Sylvester Okotie · Bibobra Ikporo

Reservoir Engineering

Fundamentals and Applications

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This book is dedicated to God Almighty for His continuous protection and for divine ideas given to us to write. Also to my lovely son, Andre Oghenemarho Ononeme Sylvester, for being such an inspiration. Also to a great uncle, Ebipuado Sapreobi, for his continuous encouragement and reassurance.

Foreword

I am delighted to write the foreword for the first edition of the *Reservoir Engineering: Fundamentals and Applications*. Having read through the content of this book as a teacher of petroleum engineering for several years, I am proud to submit that the authors have been able to provide a practical resource solution manual for academia, government advisers on oil and gas matters, the practical professionals in the oil and gas industry, and the general knowledge seekers.

The content is logically arranged, and each chapter contains practical-based background information emphasizing core areas in reservoir engineering, viz., hydrocarbon reserves classification, methods of estimating hydrocarbon reserves, aquifer fitting, material balance, decline curve analysis, inflow performance relationship, history matching, and reservoir performance prediction, among others.

The layout of each chapter includes learning objectives, abstract with keywords, nomenclature, detailed write-up of the title in its simplest form, many solved examples, and a concluding set of self-assessment questions designed to highlight and reinforce the materials in the chapter and to test the reader's understanding of the subject matter.

A lot of effort has gone into the write-up of this volume to tell the story in an intriguing and visually appealing way to make it useful and an efficient reference material for all readers. The illustrations are generous in both size, layout, and quality for maximum clarity.

I think that the authors are confident that there will be many grateful readers who will gain a broader perspective of the discipline of reservoir engineering. It is, therefore, my hope and expectation that this book will immensely provide an effective learning experience and reference resource for all readers.

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Steve Adewole

Preface

This book has been written for those who desire to have a proper understanding and grasp of *Reservoir Engineering: Fundamentals and Applications*. This book provides relevant details in logical arrangement to teach/train undergraduate and master students, postdoctoral fellow, scientists, and new employees, a refresher course to experienced engineers who are already practicing in the field and layperson who will find a body of enjoyable and useful information within the covers of this book.

To make this book more comprehensive in treating reservoir engineering fundamentals and applications, the suggestions of some of our friends and colleagues have been incorporated into the various chapters of this book. There are 11 chapters with several examples and self-assessment exercises in this book, each chapter covering a different aspect of reservoir engineering.

Chapter 1 introduces the essential features of a reservoir, the hydrocarbon phase envelope, types of reservoir fluid, flow regime, and reservoir geometry. Chapter 2 deals with the classification of hydrocarbon resources and reserves; Chapter 3 is devoted to the volumetric method of reserves estimation. Chapter 4 covers the various models for determining the amount of water encroaching the reservoir; Chapter 5 deals with material balance equation for oil and gas under different reservoir condition and primary reservoir drive mechanisms, while Chap. 6 deals with the straight-line forms of material balance equations in Chap. 5. Chapter 7 is devoted to decline curve analysis; Chapter 8 covers pressure regimes and fluid contacts. Chapters 9, 10, and 11 deal with inflow performance relationship, history matching, and reservoir performance prediction.

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Acknowledgments

This book is well written in its simplest form of understanding of the subject and the style we deliver our lectures to students. Though it might not be a smooth ride in trying to deliver the objectives of each chapter to them with rigorous assignments. We want to sincerely appreciate our students at the Federal University of Petroleum Resources and Niger Delta University over the years we have taught this course for their constructive criticism and correction of several mathematical errors. We are certain that in no distance time, they will appreciate the knowledge we have impacted.

Most of the materials were gotten from educators, lecture notes, engineers in the industry, online, manuals, and authors who have made numerous and significant contributions to the subject matter of reservoir engineering. We sincerely acknowledge their meaningful contributions.

Special appreciation goes to our lovely spouses, Mrs. Jessica O. Okotie and Engr. Yanayon Ikporo, for their encouragement, fortitude, and extraordinary understanding, which enabled us to steal many hours from our families while writing this book. We appreciate our parents, Mr. and Mrs. Daniel Okotie and Ambassador and Mrs. Felix Oboro, for providing us with education, inspiration, and confidence.

Special appreciation goes to our friends and colleagues in academia and the oil and gas industry for their constructive criticism and sharing their knowledge and experience based on their understanding of the subject matter. We appreciate Steve Ogiri for helping us with the drawing of some figures and Nengi Fiona Haribi-Botoye, Solomon Osazuwa, and Sandra Oziomachukwu Ibegbule for mathematical error checking.

We also want to thank the Federal University of Petroleum Resources, Niger Delta University, and the Department of Petroleum Engineering at both universities for giving us the platform to be teaching our students this course.

We acknowledge Springer Book Publishing for their confidence and valuable assistant in publishing this book, particularly to Amanda Quinn for initiating this project. I would like to thank the editorial staff—in particular, Kiruthika Kumar of SPi Global—for their work and professionalism. We sincerely appreciate Kanimizhi

Sekar for typesetting and production of this book. This edition of the book could not have been completed without the meaningful contributions of Prof. Steve Adewole and Ogbarode N. Ogbon.

We would like to express our sincere appreciation to all those who have in one way contributed to the success of this book, for their valuable contributions and encouragement to make sure this text is a reality.

However, the authors will be grateful to welcome constructive suggestion where necessary to improve the quality of this book and to advance knowledge. Please send such relevant information to okotie.sylvester@gmail.com, bblaye3@gmail.com and okotie.sylvester@fupre.edu.ng.

Contents

Abbreviations

AOF	Absolute Open Flow
API	American Petroleum Institute
BHP	Bottom Hole Pressure
BOPD	Barrel of Oil Per Day
BV	Bulk Volume
CBV	Cumulative Bulk Volume
DDI	Depletion Drive Index
deg	Degree
DST	Drill Stem Test
FBHP	Flowing Bottom Hole Pressure
FDI	Formation Drive Index
FGIIP	Free Gas Initially in Place
FVF	Formation Volume Factor
GDI	Gas Cap Drive Index
GIIP	Gas Initially in Place
GOC	Gas Oil Contact
GOR	Gas-Oil Ratio
GOV	Gross Oil Sand Volume Flooded by Water
GOVg	Gross Gas Sand Volume Displaced by Gas Cap
GRV	Gross Rock Volume
GWC	Gas Water Contact
IPR	Inflow Performance Relationship
MBE	Material Balance Equation
MM	Million
MMrb	Million Reservoir Barrel
OHCIP	Original Hydrocarbon in Place
OOIP	Original Oil in Place
OWC	Oil Water Contact
P.U	Planimeter Unit
Pc	Critical Pressure

Pcb	Cricondenbar
Pct	Cricondentherm Pressure
PI	Productivity Index
PM	Primary Migration
PRMS	Petroleum Resource Management System
PSS	Pseudo-Steady State
PT	Pressure-Temperature
PV	Pore Volume
PVT	Pressure Volume Temperature
rb	Reservoir Barrel
RF	Recovery Factor
RFT	Repeat Formation Tester
SBHP	Shut-in Bottom Hole Pressure
SCF	Standard Cubic Feet
SDI	Segregation Drive Index
SM	Secondary Migration
SSS	Semi-Steady State
STB	Stock Tank Barrel
STC	Stock Tank Condition
STOIIP	Stock Tank Oil Initially in Place
Tc	Critical Temperature
Tcb	Cricondenbar Temperature
Tct	Cricondentherm
TOC	Total Organic Carbon
TUW	Total Underground Withdrawal
WDI	Water Drive Index
WOR	Water-Oil Ratio

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Chapter 1
Introduction

Chapter Learning Objectives
At the end of this chapter, the students/readers should be able to:

- Understand what a petroleum reservoir is and its essential features
- Understand the job description of reservoir engineering
- Understand the concept of drainage and imbibition process
- Understand the hydrocarbon phase envelope and all its associated terminologies
- Identify various types of reservoir fluids and their respective phase envelope/diagrams
- Know the types of fluids in terms of flow regime and reservoir geometry and write the mathematical equations representing these flow.
- Understand the productivity index and factors that affect it.
- Perform basic calculations on the different flow regimes for oil and gas reservoirs
- Calculate the reservoir pressure at a specific radius and time under transient flow conditions

Nomenclature

Parameter	Symbol	Unit
Permeability	K	*mD*
Reservoir thickness	h	*ft*
Viscosity	μ	*cp*
Bottom hole (wellbore) flowing pressure	P_{wf}	*psia*
Average reservoir (Drainage) pressure	P_e	*psia*
Oil formation volume factor	β_o	*rb/stb*
Gas formation volume factor	β_g	*cuft/scf*

(continued)

