}\right). \tag{7.11}$$

Furthermore, the equivalent motor angle is given by

$$\alpha_{em} = \alpha_{esb} - \alpha_1 + \alpha_2.$$

where $\alpha_{esb}$ is calculated using equation 7.9.

### 7.4.3 Type III Motor

A type III motor configuration consists of a motor of two bends with a bent housing and a sub and a single bent sub placed with two stabilizers contacts at those points. Figure 7.1 presents the simple construct of this configuration.

The equivalent motor angle is given by

$$\alpha_{esb} = \alpha_{bh} + \alpha_{bs}\left(\frac{L_{45}}{L_{34} + L_{45}}\right)\left(\frac{L_{34} + L_{45}}{L_{23} + L_{34} + L_{45}}\right), \tag{7.12}$$

where $\alpha_{esb}$ = the equivalent angle for a single bend, $\alpha_1$ = the angle adjustment for the first under gauge stabilizer, $\alpha_2$ = the angle adjustment for the second under gauge stabilizer, and $\alpha_{bs}$ = the bent sub angle, deg.

The angle adjustments at the respective stabilizers are given by

$$\alpha_1 = \frac{360\, r_{c1}}{24\,\pi}\left(\frac{1}{L_{12}} + \frac{1}{L_{23} + L_{34} + L_{45}}\right), \tag{7.13}$$

and

$$\alpha_2 = \frac{360\, r_{c2}}{24\,\pi}\left(\frac{1}{L_{23} + L_{34} + L_{45}}\right). \tag{7.14}$$

Thus, the equivalent motor angle is given by

$$\alpha_{em} = \alpha_{esb} - \alpha_1 + \alpha_2.$$

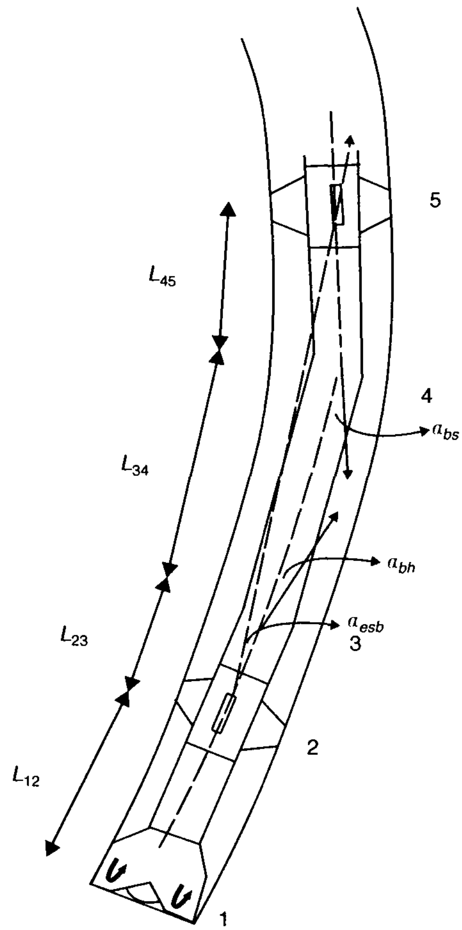



**Figure 7.1** Type 3 motor configuration.

## 7.4.4 Type IV Motor

A type IV motor configuration consists of a motor, three bends with a bent housing, and two bent subs placed between two stabilizers that act as contact points. Figure 7.2 shows a simple example of this configuration.

The equivalent motor angle is given by

$$\alpha_{esb} = \left[\alpha_{bh} + \alpha_{bs}\left(\frac{L_{45}}{L_{34}+L_{45}}\right)\right]\left(\frac{L_{34}+L_{45}}{L_{23}+L_{34}+L_{45}}\right) + \alpha_{tdb}\left(\frac{L_{12}}{L_{12}+L_{23}}\right), \tag{7.15}$$

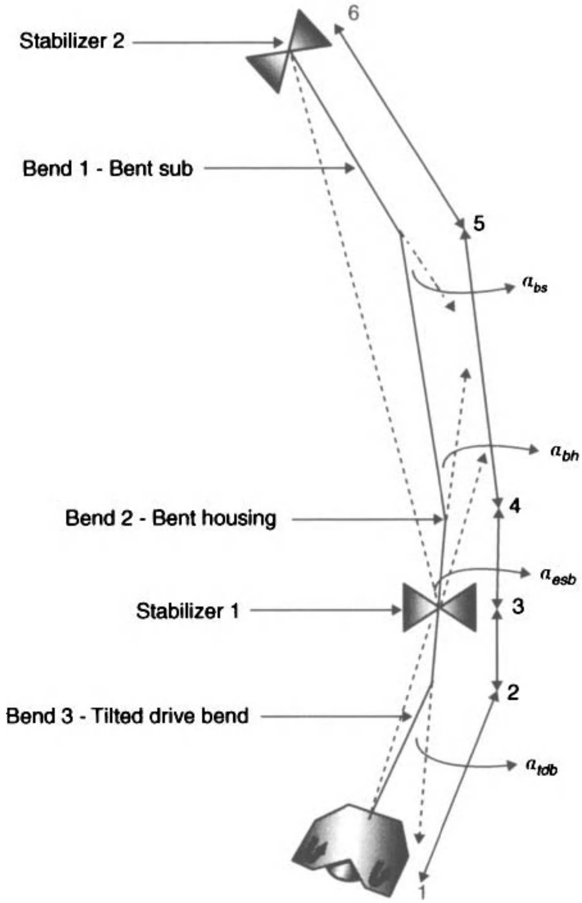



**Figure 7.2** Type IV motor Configuration.

where $\alpha_{tdb}$ = the tilted bushing angle, deg.

The angle adjustments at the respective stabilizers are given by

$$\alpha_1 = \frac{360\, r_{c1}}{24\,\pi}\left(\frac{1}{L_{12}+L_{23}} + \frac{1}{L_{23}+L_{34}+L_{45}}\right), \tag{7.16}$$

and

$$\alpha_2 = \frac{360\, r_{c2}}{24\,\pi}\left(\frac{1}{L_{23}+L_{34}+L_{45}}\right). \tag{7.17}$$

Thus, the equivalent motor angle is given by

$$\alpha_{em} = \alpha_{esh} - \alpha_1 + \alpha_2.$$

## 7.5 Stabilizer Calculations

### 7.5.1 Stabilizer Jamming Angle

A stabilizer jamming angle is the maximum tilt angle that a stabilizer blade OD can accommodate in a particular hole. Figure 7.3 demonstrates the jamming angle position.

The stabilizer jamming angle is given by

$$\alpha_{sj} = \frac{360 \times r_{chs}}{\pi L_{sh}} \text{ deg}, \tag{7.18}$$

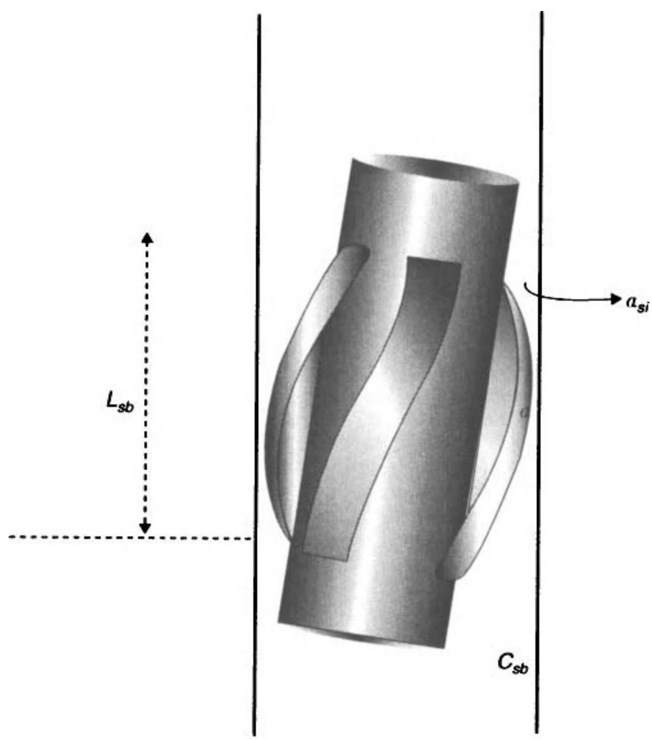



**Figure 7.3** Stabilizer jam.

where $r_{chs}$ = the radial clearance between the stabilizer blade diameter and the wellbore in inches, and $L_{sb}$ = the length of the stabilizer blade in inches.

**Problem 7.8**

Calculate the jamming angle for the following data:

- Hole diameter = 8.5 inch
- Stabilizer blade OD = 8.1 inch
- Stabilizer blade length = 2 ft

*Solution:*

The jamming angle is calculated using equation 7.18:

$$\alpha_{sj} = \frac{360 \times (8.5 - 8.1)}{\pi \times 2 \times 12 \times 2} = 0.95 \text{ deg.}$$

## 7.5.2 Alignment Angle of Stabilizers with the Wellbore

The alignment angle, shown in Figure 7.4, is of the first and second stabilizers for different motor configurations, as discussed above. It can be derived based on simple trigonometric principles and these are given below.

1. Alignment angle of the first stabilizer:

   a. Type I motor configuration is

$$\alpha_{as1} = \left( \frac{\text{BRA} \times A}{200} + \frac{180 c_1}{24 \pi A} \right) \text{deg.} \tag{7.19}$$

   For stabilizer parallel to motor,

$$\alpha_{as1} = \left( \frac{\text{BRA} \times A}{200} + \frac{180 c_1}{24 \pi A} - \alpha_{bh} \right) \text{deg.} \tag{7.20}$$

   b. Type II and II motor configuration is

$$\alpha_{as1} = \left( \frac{\text{BRA} \times A}{200} + \frac{180 c_1}{24 \pi A} \right) \text{deg.} \tag{7.21}$$

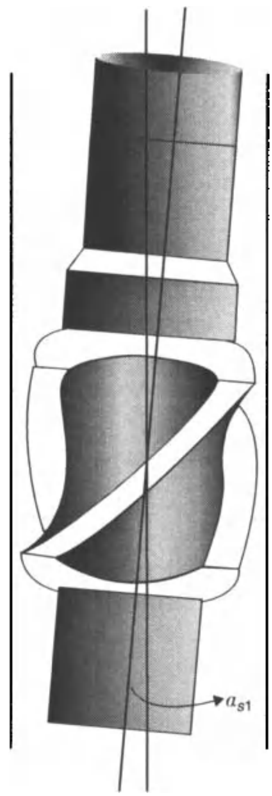



**Figure 7.4** Stabilizer Alignment Angle.

c. Type IV motor configuration is

$$\alpha_{as1} = \left[\frac{\text{BRA} \times L_{12}}{200} + \frac{180c_1}{24\pi L_{12}} - \alpha_{tdb}\left(\frac{A}{A+B}\right)\right]\text{deg}. \tag{7.22}$$

2. Alignment angle of the second stabilizer:

a. Type I motor configuration is

$$\alpha_{as2} = \left(\frac{\text{BRA} \times B}{200} + \frac{180\left(c_{1s} - c_{2s}\right)}{24\pi B}\right)\text{deg}. \tag{7.23}$$

b. Type II motor configuration is

$$\alpha_{as2} = \left( \frac{\text{BRA} \times L_{23}}{200} - \alpha_{bh} \frac{B}{L_{23}} + \frac{180\left(c_{1s} - c_{2s}\right)}{24\pi L_{23}} \right) \text{deg.} \qquad (7.24)$$

c. Type II motor configuration is

$$\alpha_{as2} = \left( \frac{\text{BRA} \times L_{23}}{200} - \alpha_{esb} - \alpha_{bh} - \alpha_{bs} - \frac{180\left(c_{1s} - c_{2s}\right)}{24\pi L_{23}} \right) \text{deg.} \qquad (7.25)$$

d. Type IV motor configuration is

$$\alpha_{as2} = \left( \frac{\text{BRA} \times L_{23}}{200} + \alpha_{esb} - \alpha_{tdb} \frac{A}{L_{12}} - \alpha_{bh} - \alpha_{bs} - \frac{180\left(c_{1s} - c_{2s}\right)}{24\pi L_{23}} \right) \text{deg.} \qquad (7.26)$$

For the above, $\alpha_{as1}$ = the alignment angle of the first stabilizer with hole positive when tilted upward, and $\alpha_{as2}$ = the alignment angle of the second stabilizer with hole positive when tilted upward toward the bit.

**Problem 7.9**

Calculate the deflection rate of a double-bent motor with the given input data:

- Lower bend angle = 1.5°
- Upper bend angle = 1.25°
- Distance of bottom stabilizer to bit segment = 3.51 ft
- Distance of lower bend angle to bottom stabilizer segment = 2.47 ft
- Distance of upper bend angle to lower bend angle segment = 13 ft
- Distance of top stabilizer to upper bend angle segment = 3.66 ft

*Solution (API Units):*

From these input data, $L_1$ = 3.51 ft., $L_2$ = 2.47 ft., $L_3$ = 13 ft, $L_4$ = 3.66 ft.,

$$L_S = 19.13\text{ft}, \; L_T = 22.33 \text{ ft},$$

and

$$\delta = \frac{200}{22.64}(0.871 \times 1.5 + 0.191 \times 1.25) = 13.64\ (°/100\ \text{ft}).$$

Therefore, the deflection rate $\delta = 13.64°/100$ ft.

*Solution (SI Units):*

From these input data, $L_1$ = 1.15 m., $L_2$ = 0.81 m., $L_3$ = 4.27 m., $L_4$ = 1.20 m.,

$$L_S = 6.28\ \text{m},\ L_T = 7.43\ \text{m},$$

and

$$\delta = \frac{60}{7.43}(0.871 \times 1.5 + 0.191 \times 1.25) = 12.48\ (°/30\text{m}).$$

**Problem 7.10**

What is the build motor angle for the following data?

- Hole diameter = 8.5 in
- Motor top and bottom stabilizers diameter = 6.5 in
- Bent housing angle = 2.5 deg
- Offset distance of the bent housing from the bit = 12 ft
- Location of the stabilizer at the bottom of te motor from the bit = 5 ft
- Location of the stabilizer at the top of the motor from the bit = 34 ft

Calculate the build rate angle for this type of build assembly

*Solution:*

Since this is a type motor 2, the equivalent motor angle is given by equation 7.9:

$$\alpha_{esb} = 2.5 \times \frac{22}{7 + 22} = 1.896°.$$

The angle adjustments at the respective stabilizers are given by equation 7.10 and equation 7.11:

$$\alpha_1 = \frac{360 \times 1}{24\pi}\left(\frac{1}{5} + \frac{1}{7+22}\right) = 1.112°,$$

and

$$\alpha_2 = \frac{360 \times 1}{24\pi}\left(\frac{1}{7+22}\right) = 0.165°.$$

The equivalent motor angle is given by

$$\alpha_{em} = \alpha_{esb} - \alpha_1 + \alpha_2 = 1.896 - 1.112 + 0.165 = 0.942°.$$

Build rate angle is calculated using equation 7.5 as =

$$\delta = \frac{200}{34} \times 0.942 = 5.541 \,\text{deg}/100\text{ft}.$$

## 7.6 Percussion Hammer

Work done is given by

$$\frac{\Delta p_m \times A_p \times \ell \times n_b}{396000}\ \text{hp} \tag{7.27}$$

where $\Delta p_m$ is the pressure drop across the piston chamber, psi, $A_p$ = the cross-sectional area of the piston, $in^2$, $l$ is the stroke length, in, and $n_b$ is the number of blows of the piston per min.

**Problem 7.11**

Calculate the work done in HP by a percussion hammer with the following details:

- Pressure drop in the chamber = 400 psi
- Diameter of the pressure changer = 4"
- Stroke length = 1 feet
- Number of blows per minute = 200

*Solution:*

$$\text{Work done} = \frac{\Delta p \times A_p \times \ell \times n_b}{396000}\ \text{hp}.$$

Substituting the values,

$$\text{Work done} = \frac{400 \times \frac{\pi}{4} \times 4^2 \times 12 \times 200}{396000} = 30.46 \text{ hp}.$$

## 7.7 Positive Displacement Motor (PDM)

Mechanical horsepower developed by the motor is given by

$$\text{MHP} = \frac{T}{550}\left(\frac{2\pi}{60}\right)N. \tag{7.28}$$

Hydraulic horsepower for an incompressible fluid is given by

$$\text{HHP} = \frac{Q\Delta P_m}{1714}, \tag{7.29}$$

where $\Delta P_m$ is the pressure drop across the motor in psi, and $Q$ is the flow rate in gal/min.

The flow rate required to rotate the shaft at $N$ rpm for a multi-lobe motor is given by

$$Q = 0.79\frac{i(1+i)}{(2-i)^2}p_h D_h^2 N. \tag{7.30}$$

Hydraulic horsepower for a multi-lobe motor is,

$$\text{HHP} = 2 \times 10^{-6}\,\Delta p_m \frac{i(1+i)}{(2-i)^2}p_h D_h^2 N \tag{7.31}$$

The overall efficiency of the motor is defined by

$$\eta = \frac{\text{usefull power at the bit}}{\text{HHP}}. \tag{7.32}$$

Motor torque and the pressure drop relationship is given by

$$T = 0.01\,\Delta p_m i\left(\frac{1+i}{(2-i)^2}\right)D_h^2 p_h \eta. \tag{7.33}$$

**Problem 7.12**

The following data pertains to a PDM of configuration 7:8. The diameter of the motor is 6.75″. The pitch of the housing is 24″. Pressure drop across the motor is 350 psi. Estimate the torque and speed when the flow rate is 400 gpm. Assume an efficiency of 70%.

*Solution:*

$i$ = winding ratio of the motor = 7/8 = 0.875.

Torque is calculated as

$$T = 0.01 \times 350 \times 0.875 \left( \frac{1.875}{(2-0.875)^2} \right) \times 36 \times 24 \times 0.7 = 2744 \text{ ft-lbf.}$$

Rotational speed of the motor can be calculated using

$$N = \frac{Q}{0.79 \dfrac{i(1+i)}{(2-i)^2} p_h D_h^2}$$

Therefore,

$$N = \frac{400 \times 230.98 \times 1.125^2}{0.79 \times 0.875 \times 1.875 \times 24 \times 36} = 104 \text{ rpm.}$$

**Problem 7.13**

Calculate the stall pressure for the following operating and geometrical conditions of a PDM.

Configuration is 1/2. The diameter of the motor is 8.25″. The speed and torque of the motor are 340 rpm and 1900 ft-lbf, respectively, at a flow rate of 600 gpm. Assume an efficiency of 80%.

*Solution:*

$i$ = winding ratio of the motor = 0.5.

Using the flow rate relationship, the pitch of the motor can be calculated as

$$p_h = \frac{Q}{0.79 \dfrac{i(1+i)}{(2-i)^2} D_h^2 N} = \frac{600 \times 230.98 \times 1.5^2}{0.79 \times 0.5 \times 1.5 \times 64 \times 340} = 24 \text{ in.}$$

Pressure drop across the motor is

$$\Delta p_m = \frac{1900}{0.01 \times 0.5\left(\frac{1.5}{(2-0.5)^2}\right) \times 64 \times 24 \times 0.8} = 460 \text{ psi}.$$

The approximate stall pressure is 810 psi.

**Problem 7.14**

The following data pertains to a motor of 3/4 configuration:

- Diameter of the motor = 8in
- Rotor diameter = 2.7 in
- Pitch = 18 in
- Pressure drop across the motor = 500 psi

Assuming a total efficiency of 80% and a volumetric efficiency of 90%, calculate the following:

A. The torque developed by the motor
B. Rotational speed for a flow rate of 500 gpm
C. Power output of the motor

*Solution:*

The motor torque and pressure drop relationship is

$$T = 0.01 \Delta p_m i \left(\frac{1+i}{(2-i)^2}\right) D_h^2 p_h \eta.$$

Substituting the values,

$$T = 0.01 \times 500 \times 0.75\left(\frac{1+0.75}{(2-0.75)^2}\right) 7.325^2 \times 18 \times 0.8 = 3245 \text{ ft-lbf}.$$

Rotational speed is

$$N = \frac{500 \times 230.98 \times 1.25^2 \times 0.9}{0.79 \times 0.75 \times 1.75 \times 18 \times 7.325^2} = 162 \text{ rpm}.$$

The horsepower developed by the motor is

$$\text{MHP} = \frac{T}{550}\left(\frac{2\pi}{60}\right)N = \frac{3245}{550}\left(\frac{2\pi}{60}\right)\times 162 = 100 \text{ hp.}$$

## 7.8 Rotor Nozzle Sizing

The following simple steps help to size the nozzle effectively:

1. Establish the differential pressure range based on the expected weight on the bit range.
2. Calculate the range of operating flow rates, $Q_{op}$, required for the run.
3. Estimate the minimum flow rate required for hole-cleaning, $Q_{min}$.
4. If the operating flow rate is less than the minimum flow rate for hole-cleaning, calculate the additional flow rate, $Q_{rn}$, that will be bypassed through the rotor nozzle.
5. Size the nozzle using the equation below:

$$A_{nr}^2 = \frac{8.311\times 10^{-5}\times Q_{rn}^2 \times \rho_m}{C_d^2 \times \Delta p_m}, \tag{7.34}$$

   where $C_d$ is the discharge coefficient, $\Delta p_m$ is the pressure drop across the motor in psi, $\rho_m$ is mud density of the circulating fluid in ppg, $Q_{rn}$ is the bypass flow rate through the rotor nozzle in gpm, and $A_{rn}$ is the area of the rotor nozzle in in$^2$.

   The proper nozzle size can be calculated by rearranging the above equation. Rotor nozzle is often expressed in 32$^{nds}$ of an inch. If the rotor nozzle is specified as "14," then the rotor nozzle has a diameter of $^{14}/_{32}$ -in.
6. Check that the diameter of the nozzle is sufficiently smaller than the shaft diameter.

**Problem 7.15**

A well is planned to be drilled with an 8.5-in class 1-1-1 bit, while the torque and rpm expected are 3000 ft-lbf and 300 rpm, respectively.

The mud weight required is 10 ppg. Determine the size of the rotor nozzle for the following conditions:

- Minimum flow rate required for hole-cleaning = 475 gpm
- Configuration: 2/3
- Diameter of the motor = 6.75 in
- Pitch of the housing = 23 in.

*Solution:*

Diameter of the housing is assumed to be 6 in.

Pressure drop expected across the motor power section is

$$\Delta p_m = \frac{3000}{0.01 \times 0.66 \left( \dfrac{1.666}{(2-0.666)^2} \right) \times 23 \times 36 \times 0.7} = 836 \text{ psi.}$$

Operating flow rate required is

$$Q_{op} = \frac{300 \times 0.79 \times 0.666 \times 1.666 \times 23 \times 36}{230.98 \times 1.333^2} = 530 \text{ gpm.}$$

Since this flow rate is higher than the minimum required flow rate of 475 gpm, there is no necessity to fit a rotor nozzle.

**Problem 7.16**

Compute the rotor nozzle size required to drill a 12¼″ hole with a bit torque of 4,000 ft-lbf and 90 rpm. The mud weight required is 10 ppg. The minimum flow rate required for hole-cleaning is 900 gpm.

- Motor configuration: 6:7
- Diameter of the motor = 8 in
- Length of the motor =16.8 ft
- Number of stages = 5.3
- Assume an efficiency of 70%.

*Solution:*

The pitch of the housing is

$$p_h = \frac{\text{length of power section}}{\text{no of stages}} = \frac{16.8 \times 12}{5.3} = 38''.$$

The diameter of the housing is assumed to be 7 in.

Pressure drop expected across the motor power section is

$$\Delta p_m = \frac{4000}{0.01\times 0.857142\left(\dfrac{1.857142}{(2-0.857142)^2}\right)\times 38\times 49\times 0.7} = 252 \text{ psi.}$$

Flow rate required is

$$Q_{op} = \frac{90\times 0.79\times 0.857142\times 1.857142\times 38\times 49}{230.98\times 1.14286^2} = 698 \text{ gpm.}$$

Since the operating flow rate is less than the minimum flow rate, the additional flow rate that needs to be bypassed is

$$Q_{rn} = Q_{\min} - Q_{op} = 900 - 698 = 202 \text{ gpm,}$$

$$\text{Bypassed flow rate} = \frac{202}{698} = \frac{Q_{rn}}{Q_{op}} = R_{bp}.$$

Assuming a discharge coefficient of 0.95, the area of the rotor nozzle can be computed as

$$A_{rn} = \sqrt{\frac{8.311\times 10^{-5}\times 202^2\times 9.5}{0.95^2\times 252}} = 0.37637 \text{ in}^2.$$

The rotor nozzle diameter is

$$d_{rn} = \sqrt{\frac{4\times 0.37637}{\pi}} = 0.692 \text{ in.}$$

The nozzle size is expressed in 32$^{\text{nds}}$ of an inch, and the closest rotor nozzle that can be selected is 22 ($0.69\times 32 \approx 22$).

## 7.9 Downhole Turbine

Torque developed is given by

$$T = 2\pi Q\rho_m \bar{r}^2 n_s N\eta \tag{7.35}$$

where

$$\eta = \frac{\eta_M \eta_H}{\eta_V},$$

$Q$ = flow rate in gpm, $\rho_m$ = mud weight in ppg, $\bar{r}^2$ = square of the mean blade radius in in$^2$, $n_s$ = the number of turbine stages, and $N$ = the rotation speed of the turbine in rpm.

Torque, stall torque, and runaway speed are related by

$$T = T_s\left(1 - \frac{N}{N_r}\right), \tag{7.36}$$

where $T_s$ = the stall torque, ft-lbf, and $N_r$ = the runaway speed in rpm.

The mechanical horsepower developed is given by

$$\mathrm{MHP} = \frac{T}{550}\left(\frac{2\pi}{60}\right)N. \tag{7.37}$$

In terms of runaway speed,

$$\mathrm{MHP} = \frac{T_s\left(N - \dfrac{N^2}{N_r}\right)}{550}\left(\frac{2\pi}{60}\right),$$

or, in terms of torque,

$$\mathrm{MHP} = \frac{N_r\left(T - \dfrac{T^2}{T_s}\right)}{550}\left(\frac{2\pi}{60}\right). \tag{7.38}$$

Stall torque of a turbine can be given by

$$T_s = 8.6595 \times 10^{-5}\,\frac{\tan\beta_e n_s \rho_m Q^2 \eta_m}{2\pi h}, \tag{7.39}$$

where $\beta_e$ = the exit angle in degree, $\eta_m$ = mechanical efficiency, $h$ = height of the vane in inches, $n_s$ = number of stages, $\eta_o = \eta_m \times \eta_{vol}$, $\eta_o$ = overall efficiency, and $\eta_v$ = volumetric efficiency.

Runaway speed of the turbine can be calculated from

$$N_r = 18.38\frac{\tan\beta Q\eta_v}{\pi h\bar{r}^2}. \qquad (7.40)$$

**Problem 7.17**

Calculate the torque developed by a hydraulic turbine with a flow rate of 400 gpm and 10 ppg mud.

- Mean blade radius = 2.05 in.
- Number of stages = 100
- Rotation speed of turbine = 100 rpm
- Overall efficiency = 65%

*Solution:*

Torque developed is calculated as

$$T = 2\pi Q\rho_m\bar{r}^2 n_s N\eta = 2\pi \times 400 \times 10 \times 2.05^2$$
$$\times 100 \times 100 \times 0.65 \text{ ft-lbf}.$$

**Problem 7.18**

Calculate the flow rate required and the torque developed at 500 rpm for a turbine pressure drop of 750 psi. Assume an efficiency of 65%. The rated mechanical horsepower of the turbine is 200 hp.

*Solution:*

Using the mechanical horsepower equation,

$$\text{MHP} = \frac{T}{550}\left(\frac{2\pi}{60}\right)N,$$

the torque developed can be written as

$$HP = \frac{T}{550}\left(\frac{2\pi}{60}\right)N \text{ ft-lbf}.$$

Efficiency is

$$\eta = \frac{200 \times 1714}{Q \times \Delta p_m}.$$

Using 65% efficiency and 750 psi pressure drop, the flow rate can be estimated as

$$Q = \frac{200 \times 1714}{0.65 \times 750} = 703 \text{ gpm}.$$

# 7.10 Jar Calculations

## 7.10.1 Force Calculations for Up Jars

The effective jar set (cock) force is given by

$$F_{es} = -\left(F_s + F_{pof}\right), \tag{7.41}$$

where $F_s$ = the set force, and $F_{pof}$ = the pump open force.

The effective jar trip force is given by

$$F_{et} = \left(F_{ts} - F_{pof}\right), \tag{7.42}$$

where $F_{ts}$ = the trip force.

Induced force is required to set the jar at the center of the jar in compression, and this requires over pulling the measured weight (hoisting/trip out). The set measured weight is given by

$$F_{emw} = \left(F_{ti} - F_s - F_{pof}\right), \tag{7.43}$$

where $F_{tis}$ = the trip in axial force, and $F_s$ = the up jar set force.

Induced force is required to trip the jar at the center of the jar in tension, and this requires slacking off the measured weight (lowering/trip in). The trip measured weight is given by

$$F_{tmw} = \left(F_{to} + F_t - F_{pof}\right), \tag{7.44}$$

where $F_{to}$ = the trip out axial force, and $F_t$ = the up jar trip force.

## 7.10.2 Force Calculations for Down Jars

The effective jar set (cock) force is given by

$$F_{es} = -\left(F_s - F_{pof}\right) \tag{7.45}$$

The effective jar trip force is given by

$$F_{es} = -\left(F_{ts} + F_{pof}\right) \tag{7.46}$$

Induced force is required to set the jar at the center of the jar in tension, and this requires over pulling the measured weight (hoisting/trip out). Set measured weight is given by

$$F_{emw} = \left(F_{to} + F_{s} - F_{pof}\right) \tag{7.47}$$

Induced force is required to trip the jar at the center of the jar in compression, and this requires slacking off the measured weight (lowering/trip in). The trip measured weight is given by

$$F_{tmw} = \left(F_{ti} - F_{t} - F_{pof}\right) \tag{7.48}$$

**Problem 7.19**

Find the jar forces using the following data:

- Well depth = 8,000 ft
- Drill pipe = 7000 ft 5″, 4″ OD, ID = 19.5 lb/ft (ppf), $S = 105$
- Drill collars = 500 ft, 8″ OD, 3″ ID
- Jar = 30 ft, 8″ OD, 3″ ID
- Drill collars = 500 ft, 8″ OD, 3″ ID
- Bit = 12.25″
- Casing = 5000 ft, 13 ⅜″
- Open hole = 12.25″
- Hoisting equipment weight = 30 kips
- Trip in axial force at the center of the jar = 150 kips
- Trip out axial force at the center of the jar = 200 kips
- Pump open force = 3000 lbs
- Type of jar: mechanical
- Up jar set (cock) force = 40 kips
- Down jar set (cock) force = 30 kips
- Measured weight when stuck = 350 kips

*Solution:*

*Up jar conditions:*

Set (cock) measured weight at the surface = 150 − 40 − 3 + 30 = 137 kips.

Change in measured weight = 137 − 350 = − 213 kips.

The trip measured weight at the surface = 200 + 40 – 3 + 30 = 267 kips.

The change in set measured weight = 267 – 137 = – 130 kips.

Reset (re-cock) measured weight at the surface = 137 kips.

The change in reset measured weight from the trip measured weight = 137 – 267 = – 130 kips.

*Down jar conditions:*

Set (cock) measured weight at the surface = 200 +30 – 3 + 30 = 257 kips.

Measured weight = 350 – 200 – 50 = 100 kips.

Since the measured weight is less than the set measured weight, the jar would have already set. For the down jar, for setting, the string has to be tripped out to 257 kips, but the present measured weight in stuck conditions is 350 kips.

The change in measured weight = 350 – 350 = 0 kips.

The trip measured weight at the surface = 150 – 40 – 3 + 30 = 137 kips.

The change in set (cock) measured weight = 137 – 350 = – 213 kips.

Reset (re-cock) measured weight at the surface = 200 + 30 – 3 + 50 = 277 kips.

The change in reset measured weight from the trip measured weight = 277 – 137 = 140 kips.

**Problem 7.20**

A well is being drilled with 10,000-ft drill pipe of 5-in, 19.5 ppf. 270 ft of drill collars are above the bumper sub of 8-in, 147 ppf. Consider regular waves of a length of 275-ft and the vertical movement of drilling vessels to be 3 ft. Assuming the following relationship between the wave length and a wave period, calculate the amplitude at the bumper sub.

$$\lambda = \frac{g}{2\pi} T^2, \qquad (7.49)$$

where $g$ = the gravitational constant.

*Solution:*

The period of vertical motion of the vessel is

$$T_p = \sqrt{\frac{275}{5.122}} = 7.32 \text{ sec.}$$

Using equation 7.49, the drill string resonant period is given by

$$T_r = \frac{4 \times 10000}{16850}\left(\frac{195000 + 39690}{195000}\right) = 2.86 \text{ sec.}$$

The magnitude of the vertical stroke at the bumper sub is

$$h_s = \frac{3}{\cos\left(\frac{2.86}{7.32} \times \frac{\pi}{2}\right)} = 3.6 \text{ ft.}$$

## 7.11 Specific Energy

Mechanical specific energy of the bit is given by

$$E_{sm} = \frac{W}{A} + \frac{120\pi NT}{AR} \text{ psi,} \tag{7.50}$$

where $A$ = the cross-sectional area of the hole drilled, in$^2$, $W$ = the weight on the bit, kips, $T$ = torque, ft-lbf, $N$ = the bit speed, rpm, and $R$ = the rate of penetration, ft/hr.

Generalized specific energy, including hydraulic energy, can be given as

$$E_s = \frac{W_{eff}}{A} + \frac{120\pi NT}{R \times A} + \frac{Q\Delta P_b}{R \times A} \tag{7.51}$$

where $Q$= flow rate, gpm, $\Delta P_b$ = bit pressure drop, psi , and $W_{eff}$ = effective weight on bit, lbf.

Effective weight can be given as

$$W_{eff} = W - \frac{Q}{58}\sqrt{\rho_m \Delta P_b}, \tag{7.52}$$

and bit pressure drop across the bit can be given as

$$\Delta P_b = \frac{8.311 \times 10^{-5} \rho_m Q^2}{C_d^2 A_n^2}, \tag{7.53}$$

where $\rho_m$ = mud weight, ppg, $C_d$ = nozzle discharge coefficient, and $A_n$ = total nozzle area, in$^2$.

Drilling efficiency is given by

$$\eta_d = \frac{E_{S\min}}{E_S},$$

where $E_{Smin}$ is roughly equal to the compressive strength of the formation being drilled.

**Problem 7.21**

Calculate the mechanical specific energy with the following data.

- Diameter of the bit = 8 ½″
- Weight on bit = 10 kips
- Bit rotational speed = 120 rpm
- Total footage drilled is 21 feet in 45 minutes.
- Torque = 2000 ft-lbf

Calculate the drilling efficiency if the formation being drilled has a compressive strength of 25 kpsi.

*Solution:*

The rate of penetration is calculated as

$$R = \frac{21}{45} \times 60 = 28 \text{ ft/hr}.$$

The mechanical specific energy is calculated as

$$E_s = \frac{W}{A} + \frac{120\pi NT}{AR} = \frac{10 \times 1000}{\frac{\pi}{4} \times 8.5^2} + \frac{120\pi \times 120 \times 2000}{\frac{\pi}{4} \times 8.5^2 \times 28}$$

$$= 57121.4 \text{ psi}.$$

The drilling efficiency is calculated as

$$\eta_d = \frac{E_{S\min}}{E_S} = \frac{25000}{57121.4} \times 100 = 44\%.$$

**Problem 7.22**

Minimum specific energy is roughly equal to the compressive strength of the formation drilled and is given as

$$E_{sm} = \left(E_{sm}\right)_{min} = \sigma$$

$$\eta_m = \frac{\left(E_{sm}\right)_{min}}{E_{sm}} \times 100.$$

Assume the torque developed by the core bit is

$$T = \frac{1}{3} \frac{W\mu\left(D_b^3 - d_b^3\right)}{\left(D_b^2 - d_b^2\right)}.$$

Using the above relationships, derive an equation for the rate of penetration for a core bit with an outside diameter, $D_b$, and an inside diameter, $d_b$.

*Solution:*

Mechanical specific energy is given as

$$E_{sm} = \frac{W}{A} + \frac{2\pi NT}{AR}.$$

Using the torque for the core bit, the mechanical specific energy can be written as

$$E_{sm} = \frac{W}{A} + \frac{2\pi N}{AR} \frac{\mu W\left(D_b^3 - d_b^3\right)}{3\left(D_b^2 - d_b^2\right)}.$$

Rearranging the equation to obtain the rate of penetration,

$$R = \frac{8\mu N\left(D_b^3 - d_b^3\right)}{3\left(D_b^2 - d_b^2\right)^2\left[\frac{E_{sm}}{W} - \frac{1}{A}\right]}.$$

Using the conditions given in terms of the compressive strength and mechanical efficiency, the rate of penetration can be obtained as

$$E_{sm} = (E_{sm})_{\min} = \sigma, \eta_{mech} = \frac{(E_{sm})_{\min}}{E_{sm}} \times 100 \text{ ,}$$

$$R = \frac{8\mu N\left(D_b^3 - d_b^3\right)}{3\left(D_b^2 - d_b^2\right)^2 \left[\frac{(E_{sm})_{\min}}{\eta_{mech} W} - \frac{1}{A}\right]}.$$

**Problem 7.23**

With the following data calculate the specific energy using hydraulic energy.

- Diameter of the hole = 12.25 inch
- Bit nozzles: 3 × 12
- Weight on bit = 20 kips
- Bit rotation = 120 RPM
- Friction = 0.2
- Rate of penetration = 20 fph
- Torque = 1333 ft-lbf
- Flow rate = 400 gpm
- Fluid weight = 10 ppg

*Solution:*

Using equation 7.50, the mechanical specific energy is calculated as

$$E_{sm} = \frac{20000}{\frac{\pi}{4} \times 12.25^2} + \frac{2\pi \times 120 \times 1333}{\frac{\pi}{4} \times 12.25^2 \times 20} = 25752 \text{ psi.}$$

Nozzle pressure drop is

$$\Delta P_b = \frac{8.311 \times 10^{-5} 10 \times 400^2}{0.95^2 \left[\frac{3\pi}{4}\left(\frac{12}{32}\right)^2\right]^2} = 1342 \text{ psi.}$$

The effective weight on the bit is

$$W_{eff} = 20000 - \frac{400}{58}\sqrt{10 \times 1342} = 19201 \text{ lbf}.$$

Using equation 7.51, the specific energy can be calculated as

$$E_s = \frac{19021}{\frac{\pi}{4} \times 12.25^2} + \frac{120\pi \times 120 \times 1333}{\frac{\pi}{4} \times 12.25^2 \times 20} + \frac{400 \times 1342}{\frac{\pi}{4} \times 12.25^2 \times 20}$$
$$= 25819 \text{ psi}.$$

*Formulas and Calculations for Drilling Operations*
by G. Robello Samuel


# 8

# Pore Pressure and Fracture Gradient

This chapter focuses on the different basic calculations involved in pore and fracture pressure estimations.

## 8.1 Formation Pressure

### 8.1.1 The Hubert and Willis Method

Matrix stress is calculated as

$$\sigma_z = \sigma_{ob} - p_p \text{ psi}, \tag{8.1}$$

where $\sigma_{ob}$ = overburden pressure, psi, and $p_p$ = pore pressure, psi.

Fracture pressure is

$$p_{ff} = \sigma_h + p_p \text{ psi}, \tag{8.2}$$

and

$$p_{ff} = \sigma_{min} + p_p \text{ psi}, \tag{8.3}$$

where $\sigma_h$ = horizontal stress, psi, and $\sigma_{min}$ = minimum horizontal stress, psi.