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Parameter	Symbol	Unit
Wellbore radius	r_w	*ft*
Reservoir radius	r_e	*ft*
Skin	s	–
Flow rate	q	*stb/day*
Area	A	*acre*
Gas deviation/compressibility factor	z	–
Length	L	*ft*
Oil compressibility	C_o	$psia^{-1}$
Standard pressure	P_{sc}	*psia*
Standard temperature	T_{sc}	°*R*
Reservoir temperature	T	°*R*
Time	t	*s, min, h*
Total fluid compressibility	C_t	$psia^{-1}$
Pressure drop due to skin	ΔP_s	*psia*
Initial reservoir pressure	P_i	*psia*
Porosity	θ	*– or %*
Radius (distance)	r	*ft*
Dimensionless pressure	P_D	–
Dimensionless radius	r_D	–
Dimensionless time	t_D	–
Productivity index	*PI or j*	*stb/day/psia*
Specific productivity index	j_s	*stb/day/psia*
Shape factor	C_A	–

1.1 Introduction

Petroleum Engineering is one of the key aspects of Engineering that is concern with the exploration and production of hydrocarbons from subsurface formations via the wellbore (a hole drilled) to the surface storage facilities for consumption by human or to meet the host country's or global energy needs, it is a broad discipline that has several areas of specializations such as Petroleum geology, Petrophysics, Drilling, Mud and Cementing, Reservoir, Production (surface & subsurface), Completion, Formation evaluation, Economics etc. Thus, all of these areas of specialty work together as an integrated team to achieve one goal; to recover the hydrocarbon in a safe and cost-effective way. Therefore, this book presents a key aspect "reservoir engineering" of petroleum engineering.

1.2 Definition of a Reservoir

A petroleum reservoir is a porous and permeable subsurface pool or formation of hydrocarbon that is contained in fractured rocks which are trapped by overlying impermeable or low permeability rock formation (cap rock, that prevents the vertical movement) and an effective seal (water barrier to prevent the lateral movement of the hydrocarbon) by a single natural pressure system. Figure 1.1 shows clearly the essential features of a reservoir which are: source rock, cap rock (non-permeable rock), reservoir (porous and permeable rock) rock, hydrocarbon (oil and gas) and aquifer (water sand).

1.2.1 Elements Required in the Definition of a Reservoir

The definition of a reservoir is not complete without mentioning the following: the source rock, migration pathway, reservoir rock which talks about porosity and permeability, cap rock, trap and a seal. These are briefly explained below.

1.2.1.1 Source Rock Hydrocarbon Generation

This is a rock in which hydrocarbon is generated from or has generated moveable quantities of hydrocarbon. It is a site where hydrocarbon liquid is formed from an organic-rich source rock with kerogen (Fig. 1.2, a precursor of petroleum) and bitumen to accumulate as oil or gas or a combination of both oil and gas.

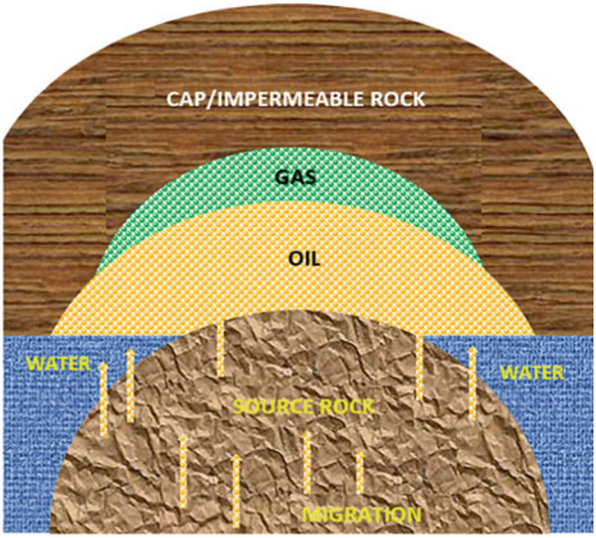


Fig. 1.1 Essential feature of a reservoir. (Source: geologylearn.blogspot.com)

Fig. 1.2 Kerogen. (Source: scientificamerican.com)

To characterize a rock as source rock, the following basic features need to be in place:

- The quantity of organic matter which is commonly assessed by a measure of the total organic carbon (TOC) contained in a rock.
- The quality which is measured by determining the types of kerogen contained in the organic matter and prevalence of long-chain hydrocarbons.
- The thermal maturity; usually estimated by using data from pyrolysis analysis.

Therefore, hydrocarbon generation is a critical phase in the development of a petroleum system which depends on three main factors:

- The presence of organic matter rich enough to yield hydrocarbons,
- Adequate temperature,
- And sufficient time to bring the source rock to maturity. On the contrary, pressure of the system, the presence of bacteria and catalysts also affect the hydrocarbon generation.

1.2.1.2 Migration

Usually, the sites where hydrocarbons are formed are not the same sites where they are accumulated to form a reservoir. They must travel a long distance before they are eventually trapped. Hence, migration can be defined as the movement of hydrocarbons from the source rock into the reservoir rocks. Hydrocarbon migration can be classified further as primary and secondary. When the newly generated hydrocarbons move out of their source rock to the reservoir rock, it is termed primary migration, also called expulsion. While the further movement of the hydrocarbon within the reservoir or area of accumulation is called secondary migration as shown in Fig. 1.3.

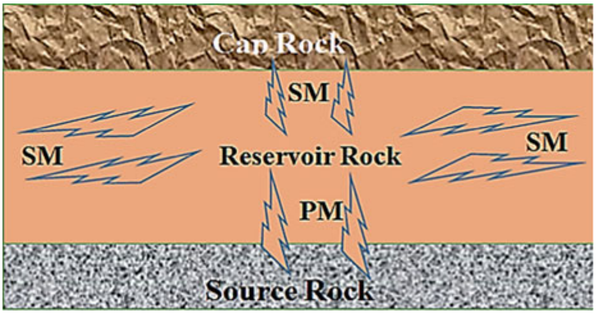


Fig. 1.3 Hydrocarbon migration. PM primary migration, SM secondary migration

1.2.1.3 Accumulation

It is the quantity of hydrocarbon that has gradually gathered or defined as the phase in the development of a petroleum system during which hydrocarbons migrate into the porous and permeable rock formation (the reservoir) and remain trapped until wells are drilled through to produce the accumulated hydrocarbons.

1.2.1.4 Porosity

This is the storage capacity of the rock to host the migrated hydrocarbon from the source rock. It can be defined as the fraction of the bulk volume of the rock that is void or open for fluid to be stored.

1.2.1.5 Seal/Cap Rock

Cap rock is a harder or more resistant rock type overlying a weaker or less resistant rock type. It is an impermeable rock that acts as a barrier to further migration of hydrocarbon liquids. The cap rock prevents vertical migration while seal prevents lateral migration of the hydrocarbon. A capillary seal is formed when the capillary pressure across the pore throats is greater than or equal to the buoyancy pressure of the migrating hydrocarbons. They do not allow fluids to migrate through them until their integrity is disrupted, causing them to leak. Sometimes the caps are not perfect seals and petroleum escapes to the Earth's surface as natural seepage, which can be spotted by oily residue on the surface soil and rocks (geologic survey). Underwater seeps can bubble up to the surface and leave an oily sheen.

1.2.1.6 Trap

This term is defined as a subsurface rock formation sealed by a relatively impermeable formation through which hydrocarbons will not migrate (Fig. 1.4). It is formed only when the capillary forces of the sealing medium cannot be overcome by the

Fig. 1.4 Trap

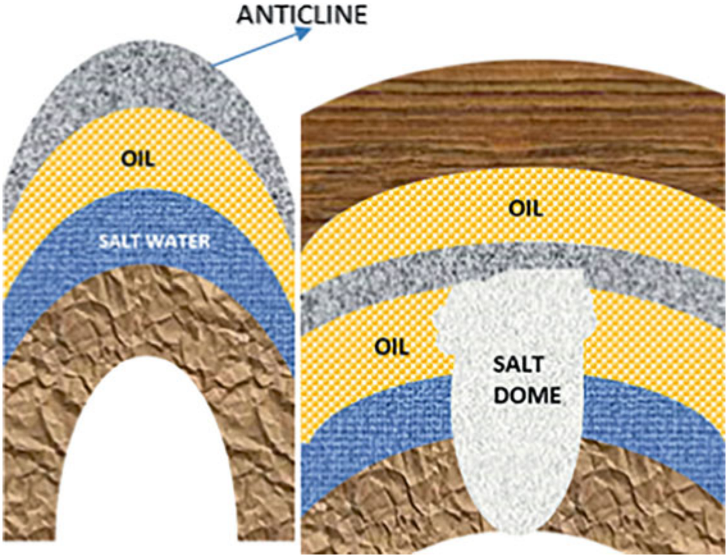


Fig. 1.5 Structural trap

buoyant forces responsible for the vertical/upward movement of the hydrocarbon through the permeable rock. There are several types of traps encountered, which can be represented as single, parallel, sealing and non-seal.