Horizontal stress is assumed to be ½ and ⅓ of the overall stress:

$$\sigma_{hmin} = \frac{1}{3}\sigma_z = \frac{1}{3}\left(\sigma_{ob} - p_p\right) \text{psi}, \tag{8.4}$$

$$\sigma_{hmax} = \frac{1}{2}\sigma_z = \frac{1}{2}\left(\sigma_{ob} - p_p\right) \text{psi}, \tag{8.5}$$

Using equation 8.4 and 8.5 fracture pressure can be given as

$$p_{ff} = \frac{1}{3}\left(\sigma_{ob} + 2p_p\right) \text{psi}, \tag{8.6}$$

$$p_{ff} = \frac{1}{2}\left(\sigma_{ob} + p_p\right) \text{psi}, \tag{8.7}$$

In terms of pressure gradient,

$$\frac{p_{ffmin}}{D} = \frac{1}{3}\left(\frac{\sigma_{ob} + 2p_p}{D}\right) \text{psi/ft}, \tag{8.8}$$

$$\frac{p_{ffmax}}{D} = \frac{1}{2}\left(\frac{\sigma_{ob} + p_p}{D}\right) \text{psi/ft}, \tag{8.9}$$

where $D$ = depth, ft.

Assume 1 psi/ft for an overburden stress gradient:

$$\frac{p_{ffmin}}{D} = \frac{1}{3}\left(1 + \frac{2p_p}{D}\right) \text{psi/ft}, \tag{8.10}$$

and

$$\frac{p_{ffmax}}{D} = \frac{1}{2}\left(1 + \frac{p_p}{D}\right) \text{psi/ft}. \tag{8.11}$$

### 8.1.2 Matthews and Kelly's Correlation

The minimum pressure required to create a fracture is at least the formation fluid pressure, and any additional pressure may be related to overcome the formation matrix:

$$\sigma_z = \sigma_{ob} - p_p \text{ psi}, \tag{8.12}$$

$$\sigma_{min} = F_\sigma \sigma_z \text{ psi}, \tag{8.13}$$

$$p_{ff} = F_\sigma \sigma_z + p_p \text{ psi}, \tag{8.14}$$

$$\frac{p_{ff}}{D} = \frac{F_\sigma \sigma_z}{D} + \frac{p_p}{D} \text{ psi/ft}, \tag{8.15}$$

where $F_\sigma$ = the matrix coefficient.

**Problem 8.1**

Calculate the minimum and maximum equivalent mud weight in ppg that can be used immediately below the casing seat at a depth of 10,000 ft for the following conditions:

- Pore pressure gradient = 0.572 psi/ft
- Overburden gradient = 0.85 + 0.015$D$ – 0.0006$D^2$ + 1.2E – 5$D^3$ psi/ft, where depth $D$ is 1,000 ft
- Matrix stress coefficient = 0.712.

Use the Mathews and Kelly method.

*Solution:*

$$\sigma_{ob} = 0.85 + 0.15 \times 10 - 0.006 \times 100 + 1.2E - 05 \times 1000$$
$$= 0.952 \text{ psi/ft}.$$

The overburden pressure at 10,000 ft = 0.952 × 10000 = 9,520 psi. Using the Mathews and Kelly method, the minimum stress is

$$\sigma_{\min} = F_\sigma \sigma_z = 0.712(9520 - 5720) = 2705.6 \text{ psi}.$$

The fracture pressure is

$$p_{ff} = 2705.6 + 5720 = 8425.6\text{psi} = 0.8425 \text{ psi/ft}.$$

The fracture pressure in equivalent mud weight is

$$p_{ff} = 0.8425 / 0.052 = 16.2 \text{ ppg}.$$

Pore pressure is given as $P_p = 0.572$ psi/ft, and in equivalent mud weight

$$P_p = 0.572/0.52 = 11 \text{ ppg}.$$

The mud density below the seat should be greater than the pore pressure and less than the fracture pressure.

**Problem 8.2**

The pore pressure on a North Sea well at 15,000 feet is equivalent to a 14.0 lb/gal mud weight. Compute the fracture gradient using the Hubert and Willis method. Assume the overburden gradient is 1 psi/ft.

*Solution:*

Pore pressure gradient = 14 × 0.052 = 0.728 psi/ft.

Using 1 psi/ft for the overburden stress gradient,

$$\frac{p_{ff\,min}}{D} = \frac{1}{3}(1 + 2 \times 0.728) = 0.81866 \text{ psi/ft}.$$

Expressing in terms of equivalent mud weight, (EMW) = 0.81866/0.052 = 15.74 ppg,

$$\frac{p_{ff\,max}}{D} = \frac{1}{2}(1 + 0.728) = 0.864 \text{ psi/ft}.$$

In equivalent mud weight, 0.864/0.052 = 16.61 ppg.

## 8.1.3 Eaton's Method

The horizontal and vertical stress ratio and the matrix stress coefficient are dependent on the Poisson's ratio of the formation.

$$\sigma_x = \sigma_y = \sigma_h = \frac{\mu}{1-\mu}\sigma_z \text{ psi}, \tag{8.16}$$

$$p_{ff} = \left(\frac{\mu}{1-\mu}\right)\left(\sigma_{ob} - p_p\right) + p_p \text{ psi}, \tag{8.17}$$

$$\frac{p_{ff}}{D} = \left(\frac{\mu}{1-\mu}\right)\left(\frac{\sigma_{ob} - p_p}{D}\right) + \frac{p_p}{D} \text{ psi/ft}, \tag{8.18}$$

where $\mu$ = Possion's ratio.

**Problem 8.3**

The pore pressure on a North Sea well at 15,000 feet is equivalent to a 15.0 lb/gal mud weight. Compute the fracture gradient using the Hubert and Willis method. Assume $\mu$ = 0.49 and $\sigma_{ob}$ = 0.99 at 15,000 ft.

*Solution:*

Pore pressure gradient = 14 × 0.052 = 0.728 psi/ft.
Using the values $\mu$ = 0.49 and $\sigma_{ob}$ = 0.99 at 15,000 ft,

$$\frac{p_{ff}}{D} = \left(\frac{0.49}{1-0.49}\right)(0.99 - 0.728) + 0.728 = 0.9797 \text{ psi/ft.}$$

Expressing in terms of equivalent mud weight, 0.9797/0.052 = 18.84 ppg.

## 8.1.4 Christman's Method

The effect of the water depth in calculating the overburden gradient is accounted for:

$$\frac{p_{ff}}{D} = \left(\frac{1}{D}\right)\left(\rho_w D_w + \rho_b D_f\right) \text{ psi/ft}. \tag{8.19}$$

Assume 1.02 gm/cc for sea water:

$$\frac{p_{ff}}{D} = \left(\frac{1}{D}\right)\left(0.44 D_w + \rho_b D_f\right) \text{ psi/ft.} \tag{8.20}$$

**Problem 8.4**

Select the required mud weight to drill the interval, 10000′ to 14000′. Under the below anticipated conditions, show that there will or will not be problems that can cause possible well blowouts. Use the following well data:

- Intermediate casing setting depth = 10000′
- Drill collar length = 1200′
- Maximum anticipated induced friction pressure losses in annulus during normal drilling: $\Delta p_{fadc}$ = 0.05 psi/ft (behind drill collar), and $\Delta p_{fadp}$ = 0.012 psi/ft (behind drill pipe)
- Maximum anticipated surge and swab pressures during tripping: $\Delta p_{swab}$ = 0.02 psi/ft, and $\Delta p_{surge}$ = 0.02 psi/ft
- Required differential pressure = 400 psi

*Solution:*

From Figure 8.1, the pressure limits at 10000 ft are 13.2 ppg and 17 ppg.

With differential pressure, the required mud

$$\text{weight} = 13.2 + \frac{400}{0.052 \times 8000} = 13.97 \text{ ppg.}$$

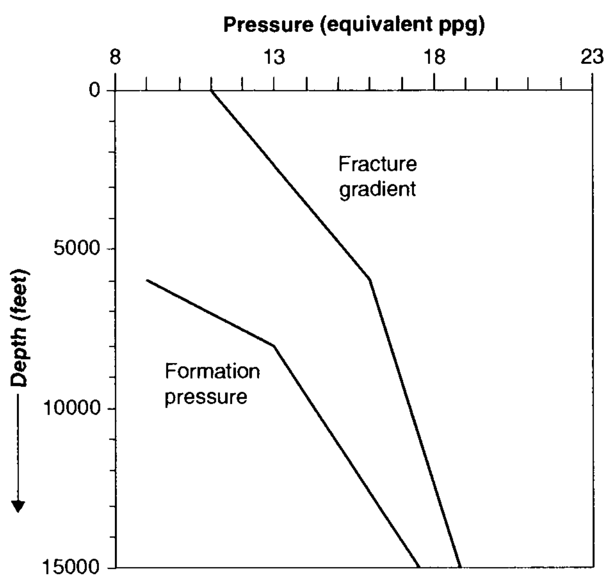



**Figure 8.1** Problem 8.1.

$$\text{ECD} = 13.97 + \frac{8800 \times 0.012 + 1200 \times 0.05}{0.052 \times 10000} = 14.28 \text{ ppg.}$$

Checking for the surge,

$$13.97 + \frac{10000 \times 0.02}{0.052 \times 10000} = 14.66 \text{ ppg.}$$

Checking for the swab,

$$13.97 - \frac{10000 \times 0.02}{0.052 \times 10000} = 13.59 \text{ ppg.}$$

Both limits are honored at 10,000 ft.

Similarly, pressure limits at 14000 ft are 16.5 ppg and 18 ppg (from the figure).

With differential pressure, the required mud

$$\text{weight} = 16.5 + \frac{400}{0.052 \times 14000} = 17.05 \text{ ppg.}$$

$$\text{ECD} = 17.05 + \frac{12800 \times 0.012 + 1200 \times 0.05}{0.052 \times 14000} = 17.34 \text{ ppg.}$$

Checking for the surge,

$$17.05 + \frac{14000 \times 0.02}{0.052 \times 14000} = 17.43 \text{ ppg.}$$

Checking for the swab,

$$17.05 - \frac{14000 \times 0.02}{0.052 \times 14000} = 16.66 \text{ ppg.}$$

Both limits are honored at this depth.

But there will be problems when the same mud weight is used to drill from 10,000 ft to 14,000 ft because it will cause kick at 14,000 ft when 13.97 ppg mud is used, and 17.05 ppg mud will cause fracture when used at 10,000 ft.

**Problem 8.5**

Compute the formation gradient for the following data:

- Air gap = 100 ft
- Water depth = 5,000 ft
- Formation depth = 15,000 ft
- Pore pressure = 7500 ft
- Seawater density = 8.5 ppg
- Average grain density = 2.59 g/cm$^3$
- $\phi_0 = 0.45$
- Porosity decline const = 8.5 x10$^{-5}$ ft$^{-1}$
- Poisson's ratio = 0.45

*Solution:*

$$\sigma_{ob} = \rho_{sw} g D_w + \rho_g g D_s - \frac{(\rho_g - \rho_{fl})\phi_0}{K}\left(1 - e^{-KD_s}\right) = 11438 \text{ psi.}$$

$$\text{Fracture pressure} = \sigma_{min} + p_p = 3222 + 7500 = 10722 \text{ psi.}$$

Gradient = 10722/15100 = 0.71 psi/ft.

**Problem 8.6**

Determine values for surface porosity $\phi_0$ and porosity decline constant $K$ for the Santa Barbara channel. Use the average bulk density data shown in Figure 8.2, an average grain density of 2.60 g/cc, and an average pore fluid density of 1.014 g/cc.

*Solution:*

Calculate porosity $\phi = \dfrac{\rho_g - \rho_b}{\rho_g - \rho_{fl}}$ for various bulk density $\rho_b$.

Use a regression fit to get

$\phi_0 = 0.372$,
$K = 0.000148$ ft$^{-1}$.

**Problem 8.7**

Compute the fracture gradient using:

A. Matthews and Kelly's correlation
B. Eaton's correlation
C. Christman's correlation

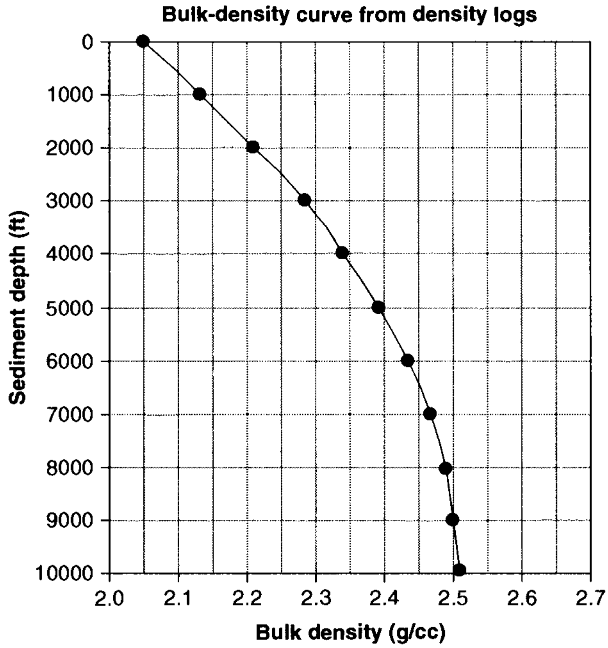



**Figure 8.2** Bulk density plot for Problem 8.6.

Use the effective stress ratio of 0.83 and the corresponding bulk density of 2.35. Use the following data:

- Formation depth = 15,000 ft
- Pore pressure = 9400 psi
- Bulk density = 2.35 g/cm$^{-3}$

Use the porosity date from Table 8.1.

*Solution:*

*A. Matthews and Kelly's correlation*

$$D_i = \frac{D - p_f}{0.535} = 10467 \text{ ft.}$$

The matrix stress coefficient = 0.85.

$$\frac{14160}{15000} = 0.944 \text{ psi/ft.}$$

**Table 8.1** Porosity for Various Depths

| $D_s$(ft) | $\rho_b$(g/cc) | $\phi$ |
|---|---|---|
| 0 | 2.05 | 0.385 |
| 1000 | 2.125 | 0.337 |
| 2000 | 2.21 | 0.284 |
| 3000 | 2.28 | 0.24 |
| 4000 | 2.34 | 0.202 |
| 5000 | 2.395 | 0.167 |
| 6000 | 2.43 | 0.145 |
| 7000 | 2.47 | 0.12 |
| 8000 | 2.49 | 0.107 |
| 9000 | 2.5 | 0.101 |
| 10000 | 2.502 | 0.1 |

*B. Eaton's correlation*

$$\sigma_{min} = \frac{\mu}{1-\mu}\sigma_z,$$

$$= \frac{0.46}{1-0.46}\left(\frac{15269-9400}{15000}\right) + \frac{9400}{15000} = 0.96 \text{ psi/ft.}$$

*C. Christman's correlation*

Using the given effective stress ratio of 0.83 corresponding to the bulk density of 2.35,

$$\sigma_{\min} = F_\sigma \sigma_z = 0.83(15269-9400) = 4871 \text{ psi},$$

$$p_{ff} = \sigma_{\min} + p_p,$$

$$= \left(\frac{4871}{15000}\right) + \frac{9400}{15000} = 0.9514 \text{ psi/ft.}$$

## 8.2 Leak-off Pressure

$$\text{Formation Breakdown Gradient} = \frac{\text{Leak-off Pressure}}{\text{TVD of Shoe}}. \tag{8.21}$$

In other words, the equivalent mud weight to fracture the formation is given as

$$\rho_{ff} = \frac{p_s}{0.052 \times D_v} + \rho_{test} \text{ ppg}, \tag{8.22}$$

where $p_s$ = the surface leak-off pressure during the test, psi, kg/cm, $D_v$ = true vertical depth of the formation tested, ft or m, and $\rho_{test}$ = the mud weight during the test, ppg, SG.

In SI units,

$$\rho_{ff} = \frac{p_s}{0.10 \times D_v} + \rho_{test} \text{ SG}. \tag{8.23}$$

The maximum fracture pressure equivalent mud weight required to drill the next formation interval is given as

$$P_s = (\rho_{ff} - \rho_{test}) 0.052 \times D_v \text{ ppg}. \tag{8.24}$$

**Problem 8.8**

Calculate the mud weight required to break down the formation with the following data taken during a leak-off test.

- Surface pressure during the leak-off = 500 psi
- Mud weight during the test = 10 ppg
- True vertical depth of the formation tested = 5002 ft

*Solution:*

Equivalent leak-off mud weight is

$$\rho_{ff} = \frac{p_s}{0.052 \times D_v} + \rho_{test} = \frac{500}{0.052 \times 5002} + 10 = 11.922 \text{ ppg}.$$

Formation pressure at the leak-off depth is

$$P_{ff} = 0.052 \times 11.922 \times 5002 = 3,101 \text{ psi}.$$

**Problem 8.9**

Calculate the maximum surface pressure to drill 5,000 ft (TVD) given the following information:

- Mud weight during the test = 10 ppg
- Maximum formation pressure equivalent to drill the next interval = 11.5 ppg

*Solution:*

Maximum pressure required during the test is

$$P_s = (\rho_{ff} - \rho_{test})0.052 \times D_v = (11.5 - 10)0.052 \times 5000 = 390 \text{ psi.}$$

**Problem 8.10**

Calculate the formation interval that can be drilled using the following information obtained during the leak-off test.

- Mud weight during the test = 10 ppg
- Surface pressure during the leak-off = 500 psi
- Maximum formation pressure equivalent to drill the next interval = 11 ppg

*Solution:*

Maximum depth that can be drilled with the given formation fracture equivalent is

$$D_v = \frac{P_s}{(\rho_{ff} - \rho_{test})0.052} = \frac{500}{(12 - 10)0.052} = 4807 \text{ ft.}$$

**Problem 8.11**

A well planner calculates the equivalent fracture gradient using the following relationship:

$$\rho_{ff} = F_s\left(\frac{\sigma_{ob}}{0.052} - \rho_f\right) + \rho_f,$$

where the overburden gradient is $\sigma_{ob} = 0.89934e^{0.0000D_v}$ and the matrix stress ratio is

$$F_s = 0.327279 + 6.94721 \times 10^{-5} D_v - 1.98884 \times 10^{-9} D_v^2.$$

Calculate the formation gradient at a depth of 9,875 ft and a water depth of 2,000 ft.

*Solution:*

The matrix stress ratio at 9875 ft is

$$F_s = 0.327279 + 6.94721 \times 10^{-5}(9875 - 2000)$$
$$- 1.98884 \times 10^{-9}(9875 - 2000)^2 = 0.7510.$$

The overburden gradient is

$$\sigma_{ob} = \frac{7875 \times 0.89934 e^{0.0000 \times 9875} + 2000 \times 8.5}{9875} = 16.95 \text{ ppg}.$$

The formation fracture gradient is

$$\rho_{ff} = 0.823(16.95 - 9) + 9 = 14.96 \text{ ppg}.$$

# 9

# Well Control

This chapter focuses on the different basic calculations involved in well control operations.

## 9.1 Kill Mud Weight

Formation pressure is calculated as

$$P_f = 0.052 \times \rho_m \times D_v + P_{sidp} \text{ psi}. \tag{9.1}$$

Kill mud weight is calculated as

$$\rho_{km} = \rho_m + \frac{P_{sidp}}{0.052 \times D_v} + \rho_{ok} \text{ ppg}, \tag{9.2}$$

where $\rho_m$ = original mud weight, ppg, $P_{sidp}$ = shut-in drillpipe pressure, psi, $D_v$ = vertical depth, ft, and $\rho_{ok}$ = overkill safety margin, ppg.

Initial circulating pressure (ICP) is calculated as

$$\mathrm{ICP} = P_{sidp} + P_p + P_o, \tag{9.3}$$

where $P_{sidp}$ = the shut-in drillpipe pressure, psi, $P_p$ = slow circulating pump pressure, psi, and $P_o$ = overkill pressure, psi.

Final circulating pressure (FCP) is calculated as

$$\mathrm{FCP} = P_p\left(\frac{\rho_{km}}{\rho_{om}}\right). \tag{9.4}$$

**Problem 9.1**

Estimate the kill fluid density for a shut-in-drillpipe pressure of 580 psi. Kick depth is 11,937 ft. The original mud density is 14.3 ppg.

*Solution:*

$$\text{Kill fluid density} = 14.3 + \frac{580}{0.052(11937)} = 15.3\,\text{ppg}.$$

$$\text{Kill fluid gradient} = 14.3 \times 0.052 + \frac{580}{(11937)} = 0.7922\,\text{psi/ft}.$$

**Problem 9.2**

Determine the initial circulating pressure for a shut-in-drillpipe pressure of 580 psi. Kick depth is 11,937 ft. The slow circulating pressure is 820 psi. Also, calculate the final circulating pressure if the original mud weight is 14.3 ppg.

*Solution:*

$$\mathrm{ICP} = P_{sidp} + P_p + P_o = 820 + 580 + 0 = 1400\,\text{psi}.$$

$$\text{Kill fluid density} = 14.3 + \frac{580}{0.052(11937)} = 15.3\,\text{ppg}.$$

$$\mathrm{FCP} = P_p\left(\frac{\rho_{km}}{\rho_{om}}\right) = 820\left(\frac{15.3}{14.3}\right) = 877\,\text{psi}.$$

## 9.2 The Length and Density of the Kick

If $V_k < V_{an/dc}$, the length of the kick is calculated as

$$L_k = \frac{V_k}{C_{an/dc}}. \tag{9.5}$$

If $V_k < V_{an/dc}$, the length of the kick is calculated as

$$L_k = L_{dc} + \left(\frac{V_k - V_{an/dc}}{C_{an/dp}}\right), \tag{9.6}$$

where $V_k$ = pit gain, bbl, $C_{an/dc}$ = the annulus capacity behind the drill collar, bbl/ft, $C_{an/dp}$ = the annulus capacity behind the drillpipe, bbl/ft, $V_{an/dc}$ = the annulus volume against the drill collar, bbl, and $L_{dc}$ = the length of the drill collar, ft.

The density of the kick is calculated as

$$\rho_k = \rho_m - \frac{P_{sidp} - P_{sicp}}{0.052 \times L_k}, \tag{9.7}$$

where $P_{sidp}$ = the initial stabilized drillpipe pressure, psi, and $P_{sicp}$ = the initial stabilized casing pressure, psi.

### 9.2.1 Type of Kick

A gas kick is given by

$$\rho_k < 0.25 \text{ psi/ft}. \tag{9.8}$$

An oil and gas mixture is given by

$$0.25 < \rho_k < 0.3 \text{ psi/ft}.$$

An oil or condensate is given by

$$0.3 < \rho_k < 0.4 \text{ psi/ft}. \tag{9.9}$$

A water kick is given by

$$0.4 < \rho_k \text{ psi/ft}.$$

### 9.2.2 Kick Classification

Kick while drilling is classified by the following:

$$\textit{Pore pressure} > \textit{Dynamic bottomhole pressure} > \textit{Static bottomhole pressure.} \tag{9.10}$$

Kick after pump shutdown is classified by the following:

$$\textit{Dynamic bottom hole pressure} > \textit{Pore pressure} > \textit{Static bottomhole pressure.} \tag{9.11}$$

Swab kick is classified by the following:

$$\textit{Dynamic bottomhole pressure} > \textit{Static bottomhole pressure} > \textit{Pore pressure.} \tag{9.12}$$

**Problem 9.3**

A kick was taken when drilling a high-pressure zone of a depth of 9,875 ft with a mud density of 9 ppg. After the well was shut-in, the pressures recorded were SIDP = 300 psi and SICP = 370 psi. The total pit gain observed was 5 bbl. The annular capacity against 900 ft of drill collar is 0.0292 bbl/ft. The overkill safety margin is 0.5 ppg. Compute the formation pressure, kick density, the type of fluid, and the required kill mud weight.

*Solution:*

Formation pressure is calculated as

$$P_f = 0.052 \times \rho_m \times D_v + P_{sidp} = 0.052 \times 9 \times 9875 + 300 = 4921 \text{ psi.}$$

Total annular volume against the drillcollar = 0.0292 × 900 = 26.28 bbl.

Therefore, the volume of the kick is

$$V_k < V_{an/dc}.$$

The length of the kick is calculated as

$$L_k = \frac{V_k}{C_{an/dc}} = \frac{5}{0.0292} = 171.23 \text{ ft.}$$

The density of the kick fluid is calculated as

$$\rho_k = \rho_m + \frac{P_{sidp} - P_{sicp}}{0.052 \times L_k} = 9 + \frac{300 - 370}{0.052 \times 171.23} = 1.13 \text{ ppg}.$$

Therefore, the kick fluid is gas.

Kill mud weight is calculated as

$$\rho_{km} = \rho_m + \frac{P_{sidp}}{0.052 \times D_v} + \rho_{ok} = 9 + \frac{300}{0.052 \times 9875} + 0.5 = 10.08 \text{ ppg}.$$

## 9.2.3 Kick Tolerance

Two conditions exist for kick tolerance.

If $D_v - L_k < D_{cs}$,

$$K_T = (\rho_{ff} - \rho_k)\frac{D_{cs}}{D_v} + \rho_k - \rho \text{ ppg}. \tag{9.13}$$

If $D_v - L_k > D_{cs}$,

$$K_T = (\rho_{ff} - \rho)\frac{D_{cs}}{D_v} + (\rho - \rho_k)\frac{L_k}{D_v} \text{ ppg}, \tag{9.14}$$

where $\rho_{ff}$ = the formation fracture gradient at the shoe, ppg, $D_{cs}$ = vertical depth of the shoe, ft, $\rho_k$ = the density of the kick, ppg, $\rho$ = the density of the wellbore mud at the time of the kick, ppg, and $L_k$ = the length of the kick, ppg.

**Problem 9.4**

Determine the kick tolerance for the following data:

- Well depth = 9,878 ft
- Casing shoe depth = 6,500 ft
- Mud density = 10.1 ppg
- Equivalent fracture mud weight at the shoe = 14.8 ppg
- Volume of the kick = 10 bbl
- Annulus capacity of 1,080 ft of drill collar = 0.0292 bbl/ft
- Kick density = 2 ppg

*Solution:*

The total annular volume against the drillcollar is 0.0292 × 1080 = 31.536 bbl.

Therefore, the volume of the kick is

$$V_k < V_{an/dc}.$$

The length of the kick is calculated as

$$L_k = \frac{V_k}{C_{an/dc}} = \frac{10}{0.0292} = 342.46 \text{ ft.}$$

Calculating $D_v - L_k = 9878 - 342.46 = 9535.53$, which is $> D_{cs} = 6500$ft. Therefore, using the second condition,

$$K_T = \left(\rho_{ff} - \rho\right)\frac{D_{cs}}{D_v} + \left(\rho - \rho_k\right)\frac{L_k}{D_v}$$

$$= (14.8 - 10.1)\frac{6500}{9878} + (10.1 - 2)\frac{342.46}{9878} = 3.37 \text{ ppg.}$$

**Problem 9.5**

A well took a kick while drilling at 9,500 ft with a mud density of 9.8 ppg, in which the casing shoe was at a depth 8,500 ft. Determine the kick tolerance for a kick volume of 25 bbl. The annulus capacity of 900 ft of drill collar is 0.0192 bbl/ft. The annulus capacity of drillpipe is 0.1215 bbl/ft. The equivalent fracture mud weight at the shoe is 15 ppg. The kick density = 2 ppg.

*Solution:*

The total annular volume against the drill collar is 0.0192 × 900 = 17.28 bbl.

Therefore, the volume of the kick is

$$V_k > V_{an/dc}.$$

The length of the kick is calculated as

$$L_k = 900 + \frac{25 - 17.28}{C_{an/dp}} = 900 + \frac{25 - 17.28}{0.1215} = 1143.54 \text{ ft.}$$

Calculating $D_v - L_k = 9500 - 1143.54 = 8356.46$, which is $< D_{cs} =$ 8500 ft.

Therefore, using the first condition,

$$K_T = (\rho_{ff} - \rho_k)\frac{D_{cs}}{D_v} + \rho_k - \rho = (15-2)\frac{8500}{9500} + 2 - 9.8 = 3.83\,\text{ppg}.$$

## 9.3 Hydrostatic Pressure due to the Gas Column

The surface pressure due to the gas column is given as

$$P_g = P_o e^{\frac{0.01877 \times \rho_g \times D}{ZT}}, \tag{9.15}$$

where $P_o$ = the formation pressure of gas, psi, $\rho_g$ = gas density, ppg, $D$ = the gas column, ft, $T$ = temperature, deg K, $Z$ = compressibility factor at temperature T, and hydrostatic pressure = $P_g - P_o$ psi.

**Problem 9.6**

A 10,000 ft well full of gas was closed at the surface. The formation pressure and temperature is estimated to be 6,200 psi and 180°F. Assume a gas gravity of 0.7 and $Z = 0.9$.

*Solution:*

Temperature = 180 + 460 = 640°K.

The surface pressure due to the gas column is given as

$$P_g = P_o e^{\frac{0.01877 \times \rho_g \times D}{ZT}} = 6200 e^{\frac{0.01877 \times 0.7 \times 10000}{0.9 \times 640}} = 7788.57\,\text{psi}.$$

Hydrostatic pressure exerted due to the gas column = $P_g - P_o$ = 7788.57 – 6200 = 1589 psi.

## 9.4 Leak-off Pressure

The formation breakdown pressure is given by

$$P_{ff} = 0.052 \times D_v \times \rho_m + P_{lop}\,\text{psi}, \tag{9.16}$$

where $P_{lop}$ = the surface leak-off pressure, and $D_v$ = true vertical depth, ft.

The equivalent mud gradient is calculated as

$$\frac{P_{ff}}{D_v}\ \text{psi/ft.} \tag{9.17}$$

The formation breakdown gradient is calculated as

$$\frac{P_{ff} - D_s \times \rho_{sw}}{D_v - D_a - D_s}\ \text{psi/ft,} \tag{9.18}$$

where $D_s$ = sea depth, ft, $\rho_{sw}$ = the sea water density, ppg, and $D_a$ = the air gap, ft.

**Problem 9.7**

A well planner calculates the equivalent fracture gradient using the following relationship:

$$\rho_{ff} = F_s\left(\frac{\sigma_{ob}}{0.052} - \rho_f\right) + \rho_f,$$

where the overburden gradient is

$$\sigma_{ob} = 0.89934e^{0.0000D_v},$$

and the matrix stress ratio is

$$F_s = 0.327279 + 6.94721 \times 10^{-5} D_v - 1.98884 \times 10^{-9} D_v^2.$$

Calculate the formation gradient at a depth 9.875 ft.

*Solution:*

The matrix stress ratio at 9,875 ft is calculated as

$$F_s = 0.327279 + 6.94721 \times 10^{-5} 9875 - 1.98884 \times 10^{-9} 9875^2 = 0.823.$$

The overburden gradient is

$$\sigma_{ob} = 0.89934e^{0.0000 \times 9875} = 0.9938\ \text{psi/ft.}$$

The formation fracture gradient is calculated as

$$\rho_{ff} = 0.823\left(\frac{0.9938}{0.052} - 9\right) + 9 = 17.32\ \text{ppg.}$$

**Problem 9.8**

Calculate the mud weight to breakdown the formation with the following data taken during a leak-off test:

- Surface pressure during the leak-off = 500 psi
- Mud weight during the test = 10 ppg
- True vertical depth of the formation tested = 5002 ft

*Solution:*

The equivalent leak-off mud weight is calculated as

$$\rho_{ff} = \frac{p_s}{0.052 \times D_v} + \rho_{test} = \frac{500}{0.052 \times 5002} + 10 = 11.922 \text{ ppg.}$$

The formation pressure at the leak-off depth is calculated as

$$P_{ff} = 0.052 \times 11.922 \times 5002 = 3{,}101 \text{ psi.}$$

## 9.5 Maximum Allowable Annular Surface Pressure (MAASP)

The maximum allowable annular surface pressure (MAASP) (static) is given by

$$P^s_{maasp} = 0.052\left(\rho_f - \rho_m\right)D_{sv} \text{ psi,} \tag{9.19}$$

where $D_{sv}$ = shoe depth (TVD), ft, $\rho_f$ = the equivalent fracture mud weight, ppg, and $\rho_m$ = the wellbore mud weight, ppg.

MAASP (dynamic) is given by

$$P^d_{maasp} = 0.052\left(\rho_f - \rho_m\right)D_{sv} - \Delta_{pa} \text{ psi,} \tag{9.20}$$

where $\Delta_{pa}$ = the annular frictional pressure loss above the shoe, psi.

**Problem 9.9**

A well is being drilled with a mud weight of 14 ppg at a depth of 15,000 ft (TVD) has a shoe at 13,100 ft (TVD). The fracture gradient at the shoe is 0.85 psi/ft. Calculate the maximum allowable annular surface pressure.

*Solution:*

This is a static condition.

The equivalent fracture mud weight is calculated as

$$\rho_f = \frac{0.85}{0.52} = 16.35\,\text{ppg}.$$

MAASP (static) is calculated as

$$P^s_{maasp} = 0.052(16.35 - 14)13100 = 1598\,\text{psi}.$$

## 9.6 Accumulators

A typical charging sequence of an accumulator is shown in Figure 9.1.

The volume delivered by the accumulator bottles is given by

$$V_d = \left( \frac{P_{pc}}{P_f} - \frac{P_{pc}}{P_s} \right) V_b, \tag{9.21}$$

where $P_{pc}$ = the nitrogen pre-charge pressure, psi, $P_f$ = the final operating pressure of the bottle, psi, $P_s$ = the accumulator system pressure, psi, and $V_b$ = the actual bottle capacity, gallons.

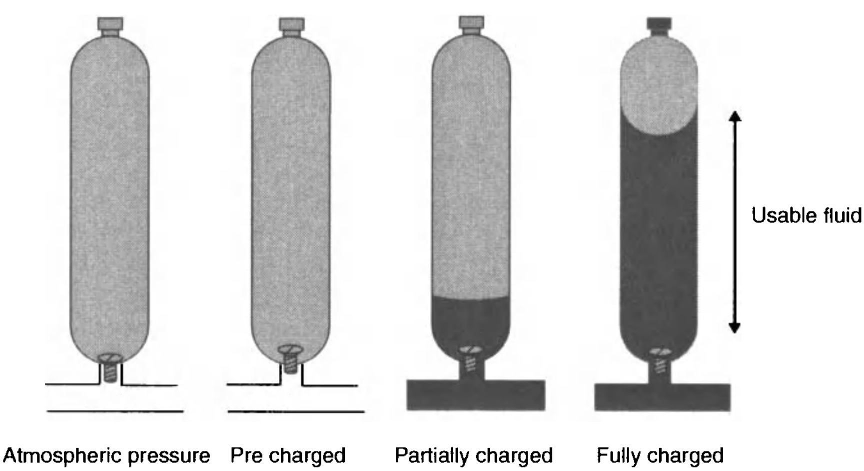



**Figure 9.1** Accumulator setup.

For sub-sea, the volume delivered is given by

$$V_d = V_d\left(\frac{460+T_2}{460+T_1}\right), \tag{9.22}$$

where $T_1$ = the ambient temperature, deg F, and $T_2$ = temperature at water depth, deg F.

$$\text{Pre-charge pressure} = P_{ipc} + L \times \text{SG} \times 0.433\,\text{psi} \tag{9.23}$$

where $P_{ipc}$ = the initial pre-charge pressure, psi, SG = the specific gravity of the fluid, and $L$ = the length of the control line, psi.

Volume delivered by the bottle is given by

$$V_d = \left(\frac{\dfrac{P_{pcc}}{Z_{pcc}}}{\dfrac{P_{fc}}{Z_{fc}}} - \frac{\dfrac{P_{pcc}}{Z_{pcc}}}{\dfrac{P_{sc}}{Z_{sc}}}\right)V_b, \tag{9.24}$$

where the corrected pre-charge pressure is

$$P_{pcc} = 0.445 \times D_w + 14.7 + P_{pc}\ \text{psi}, \tag{9.25}$$

and $Z_{pcc}$ = the compressibility factor at pressure $P_{pcc}$.

The corrected operating pressure is

$$P_{fc} = 0.445 \times D_w + 14.7 + P_f\ \text{psi}, \tag{9.26}$$

and $Z_{fc}$ = the compressibility factor at pressure $P_{fc}$.

The corrected accumulator system pressure is

$$P_{sc} = 0.445 \times D_w + 14.7 + P_s\ \text{psi}, \tag{9.27}$$

and $Z_{sc}$ = the compressibility factor at pressure $P_{sc}$.

**Problem 9.10**

Compute the total amount of fluid delivered from 8, 20-gallon bottles with a system pressure of 3,000 psi. The precharge pressure is 1100 psi, and the final pressure is 1250 psi.

*Solution:*

Volume delivered is calculated as

$$V_d = \left(\frac{P_{pc}}{P_f} - \frac{P_{pc}}{P_s}\right) V_b = 8 \times \left(\frac{1100}{1250} - \frac{1000}{3000}\right) \times 20 = 82.13 \text{gallons.}$$

**Problem 9.11**

Compute the total amount of fluid delivered from 8, 20-gallon bottles with the system pressure of 3,000 psi. The precharge pressure is 1100 psi, and the final pressure is 1250 psi. Water depth = 5,000 ft. Assume Z is 1.2 at 1100 psi, is 1.21 at 1250 psi, and is 1.3 at 3000 psi.

*Solution:*

The corrected pre-charge pressure is

$$P_{pcc} = 0.445 \times 5000 + 14.7 + 1100 = 3339.7 \text{ psi.}$$

The corrected operating pressure is

$$P_{fc} = 0.445 \times 5000 + 14.7 + 3000 = 5239.7 \text{ psi.}$$

The corrected accumulator system pressure is

$$P_{sc} = 0.445 \times 5000 + 14.7 + 1250 = 3489.7 \text{ psi.}$$

Using the compressibility factors and the calculated corrected pressures,

$$V_d = \left(\frac{\frac{3339.7}{1.2}}{\frac{5239.7}{1.3}} - \frac{\frac{3339.7}{1.2}}{\frac{3489.7}{1.21}}\right) \times 20 \times 8 = 43.92 \text{gallons.}$$

## 9.7 Driller's Method Operational Procedure

Assuming a gas kick, during the first circulation the kick fluid is circulated out of annular space using the original light mud without allowing any further kick fluid intrusion by keeping the circulating drill pipe pressure at $P_{cdpi} = P_{pr} + P_{sidp}$. Afterwards, close the choke and read $P_{sidp}$ and $P_{sic}$. They should be equal. If not, continue circulating until all gas is removed from the well. During the second circulation, a volume of the heavy kill mud that is equal to the total drillstring volume capacity is circulated while keeping the choke pressure at $P_{sidp}$. The circulating drill pipe pressure should drop from $P_{cdpi}$ at the beginning of the circulation to $P_{cdpf}$ when the new mud reaches the bit. Circulation of heavy mud is continued until all original mud in the annulus is displaced. Afterwards, close the choke and read $P_{sidp}$ and $P_{sic}$. They should be equal to 0 psi. If so, open the well and resume the drilling operations. If not, continue the well control operation.

$$V_m = V_{dp} + V_{dc} + V_{adp} + V_{adc}, \tag{9.28}$$

or in terms of the required number of strokes,

$$\overline{TNPS} = \frac{V_m}{F_p}. \tag{9.29}$$

where $F_p$ = pump factor, gal/stroke.