Traps can be described as structural traps, which are formed in geologic structures such as folds and faults. structural traps are formed chiefly as a result of changes in the structure of the subsurface rock, which may be caused by compaction, tectonic, gravitational processes or due to processes such as uplifting, folding and faulting, culminating to the formation of anticlines, folds and salt domes. Majority of the world's petroleum reserves are found in structural traps. These are shown in Fig. 1.5.

The other type of trap is the stratigraphic traps which are formed as a result of changes in rock type or pinch-outs, unconformities, or other sedimentary features such as reefs or build-ups. It can also be seen as traps formed as a result of lateral and vertical variations in the thickness, texture, porosity or lithology of the reservoir rock.

1.2.1.7 Permeability

This is defined as the ease at which the reservoir fluid flows through the porous space of the reservoir rock to the surface when penetrated by a well.

1.2.1.8 Reservoir

For the hydrocarbons that migrated from the source rock to accumulate, there must exist a subsurface body of rock (reservoir rock) having sufficient porosity to host or store the migrated hydrocarbons and also permeable enough to transmit the fluids when penetrated by a well. Therefore, a reservoir is a porous and permeable subsurface formation containing an accumulation of producible hydrocarbons (Oil and/or Gas), characterized by a single natural pressure system that is confined by impermeable rock and water barriers.

The reservoir rocks are mostly sedimentary in nature because they are more porous than most igneous and metamorphic rocks. See details in understanding the basis of rock and fluid properties textbook written by one of the same authors.

Prior to the formation of the hydrocarbon, the reservoir was actually filled with water. This will lead us to the concept of drainage and imbibition processes discussed below.

1.3 Drainage and Imbibition Process

1.3.1 Drainage/Desaturation Process

It is generally agreed that the pore spaces of reservoir rocks were originally filled with water, as hydrocarbon is being formed from the source rock, it migrates or moves into the reservoir, where it displaces the water and leave some fraction called connate or irreducible water undisplaced. Hence, when the reservoir is discovered, the pore spaces are filled with connate water and oil saturation respectively. If gas is the displacing agent, then gas moves into the reservoir, displacing the oil.

This same history must be duplicated in the laboratory to eliminate the effects of hysteresis. The laboratory procedure is performed by, saturation of the core with brine or water, then displace the water to a residual or connate water saturation with oil after which the oil in the core is displaced by gas. This flow process is called the gas drive depletion process. In the gas drive depletion process, the nonwetting phase fluid is continuously increasing with increase in saturation, and the wetting phase fluid is continuously decreasing. Therefore, drainage process is a fluid flow process in which the saturation of the nonwetting phase increases and also, the mobility increases with the saturation of the nonwetting phase.

Examples of drainage process (Onyekonwu MO, lecture note):

- Hydrocarbon (oil or gas) filling the pore space and displacing the original water of deposition in water-wet rock
- Water flooding an oil reservoir in which the reservoir is oil wet
- Gas injection in an oil or water wet oil reservoir
- Evolution of a secondary gas cap as reservoir pressure decreases

1.3.2 Imbibition/Resaturation Process

The imbibition process is performed in the laboratory by first saturating the core with the water (wetting phase), then displacing the water to its irreducible (connate) saturation by injection oil. This “drainage” procedure is designed to establish the original fluid saturations that were found when the reservoir was discovered. The wetting phase (water) is reintroduced into the core and the water (wetting phase) is continuously increased. This is the imbibition process and is intended to produce the relative permeability data needed for water drive or water flooding calculations. Therefore imbibition process is a fluid flow process in which the saturation of the wetting phase increases and also, the mobility increases with the saturation of the wetting phase.

Examples of imbibition process (Onyekonwu MO, lecture note):

- Accumulation of oil in an oil-wet reservoir
- Water flooding an oil reservoir in which the reservoir is water wet
- Accumulation of condensate as pressure decreases in a dew point reservoir Figure 1.6 schematically illustrates the difference in the drainage and imbibition processes of measuring relative permeability. It is noted that the imbibition technique causes the nonwetting phase (oil) to lose its mobility at higher values of water saturation than the drainage process does. The two processes have similar effects on the wetting phase (water) curve. The drainage method causes the wetting phase to lose its mobility at higher values of nonwetting-phase saturation than does the imbibition method.

1.4 Reservoir Engineering

It is a branch of petroleum engineering that applies scientific principles to the drainage problems arising during the development and production of oil and gas reservoirs to obtain a high economic recovery. The reservoir engineer is saddled with the responsibility like that of a medical doctor to make sure the reservoir does not go below its expected performance (fall sick) and even if it falls sick; he/she looks for a

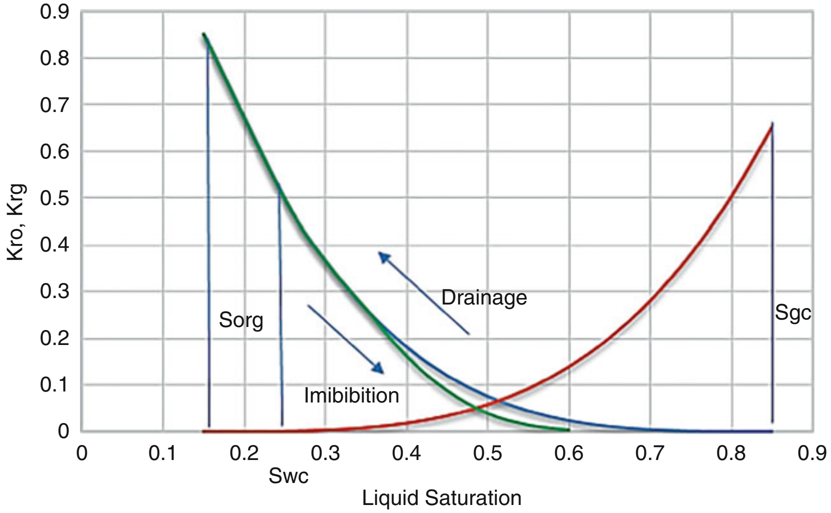


Fig. 1.6 Drainage-imbibition curve

way to bring it back to full performance throughout the entire life of the reservoir or project. Therefore, it requires an integrated input of all aspects of petroleum engineering; starting from exploration, drilling engineering to production engineering as depicted in Fig. 1.7.

1.4.1 Role or Job Description of Reservoir Engineers

Since it is usually not possible to physically ascertain what is under the ground because nobody goes into the reservoir, it implies that a Reservoir Engineer needs some techniques to adequately establish what is inside the reservoir. Therefore, it is the role of a reservoir engineer to continuously monitor the reservoir, collect relevant data and interpret these data to be able to determine the past and present conditions of the reservoir, estimate future conditions and control the flow of fluids through the reservoir rock with an aim to effectively increase recovery factor and accelerate oil recovery. It is worthy to note that the complete role/job description of a reservoir engineer to a company differs considerably from other companies, but there are key functions that are common to all. Some of the jobs description of a reservoir engineer but are not limited are stated below:

- Estimation of the original hydrocarbon in place (OHCIP)
- Calculation of the hydrocarbon recovery factor, and
- Attachment of a time scale to the hydrocarbon recovery

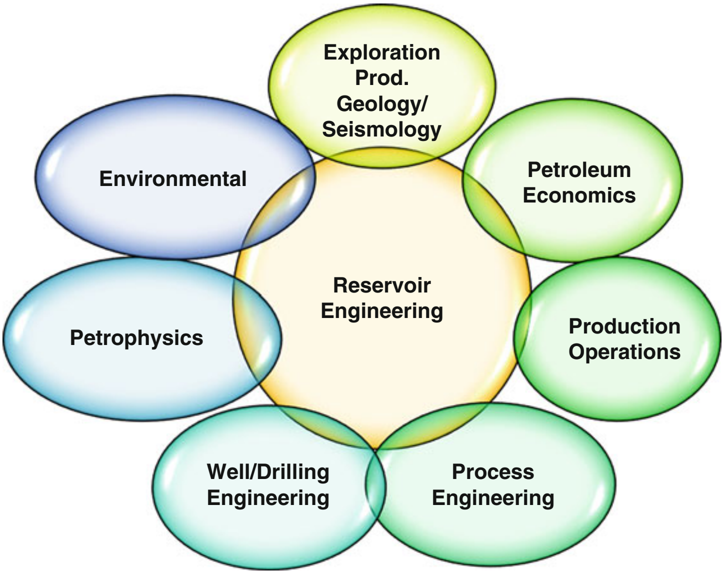


Fig. 1.7 Reservoir engineering and other aspects of petroleum engineering

- Good experience in constructing numerical reservoir simulation models (black-oil and compositional), model initialization, history matching, running sensitivities and predictions.
- Determination of reservoirs, field development strategy, production rates, reservoir monitoring plan, and economic life.
- Involvement of work with an integrated team of geologists, geophysicists, petrophysicists, and engineers from other disciplines.
- Knowledge of PVT data analysis.
- Collecting, analyzing, validating, and managing data related to the project
- Carrying out reservoir simulation studies, either for facts finding or to optimize hydrocarbon recoveries.
- Predicting reserves and performance from well proposals.
- Predicting and evaluating gas injection/waterflood and enhanced recovery performance.
- Developing and applying reservoir optimization techniques.
- Developing cost-effective reservoir monitoring and surveillance programs.
- Performing reservoir characterization studies.
- Analyzing pressure transients.
- Designing and coordinating petrophysical studies.
- Analyzing the economics and risk assessments of major development programs.
- Estimating reserves for producing properties.