**Problem 9.12**

A gas kick was taken while drilling at 10,000 ft (TVD) with a mud weight of 10 ppg. The pit gain was recorded at 15 bbls. The shut-in drill pipe and casing pressures were recorded at 400 psi and 590 psi, respectively. In addition, the following data is given:

- $C_{adp}$ = 0.0459 bbl/ft,
- $C_{adc}$ = 0.0292 bbl/ft
- $C_{dp}$ = 0.0178bbl/ft
- $C_{dc}$ = 0.0061bbl/ft
- Pump factor = $F_p$ = 2.6 gal/stroke
- Pump volumetric efficiency = 95%
- $P_{pr}$ = 1500 psi at 60 strokes/min = pump pressure at reduced flow rate
- Casing shoe depth = 6000 ft
- $L_{dc}$ = drill collar length = 1000 ft
- $k = 1$

Determine the following:

A. The pressures at the casing shoe and surface choke during the first circulation with original mud.

B. The pump pressure schedule during the second circulation with the kill mud to replace the original mud.

*Solution:*

Formation fluid pressure is calculated as

$$P_{ff} = k\rho_{mo}\mathrm{D} + P_{sidp}$$
$$= 0.052(10)1000 + 4000 = 5600 \text{ psi.}$$

The required kill mud weight is calculated as

$$\rho_{mo} + \frac{P_{sidp}}{kD} + \text{safety},$$
$$= 10 + \frac{400}{0.052(10000)} + \text{safety},$$
$$= 10.77 + \text{safety} \approx 11 \,\text{ppg}.$$

The pressure, $P_i$, at any depth, $D_i$ in the annulus is

$$P_i = \left(b + \sqrt{b^2 + 4c}\right)/2,$$

$$b = P_{ff} - 0.052\rho_{mo}(D - D_i),$$

$$c = 0.052\rho_{mo}kP_{ff}\frac{V_k}{C_{adp}},$$

$$D_i = D_{cs} = 6000 \text{ (casing shoe)} \rightarrow P_{cs}.$$

$$D_i = 0 \text{ (surface choke)} \rightarrow P_{sc}.$$

At the casing shoe,

$$b = \frac{5600 - 0.052\,(10)\,(10000 - 6000)}{3520},$$

$$c = 0.052\,(10)\,(1)\,5600\frac{15}{0.046} = 949565.$$

At surface choke:

$$b = 5600 - 0.052(10)(10000) = 400,$$

$$c = 949565.$$

Therefore,

$$P_{cs} = \left(3250 + \sqrt{3520^2 + 4(949565)}\right)/2$$
$$= 3772 \text{ psi}$$
$$= 1900 - 0.0923\overline{\text{NPS}}.$$

where $\overline{\text{NPS}} = 0.2710.$

## 9.8 Kill Methods

The Driller's Method has two circulations:

1. The kick is circulated out using the original mud weight, and
2. The kill mud is circulated.

The Wait and Weight Method (Engineer's Method) has one circulation:

1. The kill mud is circulated to circulate out the kick.

In the Concurrent Method, the original mud is weighted incrementally in stages to the kill mud weight while the kick is circulated out:

$$\rho_{m1} = \rho_{mo} + \Delta\rho_m, \tag{9.30}$$

$$\rho_{m2} = \rho_{m1} + \Delta\rho_m, \tag{9.31}$$

$$\rho_{mi} = \rho_{m(i-1)} + n\Delta\rho_m \left(i = 1, n\right). \tag{9.32}$$

In the Volumetric Method, the well kill is achieved by pumping calculated volume of kill fluid into the wellbore several times and bleeding the excess pressure every time.

In the Bullheading Method, the well kill is achieved through pumping kill fluid into the well so that the influx is pumped backed into the reservoir.

## 9.9 The Riser Margin

The riser margin is given by

$$\rho_{rm} = \frac{\rho_{emw} D_v - \rho_w D_w}{D_v - D_w - D_a}, \tag{9.33}$$

where $\rho_{emw}$ = the drilling fluid gradient to control the formation pressure with the riser, psi/ft, $D_v$ = the depth of the hole (TVD), ft, $\rho_w$ = the seawater gradient, psi/ft, $D_w$ = the water depth, ft, and $D_a$ = the air gap, ft.

**Problem 9.13**

Calculate the riser margin for the given data:

- Water depth = 3575 ft
- Air gap = 75 ft
- Depth of the hole = 15,554 ft (TVD)
- The formation gradient at the depth of hole is 0.78 psi/ft.

*Solution:*

The riser margin is

$$\rho_{rm} = \frac{\rho_{emw} D_v - \rho_w D_w}{D_v - D_w - D_a} = \frac{0.78 \times 15554 - 0.434 \times 3575}{15554 - 3575 - 75} = 0.89 \,\text{psi/ft}.$$

The safety riser margin = 0.89 – 0.78 = 0.11 psi/ft.

# 10

# Drilling Problems

This chapter focuses on the different basic calculations involved in the drilling problems such pipe sticking, keyseating, etc.

## 10.1 Stuck Point Calculations

Pipe stretch or contraction can be given as

$$\Delta L_t = \frac{12EL_p}{A_p \times F}, \tag{10.1}$$

where $\Delta L_t$ = total axial stretch or contraction, or the distance between two reference points, inches, $F$ = the tension or compression force applied, or difference in hook load lbf, $L_p$ = the length of the pipe, feet, $E$ = the modulus of elasticity, psi (for steel $E = 30 \times 10^6$ psi), and $A_p$ = the cross-section metal area of the pipe, sq. inches, or $0.7854 \times (D^2 - d^2)$.

The length of the free pipe can be given as

$$L_p = \frac{E\Delta L_t A_p}{12 \times F}. \tag{10.2}$$

For steel pipes,

$$L_p = \frac{E\Delta L_t w}{40.8F}, \tag{10.3}$$

where $w$ = the unit weight of the pipe (ppf).

$$L_p = \frac{735294\Delta L_t w}{F}. \tag{10.4}$$

**Problem 10.1**

Estimate the measured depth of the stuck point of a 4″ API steel, class 1 pipe in a 10,000 ft vertical well for the following data:

- Initial hookload = 160 kip
- Final hookload = 170 kip
- Measured increase in the stretch = 8.0 inches

*Solution:*

Since it is a class 1 pipe, it has 100% wall thickness.
The cross-section area of the drill pipe is calculated as

$$A_p = 0.7854\left(4^2 - 3.24^2\right) = 4.32 \text{ sq. in.}$$

The length of free pipe is

$$L_p = \frac{E\Delta L_t A_p}{12 \times F} = \frac{30{,}000{,}000(\text{psi}) \times 8(\text{in}) \times 4.32(\text{sq. in})}{12 \times 10{,}000(\text{lbf})} = 8{,}643 \text{ ft.}$$

**Problem 10.2**

Estimate the measured depth of the stuck point of a 4″ API steel, premium service class pipe in a 10,000 ft vertical well for the following data:

- Initial hookload = 160 kip
- Final hookload = 170 kip
- Measured increase in the stretch = 8.0 inches

*Solution:*

In order to calculate the actual pipe diameter, the pipe class and corresponding wall thickness should be used. Since it is a premium class pipe, it has 80% wall thickness (refer to Chapter 6, equation 6.4).

The new outside diameter of the pipe is calculated using equation 6.4

$$D_p = c \times D_p + D_i(1-c) = 0.8 \times 4 + 3.24(1-0.8) = 3.85 \text{ in.}$$

The cross-section area of the drill pipe is

$$A_p = 0.7854\left(3.85^2 - 3.24^2\right) = 3.38 \text{ sq. in.}$$

The length of the free pipe is

$$L_p = \frac{E \Delta L_t A_p}{12 \times F} = \frac{30{,}000{,}000\,(\text{psi}) \times 8\,(\text{in}) \times 3.38\,(\text{sq. in})}{12 \times 10{,}000\,(\text{lbf})} = 6{,}769 \text{ ft.}$$

**Problem 10.3**

While drilling a vertical well, the string got stuck at 10,000 ft. The drilling consists of a 4″, API steel, class 1 pipe. The tool joint dimensions of the pipe are 5.5″ × 2.813″. A stretch test was performed to find out the depth of the stuck point.

- Initial hookload = 160 kip
- Final hookload = 170 kip
- Measured increase in the stretch = 8.0 inches

*Solution:*

Taking the tool joint length to be 5% of the total length of the pipe length, the area ratio can be given as

$$A_p = \frac{A_p \times A_{jnt}}{0.95 A_{jnt} + 0.05 A_p} = \frac{4.32 \times 17.54}{0.95 \times 17.54 + 0.05 \times 4.32} = 4.49 \text{ sq. in}$$

The length of the free pipe is

$$L_p = \frac{E \Delta L_t A_p}{12 \times F} = \frac{30{,}000{,}000\,(\text{psi}) \times 8\,(\text{in}) \times 4.49\,(\text{sq. in})}{12 \times 10{,}000\,(\text{lbf})} = 8{,}981 \text{ ft.}$$

**Problem 10.4**

While drilling at 11,000 ft, the drill string becomes stuck. The drill string consists of 4½ inch drill pipe, 7 inch collars (900 ft), and a 9⅞ inch bit. The pipe stretch data obtained was a 36 in stretch when 45,000 lbs of pull (above the hook load) was made. If an excess annular volume factor of 50% is used, how much spotting fluid must be used to cover the stuck interval and all sections below?

*Solution:*

The pipe stretch formula is

$$L = \frac{K(30 \times 10^6)E}{P},$$

where $K$ = the pipe design constant (from Table 10.1), $E$ = the pipe stretch in inches, and $P$ = the pull on the pipe, lbf.

Therefore,

$$L = \frac{(10,8000,000)(36)}{45,500} = 8640\,\text{ft}\,(\text{stuck interval}).$$

*Spotting fluid:*

Collars = .471 bbl/ft (7 in. × 9⅞ in) × 900 ft = 42 bbl.
Drill pipe = .0751 bbl/ft (4½ in × 9⅞ in) × (11,000 − 900 − 8640) = 109 bbl.
Total annular capacity = 42 + 109 = 151 bbl + 76 bbl (50% excess) = 227 bbl.

**Table 10.1** Constants used to calculate free point.

| Tubing | K value |
|---|---|
| 2 in | 3,250,000 |
| 2.5 in | 4,500,000 |
| Drill Pipe | |
| 2.87 in (10.4 lb/ft) | 7,000,000 |
| 3.5 in (13.3 lb/ft) | 8,800,000 |
| 4.5 in (16.6 lb/ft) | 10,800,000 |

A simpler approximation of equation 10.5 is

$$L = 735,294 \frac{\Delta L W_p}{\Delta F}, \tag{10.5}$$

where $W_p$ = the unit weight of the pipe body, lbf/ft, or 2.67 × ((pipe OD, inches)$^2$ – (pipe ID, inches)$^2$), and $\Delta F$ = the differential tension pull applied to the string, lbf.

**Problem 10.5**

Estimate the free point of a 10,000 ft string of 5 inch, 19.50 lbf/ft drill pipe having an inside diameter of 4.6 inches. The drilling mud weight is 10.0 ppg mud. Overpull applied during the stretch test is 35 kips, and the measured stretch is 23 inches.

*Solution:*

The weight-per-foot of the pipe body is

$$W_p = 2.67(4.5^2 - 4.6^2) = 10.25 \text{ lbf/ft}.$$

From equation 10.5, the approximate free point is

$$L = 735,294 \frac{23 \times 10.25}{35,000} = 4954 \text{ ft}.$$

## 10.2 Differential Sticking Force

The formula for the approximate calculation of the differential sticking force due to fluid filtration is

$$F_{pull} = \mu \Delta P A_c, \tag{10.6}$$

where $\Delta P$ = the differential pressure, psi, $\Delta P = P_m - P_f$, $A_c$ = the contact surface area, in$^2$, $P_m$ = the pressure due to drilling fluid, psi, and $P_f$ = the formation pressure or pore pressure, psi.

Mud pressure is given by

$$P_m = 0.052 \times \rho_m \times D_v \text{ psi},$$

where $\rho_m$ = mud weight, ppg, and $D_v$ = the vertical depth of the calculation or stuck depth, ft.

### 10.2.1 Method 1

The contact area is given by

$$A_c = 2 \times 12 \times L_p \left\{ \left( \frac{D_h}{2} - t_{mc} \right)^2 - \left[ \frac{D_h}{2} - t_{mc} \frac{(D_h - t_{mc})}{(D_h - D_p)} \right]^2 \right\}^{\frac{1}{2}} \text{ in}^2 \tag{10.7}$$

where $D_h$ = the hole diameter, in, $D_p$ = the outer pipe diameter, in, $t_{mc}$ = the mud cake thickness, in, $L_p$ = the embedded pipe length, ft.

$D_{op}$ must be equal to or greater than $2t_{mc}$ and equal to or less than $(D_h - t_{mc})$.

### 10.2.2 Method 2

The contact area is given by

$$A_c = L_p \left( \frac{D_p D_h}{D_h - D_p} \right)^{\frac{1}{2}} \left\{ \left( \frac{1-x}{x} \right) \left( \frac{Q_f t_{mc}}{A_f} \right) \left( \frac{\Delta p}{\Delta p_f} \right) \left( \frac{\mu_f}{\mu_{df}} \right) \left( \frac{t}{t_f} \right) \right\}^{\frac{1}{4}}, \tag{10.8}$$

where $Q_f$ = the measured filtrate volume, $A_f$ = the filtration area, $\Delta p_f$ = the filtration test differential pressure, $\mu_f$ = the filtrate viscosity at test condition, $\mu_{df}$ = the filtrate viscosity at downhole conditions, $x$ = the contact ratio, $t$ = the total test time, and $t_f$ = the total stuck time.

**Problem 10.6**

Determine the pullout force to free the drill string given the following well data:

- Drill collar OD = 6.0 in
- Hole size = 9.0 in
- Mud cake thickness = 1/16 in
- Coefficient of friction = 0.15
- Length of embedded drill collar = 20 ft
- Differential pressure = 500 psi

*Solution:*

$$\mu = 0.15.$$

$$\Delta P = 500 \text{ psi.}$$

$$L_p = 20 \text{ ft.}$$

Using equation 10.7,

$$A_c = 2 \times 20 \times 12\left[\left(\frac{9}{2} - 0.0625\right)^2 - \left(\frac{9}{2} - 0.0625\frac{9 - 0.0625}{9 - 6}\right)^2\right]^{\frac{1}{2}},$$

$$= 500.4 \text{ sq. in.}$$

Using equation 10.6, pull out force is given by

$$F_{pull} = \mu \Delta P A_c,$$

$$F_{pull} = 0.15 \times 2.085 \times 500 = 37{,}530 \text{ lbf.}$$

**Problem 10.7**

Calculate the allowable mud weight at 5,000 ft for the following data:

- Cake thickness = 1/8″
- Diameter of the hole = 12 ¼″
- Pipe diameter = 6 ¼″
- Coefficient of friction = 0.25
- Pore pressure at 5,000 ft = 2340 psi
- Margin of over-pull = 75 kips
- Length of the stuck portion = 100 ft

*Solution:*

The given data are

$$\mu = 0.25.$$

$$L_p = 100 \text{ ft.}$$

$$t_{mc} = 1/8 = 0.125''$$

Using equation 10.7,

$$A_c = 2\times 100\times 12\left[\left(\frac{12.25}{2}-0.125\right)^2 - \left(\frac{12.25}{2}-0.125\times\frac{12.25-0.125}{12.25-6.25}\right)^2\right]^{\frac{1}{2}},$$

$$= 2954 \text{ sq. in.}$$

Using equation 10.6, with the maximum pull out force of 75,000 lbf the maximum differential pressure allowed is

$$\Delta P = \frac{F_{pull}}{\mu A_c},$$

since $\Delta P = P_m - P_f$ and $P_m = 0.052 \times \rho_m \times D_v$.

The above equation can be written as

$$0.052\times \rho_m \times D_v - P_f = \frac{F_{pull}}{\mu A_c}.$$

Therefore, the mud density allowed is

$$\rho_m = \frac{P_f + \dfrac{F_{pull}}{\mu A_c}}{0.052\times D_v} = \frac{2340+\dfrac{75000}{0.25\times 2954}}{0.052\times 5000} = 9.39 \text{ ppg}.$$

## 10.2.3 Method 3

Arc length at the contact point can be approximated by

$$A_c = \left(\frac{\alpha}{2}\right) D_p \times L_p, \tag{10.9}$$

$$\sin\alpha = \frac{\left(\varepsilon(X+\varepsilon)\left(X+D_p+\varepsilon\right)\left(D_p-\varepsilon\right)\right)^{\frac{1}{2}}}{\left(\frac{1}{2}X+\varepsilon\right)D_p}, \tag{10.10}$$

where $X = D_H - D_p - 2t_{mc}$, $a$ = the angle of contact between the BHA section and the mud cake in radians, and $\varepsilon$ = the deformation of the mud cake at the mid point of contact, in.

**Problem 10.8**

Estimate the approximate filter cake thickness for the following condition. 55 ft of 8″ drill collar got stuck at 10,000 ft from the bottom while drilling a 12 ¼″ hole section. The formation pressure equivalent at 10,000 is 12 ppg. 185 kips was applied to release the differential stuck pipe. Assume the coefficient of friction is 0.15 and the mud weight is 13.5 ppg.

*Solution:*

$$\mu = 0.15.$$

$$L_p = 55 \text{ ft}.$$

$$F_{pull} = 185 \text{ kips}.$$

Differential pressure = 0.052 × 10,000 × 12 – 0.052 × 10,000 × 13.5 = 780.14 psi.

Using equation 10.6, the contact area is calculated as

$$A_c = \frac{F_{pull}}{\mu \Delta P} = \frac{185{,}000}{0.15 \times 780.14} = 1580.90 \text{ in}^2.$$

$$\text{Arc length of the contact} = \frac{1580.90}{55 \times 12} = 2.395 \text{ in}.$$

Using the arc length contact in equatin 10.7, the thickness of the mud cake can be estimated by solving the following iteratively,

$$2.395 = 2 \times \left[ \left( \frac{12.25}{2} - t_{mc} \right)^2 - \left( \frac{12.25}{2} - 0.0625 \frac{12.25 - t_{mc}}{12.25 - 6} \right)^2 \right]^{\frac{1}{2}}.$$

The thickness of the cake will be $t_{mc} = 0.0625$ in.

**Problem 10.9**

Compute the sticking force required to free a differentially stuck pipe under the following conditions. An 8 ½″ hole is drilled through a depleted sand section of 100 ft with the differential pressure of 500 psi. The filter cake is estimated to be 1⁄32″. If a 6 ¼″ drill collar is stuck over the entire section of the sand, calculate the sticking with a coefficient of friction of 0.25. Estimate the force using the alternate method if the deformation of mud cake at the mid point of contact is 0.13 in.

*Solution:*

Using method 1, the contact area is given by

$$A_c = 2\times 100\times 12\left\{\left(\frac{8.5}{2}-0.03125\right)^2 - \left[\frac{8.5}{2}-t_{mc}\frac{(8.5-0.03125)}{(8.5-6)}\right]^2\right\}^{\frac{1}{2}} = 1895.77\text{ in}^2.$$

Using equation 10.6,

$$F_{pull} = \mu\Delta PA_c = 0.25\times 1895.77\times 500 = 236{,}971\text{ lbf}.$$

Using the method 2,

$$X = D_H - D_p - 2t_{mc} = 8.5 - 6 - 2\times 0.03125 = 2.4375\text{ in},$$

$$\sin a = \frac{(0.13(2.4375+0.13)(2.4375+6+0.13)(6-0.13))^{\frac{1}{2}}}{\left(\frac{1}{2}2.4375+0.13\right)6}$$

$$= 0.506279,$$

$$a = \sin^{-1}(0.506279) = 0.53086\text{ rad}.$$

The contact area is given by

$$A_c = \left(\frac{a}{2}\right)D_p\times L_p = \left(\frac{0.53086}{2}\right)6\times 100\times 12 = 1911\text{ in}^2,$$

$$F_{pull} = \mu\Delta PA_c = 0.25\times 1911\times 500 = 238889\text{ lbf}.$$

## 10.3 Spotting Fluid Requirements

The height of the spotting fluid in the annulus is given by

$$L_s = \frac{\Delta p}{0.052(\rho_m - \rho_s)} \text{ ft}, \quad (10.11)$$

where $\Delta p$ = the differential pressure due to spotting fluid, psi, $\rho_m$ = mud weight, ppg, and $\rho_s$ = the spotting fluid density, ppg.

**Problem 10.10**

Estimate the height required of an un-weighted soak solution of 7.5 ppg in order to reduce the hydrostatic pressure of 200 psi at the free point. The mud weight in the hole is 9.5 ppg.

*Solution:*

Using equation 10.11, the height of the spotting fluid in the annulus is calculated as

$$L_s = \frac{\Delta p}{0.052(\rho_m - \rho_s)} = \frac{200}{0.052(9.5 - 7.5)} = 1442 \text{ ft}.$$

**Problem 10.11**

Compute the volume required of an un-weighted spotting solution of 7.5 ppg in order to reduce the hydrostatic pressure of 200 psi at the stuck point. The mud weight in the hole is 9.5 ppg. The drill string data above the stuck point follows:

- 200 ft of drill collar with an annular capacity of 0.0313 bbl/ft
- Drill pipe with an annular capacity of 0.04591 bbl/ft

*Solution:*

Volume of the spotting solution required against the drill collar = $200 \times 0.0313 = 6.26$ bbl.

It is found from Problem 10.9 that the height of the spotting solution required, reducing the differential pressure of 200 psi, is 1442 ft.

Therefore, the length of the drill pipe to use for the annulus = 1442 – 200 = 1242 ft.