1.4.2 Types of Reservoir

The classification of a hydrocarbon reservoir is basically dependent on the composition of the hydrocarbon mixture in the reservoir, the location of the initial pressure and temperature of the reservoir and the condition at the surface (separator) production pressure and temperature. A hydrocarbon reservoir can be classified as either oil black oil or volatile oil or condensate or natural gas (associated or non-associated) reservoirs. Since the hydrocarbon system has varying fluid compositions, to appropriately classify or identify the type of reservoir system, we need to understand the hydrocarbon phase envelope (pressure-temperature diagram).

1.4.3 Phase Envelope

According to Wikipedia, a phase envelope is a type of chart used to show conditions of pressure, temperature, volume etc at which thermodynamically distinct phases occur and coexist at equilibrium. Figure 1.8 depicts a phase envelope or pressure-temperature (PT) phase diagram of a particular fluid system. It comprises of two curves (bubble point and dew point curves) which encloses an area representing the pressure and temperature combinations for which both gas and liquid phases exist; called the two-phase region. The curves or quality lines converging at the critical

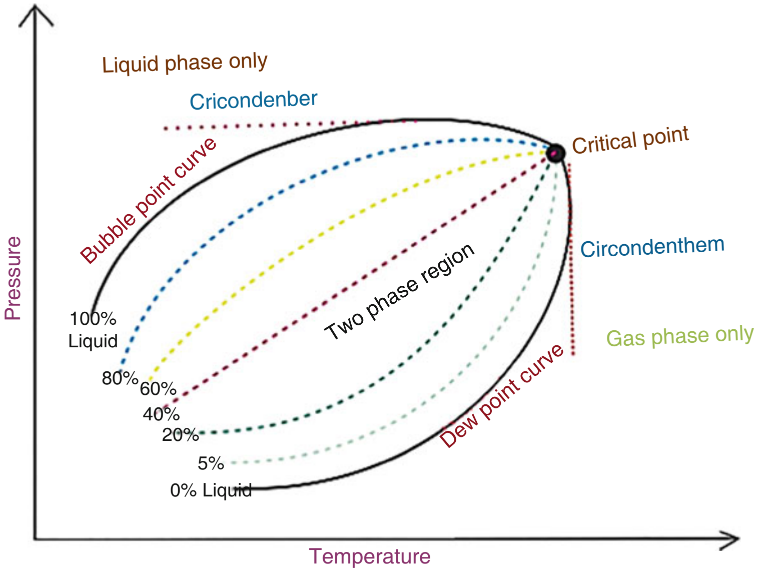


Fig. 1.8 Phase envelop

point within the two-phase envelope indicate the percentage of liquid at any given pressure and temperature of the total hydrocarbon volume of the reservoir.

Furthermore, on the phase envelope, we can place the various types of reservoirs depending on the location of the initial reservoir temperature and pressure with respect to the two-phase. Above the bubble-point curve in Fig. 1.8, we have a single liquid phase called an undersaturated reservoir while at a point beyond the dew point curve; a single gas phase occurs which may be a wet or dry gas reservoir. The various terms on the phase envelope are defined below.

1.4.3.1 Bubble-Point Curve

The bubble-point curve is defined as the line separating the liquid-phase region from the two-phase region and above which a single liquid phase exists as shown in Fig. 1.8. Note, if there is gas, it will be dissolved in the liquid.

1.4.3.2 Dew-Point Curve

The dew-point curve is defined as the line separating the vapor-phase region from the two-phase region and above which vapor phase exists as shown in Fig. 1.8.

1.4.3.3 Cricondentherm

The Cricondentherm (Tct) is defined as the temperature above which there is no existence of two-phase irrespective of the pressure or it can be defined as the maximum temperature above which a single gas phase exist and no liquid can be formed regardless of pressure (Fig. 1.8). The pressure corresponding to cricondentherm is known as the cricondentherm pressure (Pct).

1.4.3.4 Cricondenbar

The cricondenbar (Pcb) is defined as the pressure above which there is no existence of two-phase irrespective of the temperature or it can be defined as the maximum pressure above which a single liquid phase exists and no gas can be formed regardless of temperature (Fig. 1.8). The temperature corresponding to cricondenbar is known as the cricondenbar temperature (Tcb).

1.4.3.5 Critical Point

The critical point is the point where the bubble point curve, dew point curve and the quality lines converge (Fig. 1.8). At this point, one cannot distinguish between the liquid and gas properties. Hence it is referred to as the state of pressure and temperature at which all intensive properties of the gas and liquid phases are equal. The corresponding pressure and temperature at the critical point are referred to as the critical pressure (Pc) and critical temperature (Tc) of the mixture.

1.4.3.6 Quality Lines

These are dash lines enclosed by the bubble-point curve and the dew-point curve. They converge at the critical point. They also describe the pressure and temperature conditions for equal volumes of liquids as shown in Fig. 1.8.

1.4.3.7 Phase Envelope (Two-Phase Region)

This is the area enclosed by the bubble-point curve and the dew-point curve, wherein gas and liquid coexist in equilibrium; it is the region where we have the quality lines (Fig. 1.8). That is the region of greater than zero percent (0%) liquid and less than hundred percent (100%) on the phase envelope.

1.4.4 Oil Reservoirs

A reservoir can be classified as oil reservoir if the temperature of the reservoir is less than the critical temperature of the reservoir fluid. It can be further classified as a black oil or volatile oil depending on the gravity of the stock tank liquid usually the API of the crude. Also, it can be classified as undersaturated or saturated reservoir based on the location of the initial reservoir pressure.

1.4.4.1 Undersaturated and Saturated Reservoir

The fluid in the reservoir is a complex mixture of hydrocarbon molecules and as pressure and temperature reduces; that is the flow of hydrocarbon fluid from the reservoir condition to the surface separator, phase changes occur. Considering an undersaturated and a saturated reservoir as shown in Fig. 1.9 it can be seen that at the initial pressure, the reservoir is represented as a single liquid phase. As the pressure drops from the initial condition to the wellbore as a result of fluids production; the fluid remains as a single phase liquid at the wellbore. Therefore, a reservoir whose temperature is greater than the bubble point pressure is referred to as an "undersaturated reservoir".

As the pressure reduces further until it reaches the bubble point pressure (saturated pressure) where the first bubble of gas is evolved from the hydrocarbon mixture, the fluid still remains in a single liquid phase. Below the bubble point pressure, there is a two-phase region and with further reduction in pressure, the fluid is produced up the tubing and the amount of gas evolved increases until it reaches the separator. Thus,

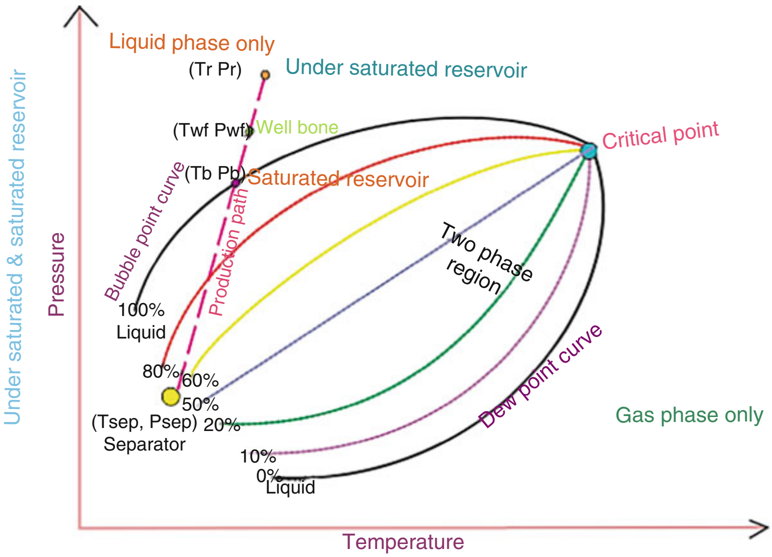


Fig. 1.9 Phase envelop of undersaturated and saturated reservoir

when the reservoir pressure is at or below the bubble point pressure, the reservoir is termed "saturated reservoir".