The volume of the spotting solution required against the drill pipe = 1242 × 04591 = 57.02 bbl.

The total volume of spotting solution required is = 6.26 + 57.02 = 63.28 bbl.

**Problem 10.12**

It was desired to spot 2 bbl of weighted spotting solution against the stuck drill collar of 600 ft every 1 hour for 12 hours. Mud weight in the hole is 9.5 ppg. The annular capacity of the drill collar is 0.0313 bbl/ft, and the annular capacity of the drill pipe is 0.04591 bbl/ft.

*Solution:*

The volume of the spotting solution required against the 500 ft of drill collar = 500 × 0.0313 = 15.65 bbl.

The volume of spotting solution required for 12 hours = 2 × 12 = 24 bbl.

The total volume of spotting solution required is = 15.65 + 24 = 39.65 bbl.

## 10.4 Loss Circulation

If $V_l < V_{an/dp}$, the length of the low-density fluid required in order to balance the formation pressure is given by

$$L_l = \frac{V_l}{C_{an/dp}}. \tag{10.12}$$

If $V_l > V_{an/dc}$, the length of the low-density fluid required to balance the formation pressure is given by

$$L_l = L_{dc} + \left( \frac{V_l - V_{an/dc}}{C_{an/dp}} \right), \tag{10.13}$$

where $V_l$ = the volume of low-density fluid pumped to balance the formation pressure, bbl, $C_{an/dc}$ = the annulus capacity behind the drill collar, bbl/ft, $C_{an/dp}$ = the annulus capacity behind the drill pipe, bbl/ft, $V_{an/dc}$ = the annulus volume against the drill collar, bbl,

$V_{an/dp}$ = the annulus volume against the drill pipe, bbl, and $L_{dc}$ = the length of the drill collar, ft.

Formation pressure is given by

$$p_{ff} = 0.052 \times D_w \times \rho_l + 0.052 \times (D_v - D_w) \times \rho_m, \qquad (10.14)$$

where $D_v$ = the vertical depth of the well where loss occurred, ft, $\rho_m$ = the density of the mud, ppg, $D_w$ = water depth, ft, $\rho_w$ = the seawater density, ppg.

**Problem 10.13**

While drilling an 8 ½″ hole at 17,523 ft (TVD) with a mud density of 11 ppg, the well encountered a big limestone cavern. Therefore, there was mud loss. Drilling was stopped, and the annulus was filled with 58 bbls of 8.4 ppg water until the well was stabilized. Calculate the formation pressure and the density that should be used to drill through the zone. The previous 9 ⅝″ casing was set at 15,500 ft. The drilling consists of 900 ft of 6″ drill collar and 5″ drill pipe. Use the capacity of the casing annulus against the drill pipe to be 0.05149 bbl/ft.

*Solution:*

The volume of the casing annulus against the drill pipe is

$$V_{an/dp} = 0.05149 \times 15500 = 798 \text{ bbl},$$

and $V_l < V_{an/dp}$.

The length of the annulus when balanced is

$$L_l = \frac{58}{0.05149} = 1126.43 \text{ ft}.$$

Using equation 10.14, the formation pressure can be calculated as

$$p_{ff} = 0.052 \times 1127 \times 8.4 + 0.052 \times (17523 - 1127) \times 11$$
$$= 492.27 + 9378.5 = 9870.8 \text{ psi}.$$

$$\text{The equivalent mud weight for drilling} = \frac{9870.8}{0.052 \times 17523}$$
$$= 10.83 \text{ ppg}.$$

## 10.5 Increased ECD Due to Cuttings

Effective mud density in the hole due to cuttings generation is given by

$$\rho_{eff} = \frac{\rho_m Q + 141.4296 \times 10^{-4} \times \mathrm{ROP} \times d_b^2}{Q + 6.7995 \times 10^{-4} \times \mathrm{ROP} \times d_b^2} \text{ ppg,} \tag{10.15}$$

where $\rho_m$ = mud density without cuttings, ppg, $Q$ = mud flow rate, gpm, ROP = the rate of penetration, ft/hr, and $d_b$ = the diameter of the bit or the diameter of the hole drilled, in.

Density due to drilled cuttings is given by

$$\rho_{eff} - \rho_f = \rho_c = \frac{ROP \times d_b^2 \times 10^{-4} \left(141.4296 - 6.7995 \times \rho_f\right)}{Q + 6.7995 \times 10^{-4} \times \mathrm{ROP} \times d_b^2} \text{ ppg.} \tag{10.16}$$

ECD due to cuttings is given by

$$\mathrm{ECD} = \rho_f + \rho_a + \rho_c \text{ ppg.}$$

where $\rho_m$ = the mud density, ppg, $\Delta\rho_a$ = the equivalent mud weight increase due to annular frictional pressure losses, and $\Delta\rho_c$ = the equivalent mud weight increase due to cuttings.

**Problem 10.14**

A 17½ hole was drilled at the rate of 120 fph with a circulation rate of 1000 gpm. The mud density was 8.8 ppg. Calculate the effective mud weight due to cuttings.

*Solution:*

Using equation 10.15, the effective mud density in the hole due to cuttings generation is

$$\rho_{eff} = \frac{\rho_m Q + 141.4296 \times 10^{-4} \times \mathrm{ROP} \times d_b^2}{Q + 6.7995 \times 10^{-4} \times \mathrm{ROP} \times d_b^2} \text{ ppg.}$$

Using the data given,

$$\rho_{eff} = \frac{8.8 \times 1000 \times 141.4296 \times 10^{-4} \times 120 \times 17.5^2}{1000 + 6.7995 \times 10^{-4} \times 120 \times 17.5^2} = 9.09 \text{ ppg.}$$

**Problem 10.15**

Calculate the maximum ROP that can be drilled under the following conditions:

- Fracture pressure = 4817 psi
- Mud density = 10 ppg
- Annular pressure loss = 88 psi
- Depth of the hole = 9,000″(TVD)
- Mud flow rate = 500 gpm
- Diameter of the hole = 12.25″

*Solution:*

Overall ECD with the cuttings is

$$\mathrm{ECD} = \rho_f + \rho_a + \rho_c = \frac{4817}{0.052 \times 9000} = 10.30 \text{ ppg}.$$

The equivalent density of the cuttings contribution can be calculated as

$$\rho_c = 10.30 - \frac{88}{0.052 \times 9000} - 10 = 0.112 \text{ ppg},$$

$$\rho_c = \frac{\mathrm{ROP} \times d_b^2 \times 10^{-4} \times \left(141.4296 - 6.7995\rho_f\right)}{Q + 6.7995 \times 10^{-4} \times \mathrm{ROP} \times d_b^2} = 0.112$$

$$= \frac{\mathrm{ROP} \times 12.25^2 \times 10^{-4} \times (141.4296 - 6.7995 \times 10)}{500 + 6.7995 \times 10^{-4} \times \mathrm{ROP} \times 12.25^2}.$$

Solving will yield the following:

$$\mathrm{ROP} = 51 \text{ fph}.$$

## 10.6 Mud Weight Increase Due to Cuttings

The volume of the cuttings entering the mud system is given by

$$V_c = \frac{(1-\phi)D_b^2 \times \mathrm{ROP}}{1029} \text{ bbl/hr}, \tag{10.17}$$

where $\phi$ the average formation porosity, $D_b$ = diameter of the bit, in, and ROP = the rate of penetration, ft/hr.

$$V_c = \frac{(1-\phi)D_b^2 \times \text{ROP}}{24.49} \text{ gal/hr,}$$

or

$$V_c = \frac{(1-\phi)D_b^2 \times \text{ROP}}{1469.4} \text{ gpm.} \tag{10.18}$$

An estimate of the average annulus mud weight ($\rho_{m,av}$) can be formulated from the above equations and is given by

$$\rho_m = \frac{\rho_{ps}Q + 0.85D_h^2\text{ROP}}{Q + 0.0408D_h^2\text{ROP}}, \tag{10.19}$$

where $\rho_{m,av}$ = average annular mud weight (lb/gal), $Q$ = flow rate (gpm), $\rho_{ps}$ = measured mud weight at the pump suction (lb/gal), $D_h$ = the diameter of hole (in), and ROP = the penetration rate based on the time the pump is on before, during, and after the joint is drilled down (fpm).

**Problem 10.16**

Calculate the volume of the cuttings generated while drilling a 12¼″ hole with the rate of penetration of 50 ft/hr. Assume the formation porosity = 30%.

*Solution:*

Using equation 10.18, the volume of the cuttings generated in barrels per hour is

$$V_c = \frac{(1-\phi)D_b^2 \times \text{ROP}}{1029} \text{ bbl/hr.}$$

Substituting the values,

$$V_c = \frac{(1-\phi)D_b^2 \times \text{ROP}}{1029} = \frac{\left(1-\dfrac{30}{100}\right) \times 12.5^2 \times 50}{1029} = 5.1 \text{ bbl/hr.}$$

The volume of the cuttings generated in barrels per hour is

$$V_c = \frac{(1-\phi)D_b^2 \times \text{ROP}}{24.49} = \frac{\left(1-\frac{30}{100}\right)\times 12.5^2 \times 50}{24.49} = 214.5\ \text{gal/hr}.$$

The volume of the cuttings generated in barrels per hour is

$$V_c = \frac{(1-\phi)D_b^2 \times \text{ROP}}{1469.4} = \frac{\left(1-\frac{30}{100}\right)\times 12.5^2 \times 50}{1469.4} = 3.57\ \text{gpm}.$$

**Problem 10.17**

A mud pump is on for 1.5 minutes while drilling a 30 foot joint, and the rate of penetration is 20.0 ft/min. Suppose the hole drilled has a 17½ in diameter, the flow rate is 800 gpm, and the mud density is 9.0 ppg. Calculate the equivalent mud density.

*Solution:*

$$\rho_{m,av} = \frac{800\times 9 + 0.85\times 20\times 17.5\times 17.5}{800 + 0.0408\times 17.5\times 17.5\times 20} = 11.8\ \text{ppg}.$$

## 10.7 Hole Cleaning – Slip Velocity Calculations

### 10.7.1 The Chien Correlation

Slip velocity is given for two conditions as follows:

$$\text{v}_s = 0.458\beta\left[\sqrt{\left(\frac{36800\,d_s}{\beta_2}\right)\left(\frac{\rho_s-\rho_m}{\rho_m}\right)+1}-1\right] \quad (\textit{for } \beta < 10) \tag{10.20}$$

$$v_s = 86.4\,d_s\sqrt{\frac{\rho_s-\rho_m}{\rho_m}} \quad \text{for } \beta < 10, \tag{10.21}$$

where $\beta = \frac{\mu_a}{\rho_m d_s}$, and $\mu_a = \mu_p + \frac{300\tau_y d_s}{V_a}$.

$\rho_m$ = mud density (ppg), $\rho_s$ = cuttings density (ppg), $\mu_a$ = the mud apparent viscosity (cp), $\mu_p$ = mud plastic viscosity (cp), $\tau_y$ = mud yield value lb/100 ft$^2$, $d_s$ = equivalent spherical diameter of cutting (in), and $V_a$ = average annular fluid velocity (fpm).

$$V_a = \frac{60Q}{2.448\left(D_h^2 - D^2\right)} \text{ ft/min.}$$

where $Q$ = flow rate in gpm, $D_h$ = inside diameter of casing or diameter of the hole, in, and $D$ = outside diameter of the pipe, in.

### 10.7.2 The Moore Correlation

Slip velocity based on the Moore correlation is given for various conditions as follows:

$$v_s = 9.24\sqrt{\frac{\rho_s - \rho_m}{\rho_m}} \quad \text{for } N_R > 2000, \tag{10.22}$$

$$v_s = 4972\left(\rho_s - \rho_m\right)\frac{d_s^2}{\mu_a} \quad \text{for } N_R \le 1, \tag{10.23}$$

where $N_R$ = the particle Reynolds number, and $\mu_a$ = apparent viscosity.

$$v_s = \frac{174\, d_s \left(\rho_s - \rho_m\right)^{0.667}}{\left(\rho_m \mu_a\right)^{0.333}} \quad \text{for } 1 \le N_R < 2000. \tag{10.24}$$

The apparent viscosity is calculated as follows:

$$\mu_a = \frac{K}{144}\left(\frac{D_h - D_{op}}{\frac{V_a}{60}}\right)^{(1-n)}\left(\frac{2+\frac{1}{n}}{0.0208}\right)^{n} \tag{10.25}$$

where $K$ = the mud consistency index, or

$$K = \frac{510 \times \theta_{300}}{511^n} \text{ eq. cP,}$$

and $n$ = the mud power-law index, $n_a = 3.32 \cdot \log\left[\frac{\theta_{600}}{\theta_{300}}\right]$, and $D_h$ = the hole diameter (in), and $D_{op}$ = the outer pipe diameter (in).

### 10.7.3 The Walker Mays Correlation

Slip velocity based on the Walker Mays correlation is given for various conditions as follows:

$$v_s = 131.4\sqrt{h_s \frac{\rho_s - \rho_m}{\rho_m}} \quad \text{for } N_R > 100, \tag{10.26}$$

$$\mu_a = 511\frac{\tau}{\gamma}, \tag{10.27}$$

$$\tau = 7.9\sqrt{h_s\left(\rho_s - \rho_m\right)}, \tag{10.28}$$

$$v_s = 1.22\,\tau\sqrt{\frac{d_s\gamma}{\sqrt{\rho_m}}} \quad \text{for } N_R < 100, \tag{10.29}$$

where $h_s$ = cutting thickness, and $\gamma$ = the shear rate corresponding to shear stress $\tau$.

## 10.8 Transport Velocity and Transport Ratio

The average cuttings transport velocity is given by

$$V_t = v_s - V_a. \tag{10.30}$$

The transport ratio is given by

$$R_t = 1 - \frac{v_s}{V_a}. \tag{10.31}$$

Transport efficiency is given by

$$\eta_t = \left(1 - \frac{v_s}{V_a}\right) \times 100. \tag{10.32}$$

**Problem 10.18**

Using the following data, calculate the slip velocity, transport velocity, and transport ratio.

- Plastic viscosity = 35 cP
- Yield point = 13 lbf/100 ft$^2$
- Flow rate = 450 gpm
- Hole diameter = 12 ¼″
- Pipe outside diameter = 5″
- Mud density = 10.7 ppg
- Assume cuttings size is 0.3 in and has a density of 21 ppg.

*Solution:*

Using the Chien correlation, the annular velocity against the drill pipe is calculated as

$$V_a = \frac{450\times 60}{2.448\times\left(12.25^2-5^2\right)} = 88.91 \text{ fpm.}$$

Apparent viscosity is

$$\mu_a = 35 + \frac{300\times 13\times 0.3}{88.19} = 50.3 \text{ cP.}$$

Calculate beta:

$$\beta = \frac{50.3}{10.7\times 0.3} = 15.67 \quad \text{as } \beta > 10$$

The slip velocity is calculated as

$$v_s = 86.4\sqrt{\frac{21-10.7}{10.7}}\times 0.3 = 24.16 \text{ fpm.}$$

The transport ratio is given by

$$R_t = 1 - \frac{24.1}{88.91} = 0.726.$$

**Problem 10.19**

Using the Moore correlation, estimate the cuttings slip velocity in a 12 ppg mud. Compute the transport efficiency when the fluid is flowing at the rate of 85 ft/min. The cuttings diameter and thickness is 0.3 in, and the specific gravity is 2.5.

- Hole size = 8.5″
- Outside pipe diameter = 4.5″
- Fluid data: $\theta_{300} = 55$, and $\theta_{300} = 85$

*Solution:*

Calculating the flow behavior index $n$ and the consistency index $K$,

$$n_a = 3.32 \cdot log\left(\frac{\theta_{600}}{\theta_{300}}\right),$$

$$n = 3.32 \cdot log\left(\frac{85}{55}\right),$$

$$n = 0.62766.$$

$$K = \frac{510 \times \theta_{300}}{511^n},$$

$$K = \frac{510 \times 55}{511^{0.737}},$$

$$K = 283 \text{ eq. cP.}$$

The apparent viscosity in the annulus is calculated as

$$\mu_a = \frac{K}{144}\left(\frac{d_h - d_p}{\frac{v_a}{60}}\right)^{1-n}\left(\frac{2+\frac{1}{n}}{0.0208}\right)^{n_p},$$

$$\mu_a = \frac{308.8}{144}\left(\frac{8.5-8.5}{85/60}\right)^{1-0.62766}\left(\frac{2+\frac{1}{0.62766}}{0.0208}\right)^{0.62766},$$

$$\mu_a = 73.39 \text{ eq. cP.}$$

Assuming an intermediate particle Re number (1<Re>2000), the slip velocity is

$$v_s = \frac{174 \times d_s\left(\rho_s - \rho_f\right)^{0.667}}{\left(\rho_s \cdot \rho_f\right)^{0.333}},$$

$$v_s = \frac{174 \times 0.3(2.5 \times 8.33 - 12.0)^{0.667}}{(20.8 \times 12.0)^{0.333}},$$

$$v_s = 35.5 \text{ fpm.}$$

The particle Re number is

$$N_{Re} = \frac{928 \times \rho_f \times v_s \times d_s}{\mu_a},$$

$$N_{Re} = \frac{928 \times 12.0 \times 0.5913 \times 0.30}{73.39}.$$

Since $N_{Re}$ = 26.91, which is between 1 and 2000, the equation to calculate the slip velocity is justified.

The transport ratio is given by

$$R_t = 1 - \frac{v_s}{v_a},$$

$$R_t = 1 - \frac{35.5}{85},$$

$$R_t = 0.582.$$

Therefore, the transport efficiency is 58%.

## 10.9 Keyseating

The side force to create a keyseat is given by

$$F_L = T \sin \beta, \tag{10.33}$$

$$F_L = 2T \sin \frac{\beta}{2}, \tag{10.34}$$

$$F_L = T \beta L, \tag{10.35}$$

where $F_L$ = the lateral force, $T$ = the tension in the drill string just above the keyseat area, lbf, $\beta$ = dogleg angle, and $L$ = the length of the dogleg, ft

The depth of the formation cut due to keyseating is given by

$$d_{key} = C_{sc} \sqrt{\frac{T \kappa L_{dl}}{OD_{tj}}} \left( \frac{L_{tj}}{\text{ROP}} \right) N, \tag{10.36}$$

where $d_{key}$ = the depth of the keyseat, $C_{sc}$ = the side cutting coefficient of the tool joint, $T$ = drill string tension at the keyseat, $\kappa$ = the dogleg curvature, $L_{dl}$ = the length of the dogleg, $OD_{tj}$ = the outside diameter of the tool joint, $N$ = rpm, $L_{tj}$ = the length of the tool joint, and ROP = the rate of penetration.

When the rate of penetration is zero, the above equation reduces to

$$d_{key} = C_{sc} \sqrt{\frac{T \kappa L_{dl}}{OD_{tj}}} N t, \tag{10.37}$$

where $t$ = total rotating time.

The side cutting coefficient $C_{sc}$ depends on the formation type, the formation stress, the deviation of the hole, and the wellbore pressure.

**Problem 10.20**

Calculate the side force in the keyseat with the dogleg angle of 4 deg/100 ft in a 100 feet keyseat interval. The effective tension above the keyseat is found to be 120 kip.

*Solution:*

Using equation 10.33, the side force at the keyseat is given as

$$F_L = T\sin\beta = 120000 \times \sin\left(4 \times \frac{\pi}{180}\right) = 8370 \text{ lbf}.$$

Using equation 10.34,

$$F_L = 2T\sin\frac{\beta}{2} = 2 \times 120000 \times \sin\left(\frac{4}{2} \times \frac{\pi}{180}\right) = 8376 \text{ lbf}.$$

Using equation 10.35,

$$F_L = T\beta L = 120000 \times \left(\frac{4}{100} \times \frac{\pi}{180}\right) \times 100 = 8377 \text{ lbf}.$$

# 11

# Cementing

In this chapter basic cementing equations and calculations will be presented.

## 11.1 Cement Slurry Requirements

The number of sacks of cement is calculated as

$$N_c = \frac{V_{sl}}{Y} \text{ sacks,} \tag{11.1}$$

where $V_{sl}$ = slurry volume, cubic ft, and $Y$ = the yield of cement, cubic ft/sack.

## 11.2 Yield of Cement

$Y$ = the yield of the slurry and is given by

$$Y = \frac{V_{sl}}{7.48} \text{ ft}^3\text{/sack,}$$

where the slurry volume is

$$V_{sl} = V_c + V_w + V_a \text{ gal},$$

in which $V_c$ = cement volume, gal, $V_w$ = water volume, gal, and $V_a$ = additive volume, gal.

The mix water requirement is calculated as

$$V_{mw} = Vms_i \times N_c \text{ ft}^3, \tag{11.2}$$

where $Vms_i$ = mix water per sack.

The number of sacks of additive is calculated as

$$N_a = N_c \times A_{\%} \text{ sacks}, \tag{11.3}$$

where $A_{\%}$ = the percentage of additive.

The weight of the additive is calculated as

$$W_a = N_a \times 94 \text{ lb}.$$

## 11.3 Slurry Density

Slurry density is given as $\rho_{sl} = \dfrac{W_c + W_w + W_a}{V_c + V_w + V_a}$, (11.4)

where $W_c$ = the weight of the cement, lb, $W_w$ = the weight of the water, lb, and $W_a$ = the weight of the additive, lb.

## 11.4 Hydrostatic Pressure Reduction

Hydrostatic pressure reduction due to the spacer is

$$\Delta p = 0.292(\rho_m - \rho_s)C_{an} \times V_s \text{ psi}, \tag{11.5}$$

where $\rho_m$ = the density of the mud, ppg, $\rho_s$ = the density of the spacer, ppg, $C_{an}$ = annular capacity, ft$^3$/ft, and $V_s$ = the spacer volume, bbl.

## 11.5 Contact Time

An estimation of the volume of cement needed for the removal of mud cake by turbulent flow is

$$V_t = t_c \times q \times 5.616 \frac{\text{ft}^3}{\text{bbl}}. \tag{11.6}$$

where $V_t$ = the volume of the fluid (in turbulent flow), ft$^3$, $t_c$ = contact time, minutes, and $q$ = displacement rate, bbl/min.

Studies have shown that a contact time (during pumping) of 10 minutes or longer provides better mud removal than shorter contact times.

**Problem 11.1**

The water requirement for API class G cement is 5.0 gal/94-lb sack, or 44%. Calculate the density of the slurry, water requirement, and slurry yield. Assume the specific gravity of cement = 3.14.

*Solution:*

Water needed for 100 g cement = $100 \times 0.44 = 44$ g.

The volume of cement is

$$V_c = 100/3.14 = 31.85 \text{ cm}^3.$$

The volume of water is

$$V_w = 44 \text{ cm}^3.$$

Total slurry volume = 75.85 cm$^3$.
Total mass of slurry = 100 + 44 = 144 g.
Slurry density is given as

$$\rho_{sl} = \frac{W_c + W_w + W_a}{V_c + V_w + V_a}.$$

The density of slurry = 144/75.85 = 1.8984 = 15.8 ppg.
The water requirement per sack is calculated as

$$426{,}38 \times 44 = 18760 \text{ cm}^3,$$

$$= 18760/3785.4 = 4.95 \text{ gal/sack}.$$

Slurry yield is calculated as

$$69.85 \text{ cm}^3/100 \text{ g} = 0.6985 \text{ cm}^3/\text{g} = 1.140 \text{ ft}^3/\text{sack}.$$

**Problem 11.2**

A cementing engineer is planning to use class H cement in order to prepare a cement slurry weighing 18 lbm/gal. He is planning to use heavy material, hematite with the ratio 0.4 gal/100 lbm with each sack of cement. The water requirement is 4.8 gal/94 lbm. Assume a specific gravity of 5.02 for hematite and 3.14 for cement.

*Solution:*

Let $x$ lbm be the hematite per sack of cement.

The total water requirement of the slurry is given by 4.8 + 0.004 $x$.

Slurry density is given as

$$\rho_{sl} = \frac{\text{total mass(lbm)}}{\text{total volume(gal)}} = \frac{W_c + W_w + W_a}{V_c + V_w + V_a},$$

$$18 = \frac{94 + x + 8.33(4.8 + 0.004x)}{\frac{94}{3.14 \times 8.33} + \frac{x}{5.02 \times 8.33} + (4.8 + 0.004x)},$$

Solving yields 28.37 lbm hematite/94 lbm of cement.

**Problem 11.3**

It is desired to increase the density of a class H cement using 30% sand and 46% water. Calculate the slurry weight, slurry yield, and mixing water volume.

The specific gravity of sand = 2.63, and the specific gravity of cement = 3.14.

*Solution:*

The weight of the cement is given as

$$W_c = 94 \text{ lbm/sack}.$$

The weight of the sand (30%) is given as

$$W_a = 28.2 \text{ lbm/sack}.$$

The weight of the water (46%) is given as

$$W_w = 43.24 \text{ lbm/sack.}$$

The volume of the cement is calculated as

$$V_c = \frac{94}{3.14 \times 8.33} = 3.6 \text{ gal.}$$

The volume of the sand is calculated as

$$V_a = \frac{28.2}{2.63 \times 8.33} = 1.29 \text{ gal.}$$

The volume of the water is calculated as

$$V_w = \frac{43.24}{8.33} = 5.19 \text{ gal.}$$

Slurry weight = (94 + 28.2 + 43.24)/(3.6 + 1.29 + 5.19) = 16.41 ppg.

$Y$ = the yield of the slurry:

$$Y = \frac{V_{sl}}{7.48} = \frac{10.08}{7.48} \text{ ft}^3\text{/sack.}$$

The yield of the slurry = 1.35 gal/sack.

**Problem 11.4**

It was desired to obtain a class H cement slurry weighing 16 ppg by adding sand and water with a slurry yield of 1.35 ft$^3$/sack and a mix water volume of 5 gal/sack. Compute the percentage of sand and water by weight to be added. The specific gravity of sand = 2.63, and the specific gravity of cement = 3.14.

*Solution:*

$Y$ = yield of the slurry and is given by

$$Y = \frac{V_{sl}}{7.48} = 1.35 \text{ ft}^3\text{/sack.}$$

The volume of the slurry is calculated as

$$V_{sl} = 1.35 \times 7.48 = 10.098 \text{ gal/sack.}$$

For one sack of cement, the slurry volume = 10.098 gal and the water volume = 5 gal.

The slurry density is given as

$$\rho_{sl} = \frac{W_c + W_w + W_a}{V_c + V_w + V_a}.$$

The weight of the water is given as

$$W_w = 5 \times 8.33 = 41.65 \text{ lbm/sack}.$$

The weight of the sand is

$$\rho_{sl}(V_c + V_w + V_a) - W_c - W_w = W_a.$$

Substituting the values yields the weight of the sand:

$$W_a = 16 \times 10.098 - 94 - 41.65 = 25.918 \text{ lbm/sack}.$$

The percentage of sand = 25.918/94 = 27.6%.

**Problem 11.5**

It is desired to increase the density of a class H cement using 30% sand, 46% water, and 1% retarder. Calculate the slurry weight and slurry yield. The specific gravity of sand = 2.63, the specific gravity of cement = 3.14, and the specific gravity of retarder = 1.96.

*Solution:*

The weight of the cement is given as

$$W_c = 94 \text{ lbm/sack}.$$

The weight of the sand (30%) is given as

$$W_a = 28.2 \text{ lbm/sack}.$$

The weight of the water (46%) is given as

$$W_w = 43.24 \text{ lbm/sack}.$$

The weight of the retarder (1%) is given as

$$W_r = 0.94 \text{ lbm/sack}.$$

The volume of the cement is calculated as

$$V_c = \frac{94}{3.14 \times 8.33} = 3.6\,\text{gal}.$$

The volume of the sand is calculated as

$$V_a = \frac{28.2}{2.63 \times 8.33} = 1.29\,\text{gal}.$$

The volume of the water is calculated as

$$V_w = \frac{43.24}{8.33} = 5.19\,\text{gal}.$$

The volume of the retarder is calculated as

$$V_r = \frac{0.94}{1.96 \times 8.33} = 0.0575\,\text{gal}.$$

Slurry weight = (94 + 28.2 + 43.24 + 0.94)/(3.6 + 1.29 + 5.19 + 0.575) = 15.62 ppg.

$Y$ = the yield of the slurry and is calculated as

$$Y = \frac{V_{sl}}{7.48} = \frac{3.6 + 1.29 + 5.19 + 0.575}{7.48} = 1.424\,\text{ft}^3/\text{sack}.$$

## 11.6 Gas Migration Potential (GMP)

The pressure reduction for the cement column is given by

$$P_r = 1.67 \times \frac{L}{D_h - D_p} \tag{11.7}$$

The gas migration potential is calculated as

$$\text{GMP} = \frac{P_{r\max}}{P_{ob}},$$

where $D_h$ = the diameter of the hole, in, $D_p$ = the outside diameter of the casing pipe, in, $P_{ob}$ = the reservoir pressure, psi, and $L$ = the length of the pipe column exposed to the cement from the reservoir zone, ft.

Ranges are as follows:

- 0–3 = Low
- 3–8 = Moderate
- >8 = High

**Problem 11.6**

Calculate the gas migration potential using the following data:

- Depth: 10000 ft, 8 ½″ hole
- Casing OD = 7″
- Mud density = 16 ppg
- Active formation depth = 9000 ft
- Reservoir pressure = 7000 psi
- Cemented depth top = 7500 ft
- Slurry density = 17.5 ppg

*Solution:*

Hydrostatic pressure = 7500 × 0.052 × 16 + 1500 × 0.052 × 17.5 = 7605 psi.
$P_{ob}$ = 7605 – 7000 = 605 psi.
$P_{r,\,max}$ = 1.67 × 1500/(8.5 – 7) = 1670 psi.
GMP = 1670/605 = 2.76 → Low.

**Problem 11.7**

A cementing engineer desires to keep the gas migration potential less than 3 while planning to cement a 5″ production casing at a depth of 18,550 ft. The hole size is 8″, and the mud density prior to cementing is 13.4 ppg. The production zone is expected to be at 18,000 ft with a reservoir pressure of 12,000 psi. The expected top of the cement is 15,000 ft. Calculate the minimum slurry density needed.

*Solution:*

Gas migration potential is given as

$$\mathrm{GMP} = \frac{P_{r\max}}{P_{ob}}.$$

$$P_r = 1.67 \times \frac{3000}{8-5} = 1670\,\mathrm{psi}.$$

$$3 = \frac{1670}{P_{ob}},$$

and therefore $P_{ob} = 557$ psi.

$$P_{ob} = P_h - 12000 = 557 \text{ psi.}$$

$$P_h = 557 + 12000 = 12557 \text{ psi.}$$

Let $S$ be the density of the slurry needed:

Hydrostatic pressure = 15000 × 0.052 × 13.4 + 3000 × 0.052 × $S$ = 12557 psi.

Solving the equation, the density of the slurry = 13.5 ppg.

**Problem 11.8**

It is desired to cement a 7″ production casing with the following conditions:

- Depth of the production 8 ½″ hole = 10005′
- Casing depth = 10000′, J-55, 26 ppf
- Excess volume factor = 20%
- Float collar is 79′ above the shoe.

Calculate the cement required. Use the yield 1.35.

*Solution:*

$$\begin{aligned}\text{Slurry volume} &= 0.005454\left(D_h^2 - D_p^2\right)\times 2000\\ &= 0.005454\left(8.5^2 - 7^2\right)\times 2000 = 254\,\text{ft}^3.\end{aligned}$$

Excess factor = 20%.
Slurry volume = 254 × 1.2 = 305 $ft^3$.
Shoe volume = 0.005454 ($6.27^2$) × 79 = 15 $ft^3$.
Total volume = 320 $ft^3$,

$$\text{Cement required} = \frac{320\text{ ft}^3}{\text{yield}} = \frac{320}{1.35} = 237\,\text{sacks.}$$

**Problem 11.9**

Calculate the number of sacks of cement needed for a 400-ft, 6 ½" open-hole abandonment cement plug job. The slurry yield is 2 cubic ft/sack. Assume an excess volume of 20%.

*Solution:*

The volume of the cement slurry needed is calculated as

$$V_{sl} = \frac{\pi}{4} \times \left(\frac{6.5}{12}\right)^2 \times 400 = 92.18\,\text{ft}^3.$$

The total volume needed with excess percentage = 92.18 × 1.20 = 110.6 ft$^3$.

The number of cement sacks needed = 110.6/2 = 56 sacks.

## 11.7 Cement Plug

The number of sacks of cement required for placing a cement plug can be estimated as

$$N_c = \frac{L_p \times V_h}{Y} \text{sacks}, \tag{11.8}$$

where $L_p$ = the length of plug, ft, $Y$ = yield, ft$^3$/sack, and $V_h$ = capacity of the hole, ft$^3$/ft.

The spacer volume ahead of the slurry is calculated as

$$V_{sa} = \frac{V_{sb} \times C_{an}}{V_{dp}} \text{bbl}, \tag{11.9}$$

where $V_{sb}$ = the volume of spacer behind the slurry, $C_{an}$ = annular volume, ft, and $V_{dp}$ = the pipe volume, ft$^3$.

The length of the plug can be calculated as

$$L_p = \frac{N_c \times Y}{V_{dp} + V_{an}} \text{ft}. \tag{11.10}$$

The volume of mud required to displace the pipe can be calculated as

$$V_m = (L_{dp} - L_p)V_{dp} - V_{sb} \text{ bbl}, \tag{11.11}$$

where $L_{dp}$ = length of drill pipe, ft.

**Problem 11.10**

A cementing engineer is planning to spot a cement plug of 200 ft with a top of the plug at 5000 ft. The hole size = 8.5″. He assumes an excess factor of 20%. The pipe used for spotting is 5″ with an internal diameter of 4.276″. A slurry yield of 1.2 cubic ft/sack is used. What would be the number of sacks of cement needed for this balanced cement plug job? Calculate the length of the plug before the pipe is pulled out.

*Solution:*

The procedure is depicted in Figure 11.1.

The length of the plug desired = 5200 − 5000 ft = 200 ft.

The slurry volume desired is calculated as

$$\frac{\left(8.5^2 \times 200\right)}{1029.5} \times 1.2 = 16.85\,\text{bbl}.$$

The sacks of cement needed is calculated as

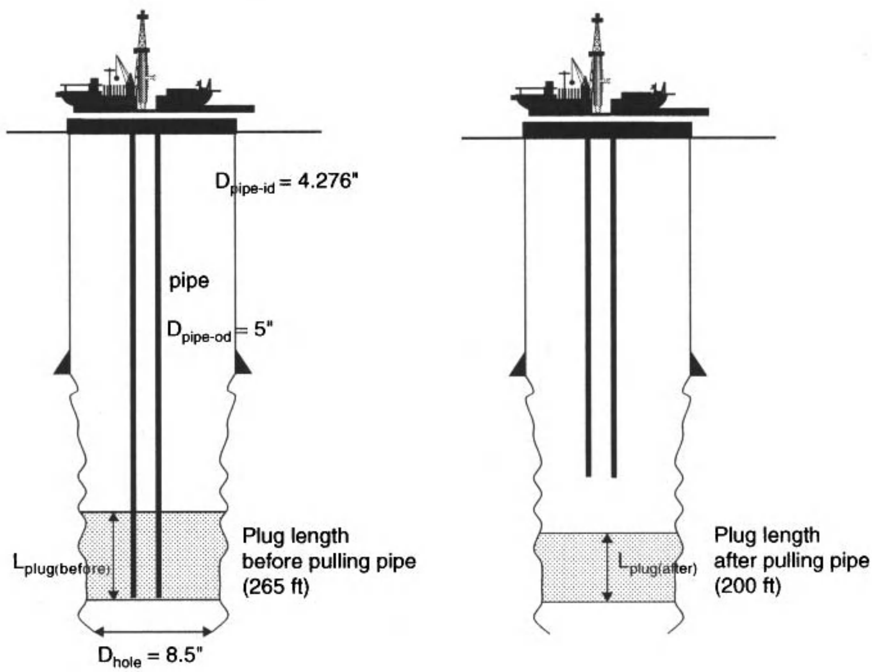



**Figure 11.1** Balanced plug for Problem 11.10.

$$\frac{(5.615\times V_{cem})}{Y}=\frac{5.615\times 16.85}{1.2}=79\,\text{sacks}.$$

The hole capacity is calculated as

$$\frac{(8.5^2)}{1029.5}\times 1.2=0.084216\,\text{bbl/ft}.$$

The annulus capacity is

$$\frac{(8.5^2-5^2)}{1029.5}=0.04590\,\text{bbl/ft}.$$

The pipe capacity is

$$\frac{(4.276^2)}{1029.5}=0.01776\,\text{bbl/ft}.$$

The plug length before the pipe is pulled out is given by

$$L_{plug(before)}=\frac{L_{plug(after)}\times \text{hole capacity}}{(\text{annulus capacity}+\text{pipe capacity})},$$
$$=\frac{200\times 0.08421}{(0.04590+0.01776)}=265\text{ ft}.$$

Alternatively, it can be calculated as

$$L_{plug(before)}=\frac{\text{Number of sacks}\times \text{Yield}}{(\text{annulus capacity}+\text{pipe capacity})},$$
$$=\frac{79\times 1.2}{5.615\times(0.04590+0.01776)}=265\text{ ft}.$$

**Problem 11.11**

After cementation it was found that the float equipment in the float collar was not holding the pressure. Calculate the surface pressure needed to prevent backflow of cement if the casing was cemented with 15 ppg cement slurry. The top of the cement is expected to be around 7,000 ft from the surface. The total length of the cemented casing is 12,500 ft. The mud density before cementation was 10 ppg, and the displaced mud was 11 ppg.

*Solution:*

The surface pressure is calculated as

$$P_s = 0.052 \times 7000 \times 10 + 0.052 \times (12500 - 7000) \times 15 - 0.052 \times 12500 \times 11 = 780 \text{ psi}.$$

**Problem 11.12**

Compute the length of the plug before the pipe is pulled out from the slurry. A slurry yield of 1.5 cubic ft/sack is used. The plug length is 250 ft inside a hole size of 8". Assume an excess volume of 20%. The capacity of the annulus against the tubing pipe is 0.323 cubic ft/ft, and the capacity of the pipe is 0.0221 cubic ft/ft.

*Solution:*

The length of the plug desired = 250 ft.

The slurry volume desired is calculated as

$$\frac{\left(8.0^2 \times 250\right)}{1029.5} \times 1.2 = 18.65 \,\text{bbl}.$$

The number of sacks of cement needed is calculated as

$$\frac{\left(5.615 \times V_{cem}\right)}{Y} = \frac{5.615 \times 18.65}{1.5} = 70 \,\text{sack}.$$

The plug length before the pipe is pulled out is given by

$$L_{plug(before)} = \frac{\text{Number of sacks} \times \text{Yield}}{\left(\text{annulus capacity} + \text{pipe capacity}\right)},$$

$$= \frac{70 \times 1.5}{(0.323 + 0.0221)} = 303 \,\text{ft}.$$

# 12

# Well Cost

This chapter focuses on the different basic calculations involved in the well cost.

## 12.1 Drilling Costs

### 12.1.1 Cost Per Foot

The overall well cost excluding the production is calculated as

$$C_{wdo} = C_d + C_o, \tag{12.1}$$

where $C_d$ = the cost of drilling a foot of hole that includes bit cost, downhole drilling tools cost, and rig rental costs only. $C_o$ = all other costs of making a foot of hole, such as casings, mud, cementing services, logging services, coring services, site preparation, fuel, transportation, completion, etc.

Bit run costs may be expressed as

$$C_{di} = \frac{C_{bi} + C_r\left(T_{di} + T_{ti} + T_{ci}\right)}{\Delta D_i}, \tag{12.2}$$

where for the drill bit number $i$ $C_{di}$ = the drilling cost, \$/ft, $C_{bi}$ = the bit cost, \$, $C_r$ = the rig cost, \$/hr, $T_{di}$ = drilling time, hrs, $T_{ti}$ = trip time, hrs, $T_{ci}$ = connection time, and $\Delta D_i$ = the formation interval drilled, ft, by bit number $i$.

The trip time is given as

$$T_{ti} = 2\left(\frac{t_s}{L_s}\right)D_i, \tag{12.3}$$

where $T_{ti}$ = the trip time required to change a bit and resume drilling operations, hrs, $t_s$ = the average time required to handle one stand of drill string, hrs, $L_s$ = the average length of one stand of drill string, ft, and $D_i$ = the depth where the trip was made, ft.