1.4.5 Types of Reservoir Fluids

1.4.5.1 Black Oil Reservoir

Figure 1.10 represents a black oil system which is made up of heavy hydrocarbons and non-volatile hydrocarbons. It is characterized by a dark or deep color liquid having initial gas-oil ratios of 500 scf/stb or less, oil gravity between 30° and 40° API. The pressure and temperature conditions existing in the separator indicate a high percentage of about 85% of liquid produced. The oil remains undersaturated within the region above the bubble point pressure, this means that the oil could dissolve more gas if present in the hydrocarbon mixture. At the bubble point pressure, the reservoir is said to be saturated and this implies that the oil contains the maximum amount of dissolved gas and cannot hold any more gas. Further reduction in pressure causes some shrinkage in the volume of oil as it moves from the reservoir (two-phase region) to the surface (separator). Therefore, black oil is often called low shrinkage crude oil or ordinary oil.

1.4.5.2 Volatile Oil Reservoir

A volatile oil reservoir is one whose reservoir temperature is below the critical point or critical temperature of the fluid as shown in Fig. 1.11. It contains relatively low

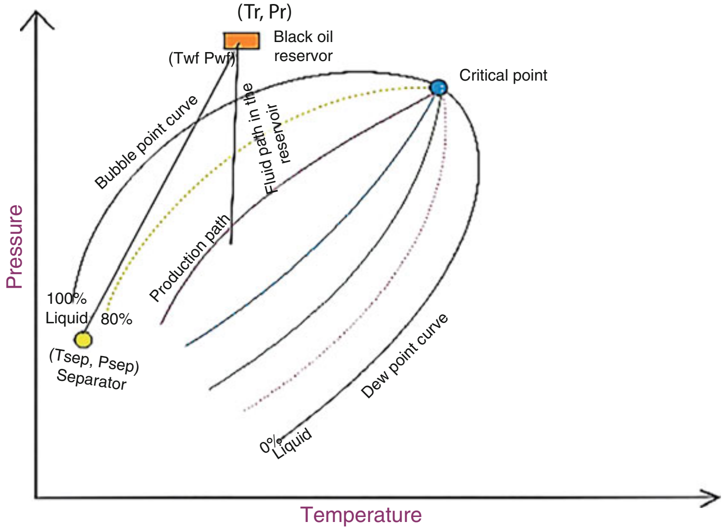


Fig. 1.10 Black oil reservoir

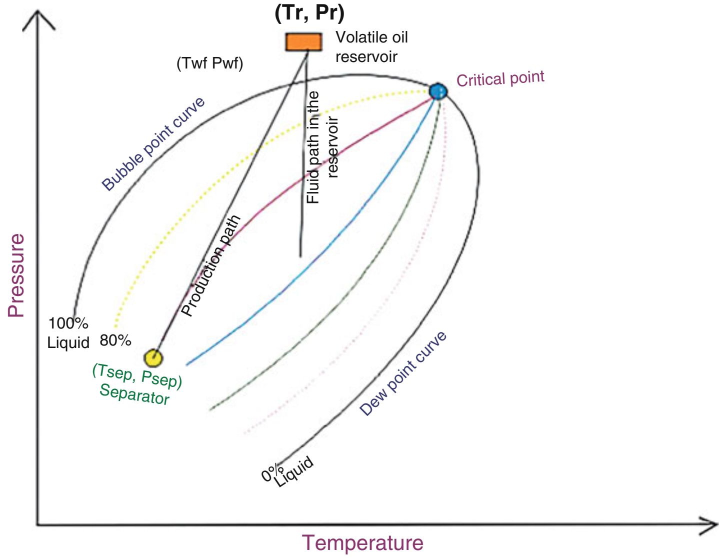


Fig. 1.11 Volatile oil reservoir

liquid content as it approaches the critical temperature, as compared to black oil reservoir that is far away from the critical point; a volatile oil reservoir is made up of fewer heavy hydrocarbon molecules and more intermediate components (ethane through hexane) than black oils. Volatile oils are generally characterized with stock tank gravity between 40° and 50° API, with a lighter color (brown, orange, or green) than black oil. In the case of volatile oil, 65% of the reservoir fluid is liquid at the separator condition. This means that relatively large volume of gas is evolved from the hydrocarbon mixture leaving a smaller portion as liquid. It is a high shrinkage oil as compared to black oil.

1.4.5.3 Condensate (Retrograde Gas)

A condensate reservoir fluid is a gas at the initial reservoir pressure. It occurs as shown in Fig. 1.12 when the temperature of the reservoir lies between the critical temperature and cricondentherm of the reservoir fluid. It contains lighter hydrocarbons and fewer heavier hydrocarbons than volatile oil, its oil gravity is above 40° API and up to 60° API (i.e. between 40° and 60° API), the gas-oil ratio increases with time due to the liquid dropout, and the loss of heavy components in the liquid whose GOR is up to 70,000 scf/stb, it has about 5–10% liquid at the surface depending on the reservoir. The reservoir fluid is water-white or slightly colored oil at the stock tank.

In Fig. 1.12, at the initial condition, the reservoir is in a single gas phase and as the pressure drops, the fluid goes through the dew point which then condenses large volumes of liquid as it passes through the two phase region in the reservoir. Consequently, as the reservoir further depletes and the pressure drops, liquid condenses from the gas to form a free liquid inside the reservoir.

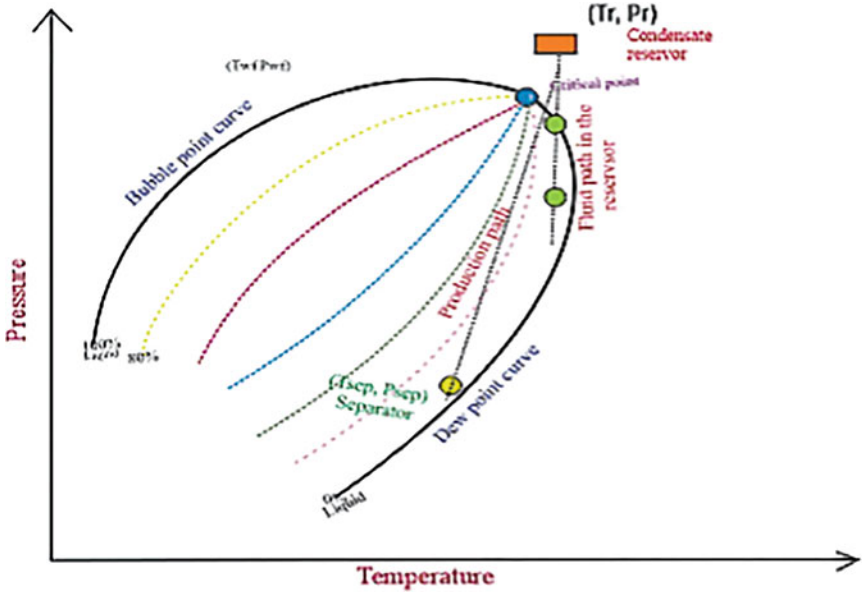


Fig. 1.12 Condensate or retrograde gas reservoir

Table 1.1 Comparison of oil reservoir fluid properties

Parameter	Black oil	Volatile oil	Condensate
Alternative name	Low shrinkage oil	Intermediate or high shrinkage oil	Gas in the reservoir but liquid at the surface due to pressure reduction
API range	30–40° API	40–50° API	60–120° API
Gas-oil ratio (GOR)	$\leq$500	$\leq$8000 scf/stb	8000–70,000 scf/stb
Oil formation volume factor (Bo)	<1.2 bbl/stb	<2 bbl/stb	
Evolved gas from the oil	Little	High	Very high
Percentage of separator liquid	85%	65%	<10%
Colour	Dark or deep color	Brown, orange, or green	Water-white or slightly colored oil at the stock tank
viscosity	High	Medium	Low

In the production of a gas condensate field, gas is mostly produced with some liquid dropout as the pressure drops below dew point pressure; occurring mostly in the separator and can still be produced in the wellbore which ultimately leads to a restriction in the flow of gas. The temperature and pressure may change once the reservoir fluids enter into the wellbore, thereby causing liquid dropout within the wellbore. Thus, if the gas having the larger fraction does not have enough energy to lift the dropout liquid to the surface, a fallback in the wellbore occurs or liquid loading. If this is continuous, the percentage of the liquid will increase and may eventually restrict the gas production. This challenge can be adequately handled with artificial lift technologies such as gas lift. Table 1.1 shows the comparison of blackoil, volatile and condensate reservoir fluid properties.

1.4.5.4 Gas Reservoirs

Hydrocarbon reservoir can be called gas reservoir, if the temperature of the reservoir is greater than the cricondentherm of the hydrocarbon fluid. This is only applicable to non-associated gas reservoirs which can either be wet or dry gas depending on the phase present in the reservoir and at the surface separator.

1.4.5.5 Wet-Gas Reservoirs

A natural gas system which contains a significant amount of propane, butane and other liquid hydrocarbons is known as wet gas or rich gas. It contains less amount of methane (85%) and more ethane than dry gas. Figure 1.13 shows a wet gas reservoir which exists solely as a gas in the reservoir throughout the reduction in reservoir

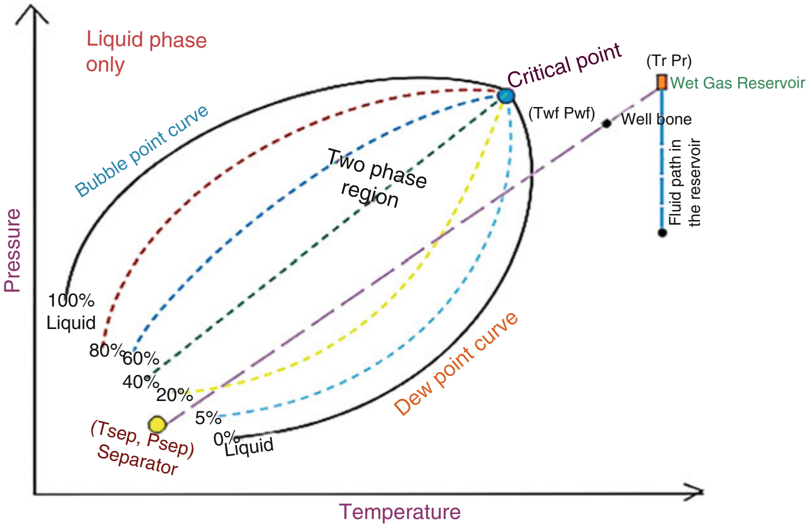


Fig. 1.13 Wet gas reservoir

pressure. It temperature lies above the cricondentherm of the hydrocarbon mixture similar to a dry gas reservoir. The reservoir fluid will always remain in the vapor phase region as the reservoir is depleted isothermally, along with the production path unlike retrograde condensate; no liquid is formed inside the reservoir. However, separator conditions lie within the phase envelope, causing some liquid to be formed at the surface. This surface liquid is normally called condensate. Wet-gas reservoirs are characterized by gas oil ratios between 60,000 to 100,000 scf/STB, stock-tank oil gravity above 60° API, the liquid is water-white in color and separator conditions lie within the two-phase region.

1.4.5.6 Dry Gas Reservoir

The hydrocarbon mixture of a dry gas exists as a gas in the reservoir (even in the two phase region) and in the surface separator characterized with a gas-oil ratio greater than 100,000 scf/STB. It contains mainly methane with some intermediates. The pressure or production path does not enter into the phase envelope (two phase region) as shown in Fig. 1.14, this means that the surface separator conditions fall outside the phase envelope which is in contrast to wet gas reservoir; hence there is no traces of liquid formed at the surface separator.

Natural gas which occurs in the absence of condensate or liquid hydrocarbons, or gas that had condensable hydrocarbons removed, is called dry gas. It is primarily methane with some intermediates. The hydrocarbon mixture is solely gas in the reservoir and there is no liquid (condensate surface liquid) formed either in the reservoir or at the surface. The pressure path (line) does not enter into the phase

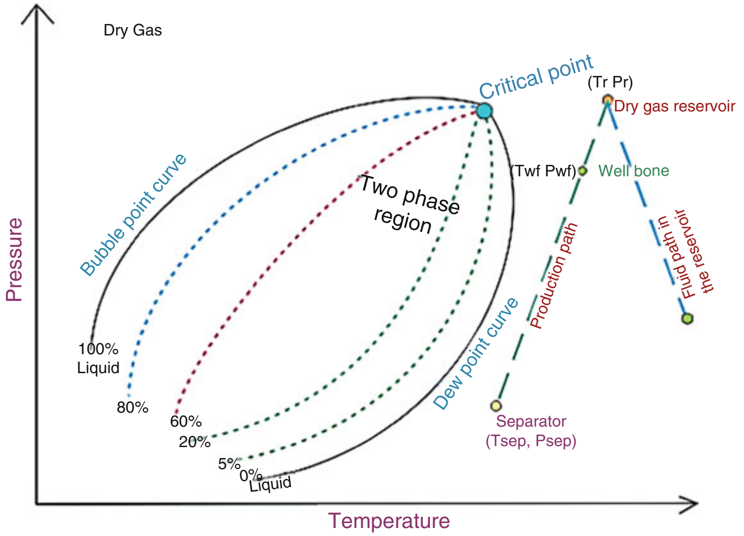


Fig. 1.14 Dry gas reservoir

Table 1.2 Properties of wet and dry gas reservoir fluid

Parameter	Wet gas	Dry gas
Effect of pressure reduction	There are tracies of liquid at the surface	No tracy of liquid at the surface
Gas-oil ratio (GOR)	≤100,000 scf/stb	>100,000 scf/stb
Color	Light straw to water white	Water white
Viscosity	Low	Very low
API	>60	

envelope (two-phase region) in the phase diagram, thus there is only dry gas in the reservoir (Table 1.2).

1.5 Types of Fluids in Terms of Flow Regime and Reservoir Geometry

The fluid in hydrocarbon reservoirs can be classified in terms of pressure change occurring as fluid flow from various path of the reservoir system to the wellbore. They are further classified in terms of flow regime and reservoir geometry.

The reservoir fluid can either by incompressible, slightly compressible or compressible depending on the state of the pressure change in the reservoir. When the volume or density of the fluid does not change with pressure, it is called an incompressible fluid. This implies that as the pressure within the system changes, the volume of the fluid remains the same. This fluid behavior hardly exists but it is an assumption for easy derivation for fluid flow equations. For the case of a slightly compressible fluid, there is a little change in volume or density as pressure changes. Also, for fluid such as gase's are easily compressible and expand to fill the volume of its container; this makes gases to experience large changes in volume as a function of pressure. This is termed a compressible fluid.

1.5.1 Reservoir Geometry

Petroleum reservoir is usually trapped with fluids that are looking for ways to flow out; once a well is drilled, cased and perforated, the trapped fluid then flows from all directions in the reservoir to the wellbore where it is produced to the surface facilities. The movement of hydrocarbon fluid towards the wellbore is either characterized as radial or linear depending on the flow direction.

1.5.1.1 Linear Flow

Linear flow occurs when the paths at which the fluids flow are parallel to each other such that the movement is in a single direction. In this type of flow, the cross-sectional area is assumed to be constant, thereby creating a laminar flow. This is represented in Fig. 1.15.

1.5.1.2 Radial Flow

On the other hand, radial flow occurs when fluids move in a multi-direction within the reservoir towards the perforations at the wellbore, thus creating an iso-potential lines. The radial flow system is shown in Fig. 1.16.

Fig. 1.15 Linear flow system

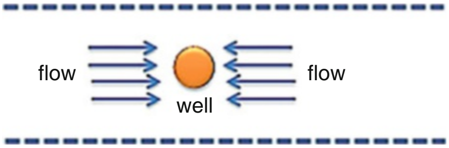
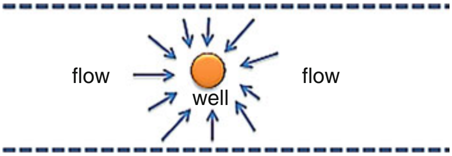


Fig. 1.16 Radial flow system



1.5.2 Flow Regimes

1.5.2.1 Steady-State Fluids Flow

This type of flow is referred to the condition at any single or given point in the reservoir where the properties such as pressure, temperature and velocity of the fluid does not change with time. It can be defined as the flow at which the rate of change of pressure (P) with respect to time (t) at any location *i* in the reservoir is zero as shown in the equation below. At this state of flow, all the boundaries effects have been felt but there is no decline in the static pressure at the boundary (called constant pressure boundary). This implies that in a system of mass flow rate, there is no accumulation of mass within any component in the system. Steady state flow is more applicable to laboratory displacement experiments than to petroleum reservoir conditions which are hardly seen. This scenario can only be seen in reservoirs undergoing pressure maintenance either by water or gas injection or when the reservoir is completely recharging and supported by a strong aquifer. This is to say that; there is a flow of fluid across the boundaries of the reservoir (unbounded reservoir).

$$\frac{\partial P}{\partial t} = 0$$

1.5.2.2 Linear Flow Equation for Steady-State Incompressible Fluid

For oil

$$q = \frac{kA}{\mu} \frac{\left(P_e - P_{wf}\right)}{L}$$

In Darcy units it is given as

$$q\left({}^{cm^3}\!/_{s}\right) = \frac{k(D)A(cm^2)}{\mu(cp)} \frac{\left(P_e - P_{wf}\right)(atm)}{L(cm)}$$

In oil field units

$$1D = 1000\,mD,$$

$$(0.3048)^2\,m^2 = (30.48)^2\,cm^2 = 1\,ft^2$$

$$1\ atm = 14.7\ psi$$

Converting the left hand side of the equation (flow rate)

$$q\left({}^{cm^3}\!/_{s}\right) = q\left({}^{STB}\!/_{day}\right) \times \beta_o\left({}^{bbl}\!/_{STB}\right) \times \frac{1\ day}{24\ h} \times \frac{1\ h}{3600\ s} \times \frac{0.1589973\ m^3}{bbl} \times \frac{100^3\ cm^3}{m^3}$$
$$= 1.84013\ q\beta_o\left({}^{STB}\!/_{day}\right)\left({}^{bbl}\!/_{STB}\right)$$