Assuming that the average bit life for a given bit group in a given hole interval is $T_b$, the depth of the next trip is given by the following equation:

$$D_{i+1} = \frac{1}{a}\ln(ak\overline{T}_b + e^{aD_i}), \tag{12.4}$$

Penetration rate can be given as

$$\frac{dD}{dT_d} = ke^{-aD},$$

where k and $a$ are constants and must be determined from past field data.

The drilling time it takes for a given bit to drill from depth $D_i$ to depth $D_i + L$ may also be given as

$$T_{di} = \frac{1}{ka}[e^{aD_{i+1}} - e^{aD_i}]. \tag{12.5}$$

**Problem 12.1**

Using the following data and the data listed in Table 12.1 for different bits with their respective drilling performance, determine the cost analysis and select the bit that will result in the lowest drilling cost.

- Operating cost of the rig is \$12000/day.
- Trip time is 10 hours.
- Connection time is 1 minute per connection.

**Table 12.1** Bit performance.

| Bit | Bit Cost ($) | Rotating Time (hours) | Connection Time (hours) | ROP (ft/hr) |
|---|---|---|---|---|
| A | 1000 | 15 | 0.1 | 14 |
| B | 3000 | 35 | 0.2 | 13 |
| C | 4000 | 45 | 0.3 | 10 |
| D | 4500 | 55 | 0.3 | 11 |

*Solution:*

The cost per foot for each of the bits can be calculated using equation 12.2.

For bit A,

$$C_f = \frac{1000 + \frac{12000}{24}(15 + 0.1 + 10)}{14 \times 15} = \$65/\text{ft}.$$

For bit B,

$$C_f = \frac{2000 + \frac{12000}{24}(35 + 0.2 + 10)}{13 \times 35} = \$54/\text{ft}.$$

For bit C,

$$C_f = \frac{4000 + \frac{12000}{24}(45 + 0.3 + 10)}{10 \times 45} = \$70/\text{ft}.$$

For bit D,

$$C_f = \frac{4500 + \frac{12000}{24}(55 + 0.3 + 10)}{11 \times 65} = \$52/\text{ft}.$$

**Problem 12.2**

Find the cost ($/ft) to drill between 9,000 ft and 10,000 ft for a prospect well whose data is given as follows:

- Well depth = 17,000 ft
- Casing setting for phases = 500 ft, 4000 ft, 9000 ft, 14,000 ft, and 17,000 ft

- Rig can handle triples, and the rental cost is $12,000/day.
- Tripping time per stand = 2.5 minutes

Bits used for different phases are given Table 12.2.
Control well data for this region is given below in Table 12.3.

The formation constants for the 3rd phase are given by $K$ = 300 and $a$ = 0.00035.

*Solution:*

Calculating the time to drill from 9,000 ft to 10,000 ft,

$$T_d = \frac{1}{K \cdot a} \cdot \left(e^{a \cdot D + \Delta D} - 1\right) - \frac{1}{K \cdot a} \cdot \left(e^{a \cdot D} - 1\right),$$

**Table 12.2** Bit use.

| Phase | Size(in) | Average Life(hours) | Cost($) |
|---|---|---|---|
| 1 | 22 | 40 | 4000 |
| 2 | 17 ½ | 35 | 3000 |
| 3 | 13 ½ | 24 | 2400 |
| 4 | 8 ⅜ | 20 | 2000 |
| 5 | 5 ¾ | 10 | 1500 |

**Table 12.3** Well data.

| Parameter | Value |
|---|---|
| D | 9,000 ft |
| ΔD | 1,000 ft |
| Ts | 2.5 min |
| Ts | 0.0417 hrs |
| Ls | 93 ft |
| Tb | 20 hrs |
| Cr | 12,000 $/day |
| Cr | 500 $/hour |
| Cb | 2,000 $ |

$$T_d = \frac{1}{(300)\cdot(0.00035)}\cdot\left(e^{(0.00035)(9,000+1,000)}-1\right)$$
$$-\frac{1}{(300)\cdot(0.00035)}\cdot\left(e^{(0.00035)(9,000)}-1\right),$$
$$= 93.14\,\text{hrs.}$$

Defining the trip time,

$$T_t = 2\cdot\left(\frac{t_s}{L_s}\right)\cdot D,$$

where $D$ is the mean depth, or

$$\frac{(D+\Delta D)+D}{2} = \frac{(9000+1000)+9000}{2} = 9500\,\text{ft.}$$

$$T_t = 2\cdot\left(\frac{0.0417}{93}\right)\cdot 9,500,$$
$$T_t = 8.52\ \text{hours/bit.}$$

The number of bits is

$$N_b = \frac{T_d}{T_b},$$

$$N_b = \frac{93.14}{20},$$

$$N_b = 4.66\ \text{bits (5 bits).}$$

Therefore,

$T_t$ = 8.52 (hours/bit) / 4.66 (bit).

Total trip hours = 8.8 x 4 = 35.2 hrs.

Determining the connection time, the average of stands is calculated as

$$N_s = \frac{\bar{D}}{L_s},$$

$$N_s = \frac{\frac{10,000+9,000}{2}}{93},$$

$$N_s = 102.15.$$

$$T_c = N_s \cdot T_s,$$

$$T_c = 102.15 \times 0.0417 = 4.26 \text{ hrs.}$$

Calculating the total cost of bits, the number of bits calculated is 4.66. Therefore, we will use 5 bits:

$$C_b = C_b \cdot N_b,$$

$$C_b = 2,000 \times 5 = 10,000 \text{ dollars.}$$

The total cost of the rental of the rig is

$$C_r = C_r \cdot T_d,$$

$$C_r = 500 \times 93.14 = 46,570 \text{ dollars.}$$

The cost per foot to drill 1,000 ft is calculated as

$$C_d = \frac{C_b + C_r \cdot (T_d + T_t + T_c)}{\Delta D},$$

$$C_d = \frac{10,000 + 500 \cdot (93.14 + 39.70 + 4.26)}{1,000} = 78 \text{ \$/ft.}$$

### 12.1.2 Coring Costs

Core recovery is given as the ratio length of the core recovered to the length of the core cut. It is usually expressed as a percentage. Similar to drilling costs, coring costs per foot can be given as

$$C_c = \left( \frac{C_b + C_r\left(t_t + t_r + t_{cc} + t_{rc} + t_c\right)}{\Delta D} \right) \frac{1}{R_c}, \quad (12.6)$$

where $C_b$ = core bit cost, \$, $C_r$ = rig daily rental cost, \$, $t_t$= trip time, hr, $t_r$ = rotating time, hr, $t_{cc}$ = connection time, hr, $t_{rc}$ = core recovery, laying down core barrel, hr, $t_c$ = coring time, hr, $R_c$ = core recovery percentage, and $\Delta D$ = the formation cored which is a function of rate of penetration,

$$\Delta D = \int_0^{t_r} \text{ROP}\, dt.$$

where ROP = rate of penetration.

**Problem 12.3**

A coring job is planned from 16,090 ft to 17,020 ft. Calculate the coring cost per foot with the following data:

- Trip time = 10 hr
- Rotating time = 7 hr
- Miscellaneous time = 1 hr
- Rig cost = 15,000 $/day
- Bit cost = $2,500

*Solution:*

Using equation 12.6, calculate the cost per foot of coring:

$$C_c = \left( \frac{2,500 + \frac{15,000}{24}(10+7+1)}{30} \right) \frac{1}{0.90} = 509 \ \$/\text{hr}.$$

## 12.2 Future Value (FV)

Future value is calculated as

$$\text{FV} = \text{PV}\left(1 + \frac{r}{n}\right)^{n \times m} \tag{12.7}$$

where FV = future value, PV = present value, $r$ = periodic interest rate or growth rate in fraction, $n$ = the number of payments per year, and $m$ = the number of years.

For continuous compounding,

$$\text{FV} = \text{PV}e^{r \times n}. \tag{12.8}$$

**Problem 12.4**

A company bought an external casing packer (ECP) five years ago for $10,000 and did not use it. What would its value be today to break-even assuming an average rate of return for the past five years is 14%.

*Solution:*

Past value = $10000.
Interest rate = 14% per year.
$n$ = 4 periods per year.

$$\mathrm{FV} = 10000\left(1+\frac{0.14}{4}\right)^{4\times5} = \$\,19897.$$

## 12.3 Expected Value (EV)

Expected value is calculated as

$$\mathrm{EV} = \sum_i p_i C_i,$$

$$\sum_i p_i = 1, \tag{12.9}$$

where $p_i$ = probability of the $i^{th}$ event, and $C_i$ = the cost of the $i^{th}$ event.

## 12.4 Price Elasticity

Price elasticity is used to measure the effect of economic variables such as demand or supply of rigs or wells drilled with respect to change in the crude oil price. It enables one to find out how sensitive one variable is with the other one, and it is also independent of units of measurement. It is the ratio of the percentage of change of wells and footage drilled to the percentage change in the crude price. It describes the degree of responsiveness of the rig in demand or rig in supply to the change in the crude price.

$$\text{Drilling Price Elasticity} = \frac{\text{Percentage change in drilling wells (R)}}{\text{Percentage change in crude oil price (P)}}. \tag{12.10}$$

Elasticity, $E$, is given as

$$E = \frac{\%\Delta R}{\%\Delta P} = \frac{\frac{dR}{R}}{\frac{dP}{P}} = \frac{dR}{dP} \times \frac{P}{R}. \tag{12.11}$$

**Problem 12.5**

If the oil price increases from \$30.00 to \$34.00 and if the drilling rigs increase from 1220 to 1240, then the elasticity of drilling rigs would be calculated as

$$\frac{\frac{(1120-1140)}{1120} \times 100}{\frac{(30-34)}{30} \times 100} = \frac{1.79\,\%}{13.33\,\%} = 0.133.$$

The midpoint formula is another method and gives the answer regardless of the direction of change:

$$\text{Price Elasticity} = \frac{(R_2 - R_1)/[(R_2 + R_1)/2]}{(P_2 - P_1)/[(P_2 + P_1)/2]},$$

$$E = \frac{\frac{(1140-1120)}{1130}}{\frac{(34-30)}{32}} = 0.14.$$

## 12.4.1 Ranges of Elasticity

In order to compare the calculated elastic values, the elasticity can be classified as follows:

1. Inelastic: Elasticity < 1
   The number of drilling rigs does not respond strongly to the oil price.
2. Elastic: Elasticity > 1
   The number of drilling rigs responds strongly to the oil price.

3. Perfectly Inelastic: Elasticity = 0
   The number of drilling rigs does not respond to the oil price change.
4. Perfectly Elastic: Elasticity $\alpha$
   The number of drilling rigs responds infinitely to the oil price change.
5. Unit Elastic: Elasticity = 1
   The number of drilling rigs responds by the same percentage as the oil price change.

**Problem 12.6**

An oil company is planning to drill wells where the prospective formations are classified as prospect 1, 2, and 3. The probabilities of striking oil in the three prospective formations are 0.30, 0.45, and 0.25, respectively. From the offset wells drilled earlier, striking oil is 40% in the prospect 1 formation, 20% in the prospect 2 formation, and 30% in the prospect 3 formation. If the oil is struck at this site, determine the probability of it being from the prospect 1 formation.

O = "Oil"

*Solution:*

| Prospect | Probability assigned of striking oil | Probability of striking oil in offset wells | | Probability of striking oil | Probability of it being from the respective formation |
|---|---|---|---|---|---|
| P(I) | 0.3 | *P(O* \| *I)* | 0.40 | 0.120 | 0.421 |
| P(II) | 0.45 | *P(O* \| *II)* | 0.20 | 0.090 | 0.316 |
| P(III) | 0.25 | *P(O* \| *III)* | 0.30 | 0.075 | 0.263 |

Or,

$$P(I/O)=\frac{0.4\times0.3}{0.4\times0.3+0.20\times0.45+0.30\times0.25}=0.421.$$

# Appendix: Useful Conversion Factors

### Length

1 meter = 39.37 in
1 inch = 2.54 cm
1 feet = 30.48 cm = 0.3048 m
1 mile = 5280 ft = 1720 yard = 1609.344 meter
1 nautical mile = 6076 ft

### Mass

1 lbm = 453.6 g = 0.4536 kg = 7000 gr (grain)
1 Kg = 1000 g = 2.2046 lbm
1 slug = 1 lbf $s^2/ft$ = 32.174 lbm
1 US ton = 2000 lbm (also called short ton)
1 long ton = 2240 lbm (also called British ton)
1 tonne = 1000 kg (also called metric ton) = 2204.6 lbm
1 kip = 1000 lb

### Force

1 lbf = 4.448 N = $4.448 \times 10^5$ dynes
1 lbf = 32.174 poundals = 32.174 lbm $ft/s^2$

**Gravitational Acceleration**

$g = 32.2 \text{ ft/s}^2 = 9.81 \text{ m/s}^2$

$$g_\lambda = 9.7803267714\left(\frac{1+0.00193185138639\sin^2\lambda}{\sqrt{1-0.00669437999013\sin^2\lambda}}\right)\text{m/s}^2,$$

where λ = the geographic latitude of the earth ellipsoid measured from the equator in deg.

**Pressure**

1 atm = 14.69595 psia = 2118 $\text{lbf/ft}^2$
= 29.92 in. Hg = 760 mm = 1.013 bars
= 33.93 ft $H_2O$ = 1.013 × 10 Pa = 101.3 kPa
1 Pa = 1 $\text{N/m}^2$ = $10^{-5}$ bars

**Volume**

1 $\text{ft}^3$ = 7.481 US gal = 6.31 Imperial gal = 28.316 L
1 $\text{m}^3$ = 1000 L = $10^6$ $\text{cm}^3$ = 264.2 US gal = 35.31 ft = 264.2 gal = 35.31 $\text{ft}^3$
1 bbl = 42 US gal = 5.61 $\text{ft}^3$
1 bbl = 9694.08 $\text{in}^3$

**Density**

Water = 62.4 $\text{lbm/ft}^3$ = 1000 $\text{kg/m}^3$ = 1 $\text{g/cm}^3$ = 8.33 lbm/US gal
°API, 60° F = (141.5/SG,60°F) –. 131.5
SG, 60° F = (141.5)/(°API,60°F + 131.5)

**Velocity**

1 knot = 1 nautical mile/hr
60 mph = 88 ft/s

$$\left(\frac{\text{lb} \times \text{s}^n}{\text{ft}^2}\right) = 0.002088543 \times \text{eq.cP}$$

**Temperature**

°F = 1.8 (°C) + 32
°R = °F + 459.67 = 1.8 (K)

**Energy**

1 J = 1 kg m²/s² = $10^7$ dyne
1 BTU = 777 ft lbf = 252 cal = 1055 J
1 hp hr = 2545 Btu
1 kW hr = 3412 Btu = 1.341 hp hr

**Power**

1 hp = 550 ft lbf/s = 33,000 ft lbf/min

**Gas Constant**

R = 1.987 Btu/lb mol °R = 1.987 cal/g mol K
= 0.7302 atm $ft^3$/lb mol °R = 1545 ft lbf/lb mol °R
= 0.08206 L atm/g mol K
= 82.06 atm $cm^3$ mol K
= 8314 Pa $m^3$ mol K ox. J/kg mol K
= 8.314kJ/kgmOl K

**Viscosity**

1 cP = 0.01 Poise = 0.01 g/cm s = 0.01 dyne s/cm
= 0.001 kg/ms = mPa s = 0.001 N s/$m^2$
= 2.42 lbm/ft hr = 0.0752 slug/ft hr
= $6.72 \times 10^{-4}$ lbm/ft s
= $2.09 \times 10^{-5}$ lbf s/$ft^2$
1 Pa s = 0.0209 lbf s/$ft^2$ = 0.672 lbm/ft s

$$\left(\frac{\text{lb} \times \text{s}^n}{\text{ft}^2}\right) = 0.002088543 \times \text{eq.cP}$$

$$\left(\frac{\text{lb} \times \text{s}}{\text{ft}^2}\right) = 4.79 \times 10^4 \,\text{cP}$$

**Acceleration Due to Gravity**

$g$ = 32.2 ft/$s^2$ = 9.81 m/$s^2$

**Trigonometric Relationships**

$$\csc\theta + \cot\theta = \frac{1+\cos\theta}{\sin\theta}$$

$\csc\theta + \cot\theta = \cot(\theta/2)$
$\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$
$\cos\theta = \cos^2(\theta/2) - \sin^2(\theta/2)$
$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$$\sin\theta = \frac{2e^t}{1+e^t} = \frac{2}{e^t + e^{-t}}$$

$$\cos\theta = \frac{1-e^{2t}}{1+e^{2t}} = \frac{e^{-t} - e^t}{e^t + e^{-t}}$$

$\sin\theta = \operatorname{sech} t$
$\cos\theta = -\tanh t$
$\cosh^2 t - \sinh^2 t = 1$
$\tanh^2 t + \operatorname{sech}^2 t = 1$

$$\cosh t = \frac{e^t + e^{-t}}{2}$$

$$\sinh t = \frac{e^t - e^{-t}}{2}$$

$\sinh^2\theta + \cosh^2\theta = \cosh 2\theta$
$\cosh(a \pm b) = \cosh a \cosh b \mp \sinh a \sinh b$

$$\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

**Trigonometric Approximations**

$\sin\theta \cong \theta$

$\cos\theta \cong 1$

For higher order approximation,

$$\cos\theta \cong 1 - \frac{\theta^2}{2}.$$

### Useful Relation

$a^x = e^{x \ln a}$

### Integration Techniques

Integration by parts,

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du.$$

### Taylor Series

If $f(c)$ is a continuous function in an open interval, it's value at neighboring points can be expressed in terms of the Taylor series as

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c) = f(c) + f'(c)(x-c) + f''(c)\frac{(x-c)^2}{2!}$$

$$+ \ldots\ldots + f^{(n)}(c)\frac{(x-c)^n}{n!} + \cdots,$$

where $f^n$ denotes $n^{\text{th}}$ derivative.

If $c = 0$, then the series is Maclaurin series for $f$.

### Prefix Scale (Figure A.1)

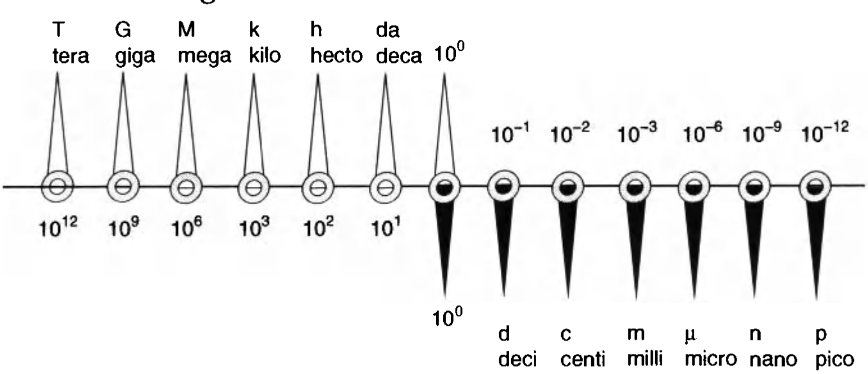



**Figure A.1** Metric prefix scale.

## SI Metric Conversion Factors

| | |
|---|---|
| bbl/ft × 5.216 | E − 01 = $m^3$/m |
| cP × 1.0 | E − 03 = Pa.s |
| ft × 3.048 | E − 03 = m |
| gal × 7.46043 | E − 03 = $m^3$ |
| hp × 7.46043 | E − 01 = kW |
| in. × 2.54 | E + 00 = cm |
| lbf × 9.869233 | E − 00 = Nm |
| lbf/$ft^2$ × 4.788026 | E − 02 = kPa |
| lbm × 4.535924 | E − 01 = kg |
| lbm/$ft^3$ × 1.601846 | E + 01 = kg/$m^3$ |
| lbm/gal × 1.198264 | E + 02 = kg/$m^3$ |
| md × 6.894757 | E − 04 = $\mu m^2$ |
| psi × 6.894757 | E + 00 = kPa |
| psi/ft × 2.262059 | E + 01 = kPa/m |
| sq.in × 6.451 | E + 00 = $cm^2$ |

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