Converting the right side of the equation

$$= \frac{k(mD) \times \frac{1D}{1000\ mD} \times A\left(ft^2\right) \times \frac{(30.48)^2\ cm^2}{1\ ft^2} \times \left(P_e - P_{wf}\right)(psi) \times \frac{1\ atm}{14.7\ psi}}{\mu(cp)L(ft) \times \frac{30.48\ cm}{1\ ft}}$$

$$= 0.002073469 \frac{k(mD)\ A\left(ft^2\right)\ \left(P_e - P_{wf}\right)(psi)}{\mu(cp)\ L(ft)}$$

Combining both sides of the equation

$$1.84013\ q\beta_o\left({}^{STB}\!/_{day}\right)\left({}^{bbl}\!/_{STB}\right) = 0.002073469 \frac{k(mD)\ A\left(ft^2\right)\ \left(P_e - P_{wf}\right)(psi)}{\mu(cp)\ L(ft)}$$

$$q\left({}^{STB}\!/_{day}\right) = \frac{0.002073469}{1.84013} \cdot \frac{k(mD)\ A\left(ft^2\right)\ \left(P_e - P_{wf}\right)(psi)}{\mu(cp)\beta_o\left({}^{bbl}\!/_{STB}\right)\ L(ft)}$$

$$\therefore q\left({}^{STB}\!/_{day}\right) = 1.127 \times 10^{-3}\ \frac{k(mD)\ A\left(ft^2\right)\ \left(P_e - P_{wf}\right)(psi)}{\mu(cp)\beta_o\left({}^{bbl}\!/_{STB}\right)\ L(ft)}$$

Gas flow equation in oil field units under steady-state flow

$$q\left({}^{Scf}\!/_{day}\right) = 3.164 \times 10^{-3}\ \frac{T_{sc}k(mD)\ A\left(ft^2\right)\ \left({P_e}^2 - {P_{wf}}^2\right)(psi)}{z\ P_{sc}\ T\ \mu_g(cp)\ L(ft)}$$

1.5.2.3 Radial Flow Equation for Steady-State (Unbounded Reservoir) Incompressible Fluid

By derivation for oil flow

$$U_r = \frac{q}{A} = -\frac{k}{\mu}\frac{dP}{dr}$$

$$q = -\frac{kA}{\mu}\frac{dP}{dr} \qquad where\ A = 2\pi rh$$

$$q = \frac{k(2\pi rh)}{\mu}\frac{dP}{dr}$$

$$\frac{q\mu}{2\pi kh}\frac{dr}{r} = dP$$

Integrating from the wellbore conditions to the reservoir boundary conditions That is from $r = r_w \rightarrow r_e \quad P = P_{wf} \rightarrow P_e$

$$\frac{q\mu}{2\pi kh}\int_{r_w}^{r_e}\frac{dr}{r} = \int_{P_{wf}}^{P_e} dP$$

$$\frac{q\mu}{2\pi kh}\ln(r)|_{r_w}^{r_e} = \mathrm{P}|_{P_{wf}}^{P_e}$$

$$\frac{q\mu}{2\pi kh}(\ln r_e - \ln r_w) = P_e - P_{wf}$$

$$\frac{q\mu}{2\pi kh}\ln\frac{r_e}{r_w} = P_e - P_{wf}$$

$$q = \frac{2\pi kh\left(P_e - P_{wf}\right)}{\mu \ln\frac{r_e}{r_w}}$$

Oil field units

$$q = \frac{7.08 \times 10^{-3}\ k(mD)h(ft)\left(P_e - P_{wf}\right)(psi)}{\mu\beta \ln\frac{r_e}{r_w}}$$

In terms of average pressure, it is represented as

$$q = \frac{2\pi kh\left(\bar{P}_r - P_{wf}\right)}{\mu\left[\ln\frac{r_e}{r_w} - \frac{1}{2}\right]}$$

$$q = \frac{7.08 \times 10^{-3}\ k(mD)h(ft)\left(\bar{P}_r - P_{wf}\right)(psi)}{\mu\beta\left[\ln\frac{r_e}{r_w} - \frac{1}{2}\right]}$$

1.5.2.4 Steady State with the Effect of Skin

Practically, during drilling and completion operations, the permeability around the wellbore of most wells have been damaged or reduced thereby causes an impairment to flow of fluid and thus create an additional pressure drop near the wellbore. This impairment to flow is known as skin. Incorporating it into the flow equation gives:

$$q = \frac{2\pi kh\left(P_e - P_{wf}\right)}{\mu\left[\ln\frac{r_e}{r_w} + s\right]} = \frac{2\pi kh\left(\bar{P}_r - P_{wf}\right)}{\mu\left[\ln\frac{r_e}{r_w} - \frac{1}{2} + s\right]}$$

Where s = skin
Oil field units

$$q = \frac{7.08 \times 10^{-3}\ k(mD)h(ft)\left(P_e - P_{wf}\right)(psi)}{\mu\beta\left[\ln\frac{r_e}{r_w} + s\right]}$$

1.5.2.5 Radial Flow Equation for Steady-State (Unbounded Reservoir) Slighty Compressible Fluid

$$q = \frac{7.08 \times 10^{-3}\ kh\left(P_e - P_{wf}\right)}{\mu\beta\left[\ln\frac{r_e}{r_w} + s\right]}$$

Recall

$$P = P_{wf}e^{C_o\left(P - P_{wf}\right)}$$

$$\therefore\ q = \frac{7.08 \times 10^{-3}\ kh\ \ln\left[1 + C_o\left(P_e - P_{wf}\right)\right]}{\mu_o \beta_o c_o \left[\ln \frac{r_e}{r_w} + s\right]}$$

Example 1.1

Calculate the flow rate of a vertical well at steady state producing oil from the center of a cylindrical drainage area with the following parameters:

Wellbore radius, r_w	*14 cm*
Drainage radius, r_e	*98 m*
Reservoir height, h	*35 m*
Pressure at the outer boundary, P_e	*182 bar*
Wellbore flowing pressure, P_{wf}	*168 bar*
Formation permeability, k	*0.715 D*
Oil viscosity, μ_o	*0.71 cp*
Oil formation volume factor, β_o	*1.25 rb/stb*
Mechanical skin, s	*5*

Solution

$$q = \frac{7.08 \times 10^{-3}\ k(mD)h(ft)\left(P_e - P_{wf}\right)(psi)}{\mu\beta\left[\ln \frac{r_e}{r_w} + s\right]}$$

Conversion to oil field units

$$r_e = 98\ m \times \frac{3.28084\,ft}{1\ m} = 321.5223$$

$$r_w = 14\ cm = 0.14\ m = 0.14\ m \times \frac{3.28084\,ft}{1\ m} = 0.4593\,ft$$

$$h = 35\ m = 35\ m \times \frac{3.28084\,ft}{1\ m} = 114.8294\,ft$$

$$P_e = 182\ bar = 182\ bar \times \frac{14.5037738\,psi}{1\ bar} = 2639.6868\,psi$$

$$P_{wf} = 168\ bar = 168\ bar \times \frac{14.5037738\,psi}{1\ bar} = 2436.6339\,psi$$

$$k = 0.715\ D = 0.715\ D \times \frac{1000\ mD}{1\ D} = 715\ mD$$

$$q_o = \frac{7.08 \times 10^{-3} \times 715 \times 114.8294 \times (2639.6868 - 2436.6339)}{0.71 \times 1.25 \times \left[\ln\left(\frac{321.5223}{0.4593}\right) + 5\right]}$$
$$= 11513.5483\ stb/days$$

Example 1.2

Given the following information of a vertical well producing at steady state from a cylindrical region:

Wellbore radius, r_w	*0.235 ft*
Drainage radius, r_e	*250 ft*
Reservoir height, h	*200 ft*
Pressure at the outer boundary, P_e	*3400 psi*
Wellbore flowing pressure, P_{wf}	*3062 psi*
Formation absolute permeability, k	*280 mD*
Oil relative permeability, K_{ro}	*0.85*
Oil viscosity, μ_o	*0.65 cp*
Oil formation volume factor, β_o	*1.37 rb/stb*
Skin, s	*3.7*
Solution gas-oil ratio, R_s	*500 scf/stb*

Calculate

(a) The oil flow rate at reservoir condition
(b) The oil and gas flow rate at stock tank condition (STC)

Solution
Oil flow rate

$$Q_o = \frac{7.08 \times 10^{-3}\ k(mD)h(ft)\left(P_e - P_{wf}\right)(psi)}{\mu\left[\ln\frac{r_e}{r_w} + s\right]}$$

$$k_o = kk_{ro}$$

$$Q_o = \frac{7.08 \times 10^{-3} \times (0.85 \times 280) \times 200 \times (3400 - 3062)}{0.65 \times \left[\ln\left(\frac{250}{0.235}\right) + 3.7\right]} = 16424.576\ rb/d$$

Oil flow rate at STC

$$Q_o(STC) = \frac{Q_o(RC)}{\beta_o}$$

$$Q_o(STC) = \frac{16424.576}{1.37} = 11988.7416\ stb/d = 11.988\ Mstb/d$$

Gas flow rate at STC

$$R_s = \frac{Q_g(STC)}{Q_o(STC)}$$

$$\therefore\ Q_g(STC) = R_s Q_o(STC)$$

$$= 500 \times 11988.7416 = 5994370.802\ scf/d = 5.994\ MMscf/d$$

Example 1.3

Given the following data:

Wellbore radius, r_w	*0.5 ft*
Drainage radius, r_e	*850 ft*
Reservoir height, h	*27 ft*
Pressure at the outer boundary, P_e	*5000 psi*
Wellbore flowing pressure, P_{wf}	*4805 psi*
Formation absolute permeability, k	*108 mD*
Oil viscosity, μ_o	*0.47 cp*
Oil formation volume factor, β_o	*1.347 rb/stb*
Skin, s	*3.89*

(a) Calculate the well flow rate at steady state radial flow
(b) The flow rate after a successful stimulation job to reduce the skin factor to 0.93

Solution

$$q = \frac{7.08 \times 10^{-3}\ k(mD)h(ft)\left(P_e - P_{wf}\right)(psi)}{\mu\beta\left[\ln\frac{r_e}{r_w} + s\right]}$$

(a) The pre-stimulation flow rate is

$$Q_o = \frac{7.08 \times 10^{-3} \times 108 \times 27 \times (5000 - 4805)}{0.47 \times 1.347 \times \left[\ln\left(\frac{850}{0.5}\right) + 3.89\right]} = 561.3348\ stb/d$$

(b) The post-stimulation flow rate is

$$Q_o = \frac{7.08 \times 10^{-3} \times 108 \times 27 \times (5000 - 4805)}{0.47 \times 1.347 \times \left[\ln\left(\frac{850}{0.5}\right) + 0.93\right]} = 759.8859\ stb/d$$

The stimulation job improved the well flow rate for about 26% which implies a successful stimulation operation.

Example 1.4
Given the following data:

Wellbore radius, r_w	*0.3333 ft*
Drainage radius, r_e	*1000 ft*
Reservoir height, h	*27 ft*
Pressure at the outer boundary, P_e	*3500 psi*
Wellbore flowing pressure, P_{wf}	*2000 psi*

Calculate the following assuming steady state flow.

I. The reservoir pressure at a radius of 50 ft
II. The pressure gradient at 50 ft

Solution

$$P_e - P_{wf} = \frac{q\mu\beta \ln\frac{r_e}{r_w}}{7.08 \times 10^{-3}\ kh}$$

At wellbore radius of 0.3333 ft

$$3500 - 2000 = \frac{q\mu\beta \ln\left(\frac{1000}{0.3333}\right)}{7.08 \times 10^{-3}\ kh}$$

$$1500 \times 7.08 \times 10^{-3}\ kh = q\mu\beta \ln(3000.30) \tag{1.1}$$

At radius of 50 ft, the pressure is designated as P_{50}

$$3500 - P_{50} = \frac{q\mu\beta \ln\left(\frac{1000}{50}\right)}{7.08 \times 10^{-3} kh}$$

$$(3500 - P_{50}) 7.08 \times 10^{-3}\, kh = q\mu\beta \ln(20) \tag{1.2}$$

Divide Eq. 1.1 by 1.2

$$\frac{1500 \times \cancel{7.08 \times 10^{-3} kh}}{(3500 - P_{50})\cancel{7.08 \times 10^{-3} kh}} = \frac{\cancel{q\mu\beta} \ln(3000.30)}{\cancel{q\mu\beta} \ln(20)}$$

$$\frac{1500}{(3500 - P_{50})} = \frac{\ln(3000.30)}{\ln(20)} = \frac{8.00647}{2.99573} = 2.6726$$

$$3500 - P_{50} = \frac{1500}{2.6726} = 561.2512$$

$$\therefore\ P_{50} = 3500 - 561.2512 = 2938.7488\, psia$$

Pressure gradient at 50 ft

$$\Delta P = \frac{q\mu\beta \ln\frac{r_e}{r_w}}{7.08 \times 10^{-3}\, kh}$$

By integrating

$$\int_{r_w}^{r_e} \frac{1}{r} dr = \ln\frac{r_e}{r_w}$$

$$dP = \frac{q\mu\beta \frac{1}{r} dr}{7.08 \times 10^{-3}\, kh}$$

$$\frac{dP}{dr} = \frac{q\mu\beta \frac{1}{r}}{7.08 \times 10^{-3}\, kh} = \frac{q\mu\beta}{7.08 \times 10^{-3}\, kh} . \frac{1}{r} \tag{1.3}$$

From Eq. 1.1

$$\frac{q\mu\beta}{7.08\times10^{-3}\,kh}=\frac{1500}{\ln\left(3000.30\right)}=187.3485$$

Substitute into Eq. 1.3

$$\frac{dP}{dr}=187.3485.\frac{1}{r}$$

At the desired radius of 50 ft

$$\frac{dP}{dr}=\frac{187.3485}{50}=3.7469\,psia/ft$$

Example 1.5

The follow information was gotten from an ABC field in the Niger Delta region undergoing a steady state flow:

Wellbore radius, r_w	*0.4583 ft*
Drainage radius, r_e	*850 ft*
Reservoir height, h	*46 ft*
Formation permeability, k	*235 mD*

After the wellbore was under-reamed, the diameter increased to 22 in (1.833 ft).

- What is the production rate before and after under-reaming?
- What is the productivity ratio?

Solution

Production rate before under-reaming

$$q_o=\frac{7.08\times10^{-3}\,kh\left(P_e-P_{wf}\right)}{\mu\beta\left[\ln\frac{r_e}{r_w}\right]}$$

$$q_o=\frac{7.08\times10^{-3}\times235\times46\times\left(P_e-P_{wf}\right)}{\mu\beta\left[\ln\left(\frac{850}{0.4583}\right)\right]}$$

$$q_o=10.1701\frac{\left(P_e-P_{wf}\right)}{\mu\beta}\quad(bbl)$$

Production rate after under-reaming

$$q_{Und} = \frac{7.08 \times 10^{-3}\, kh\left(P_e - P_{wf}\right)}{\mu\beta\left[\ln \frac{r_e}{r_{und}}\right]}$$

$$q_{Und} = \frac{7.08 \times 10^{-3} \times 235 \times 46 \times \left(P_e - P_{wf}\right)}{\mu\beta\left[\ln\left(\frac{850}{1.833}\right)\right]}$$

$$q_{Und} = 12.4664\frac{\left(P_e - P_{wf}\right)}{\mu\beta} \quad (bbl)$$

$$Productivity\ ratio = \frac{productivity\ index\ of\ damage\ well}{productivity\ index\ of\ undamage\ well}$$

$$Productivity\ ratio = \frac{\frac{q_{Und}}{q_o} = 12.4664\frac{\left(P_e - P_{wf}\right)}{\mu\beta}}{10.1701\frac{\left(P_e - P_{wf}\right)}{\mu\beta}}$$

$$= \frac{12.4664\,\cancel{\left(P_e - P_{wf}\right)}}{10.1701\,\cancel{\mu\beta}} \cdot \frac{\cancel{\mu\beta}}{\cancel{\left(P_e - P_{wf}\right)}} = 1.2258$$

1.5.2.6 Radial Flow Equation for Steady-State (Unbounded Reservoir) Compressible Fluid (Gases)

Low pressure approximation

$$q = 1.988 \times 10^{-2}\,\frac{T_{sc}kh\left(P_e{}^2 - P_{wf}{}^2\right)}{P_{sc}\,T\left(\mu_g z\right)_i\,\ln\frac{r_e}{r_w}} \quad (scf/day)$$

Or

$$= \frac{kh\left(P_e{}^2 - P_{wf}{}^2\right)}{1422\,T\left(\mu_g z\right)_{avg}\,\ln\frac{r_e}{r_w}} \quad (Mscf/day)$$

High pressure approximation

$$q = 3.976 \times 10^{-2}\,\frac{T_{sc}kh\,\bar{P}_r\left(P_e - P_{wf}\right)}{P_{sc}\,T\left(\overline{\mu_g z}\right)\,\ln\frac{r_e}{r_w}} \quad (scf/day)$$

Table 1.3 The graphical method is represented below for different pressure drops

P	μ	z	$^2p/_{(\mu z)}$	Area	$m(p)$
P_0	μ_0	z_0	$^2p/_{(\mu z)_0} = Y_0$	–	
P_1	μ_1	z_1	$^2p/_{(\mu z)_1} = Y_1$	$A_1 = 0.5(Y_0 + Y_1)(P_1 - P_0)$	A_1
P_2	μ_2	z_2	$^2p/_{(\mu z)_2} = Y_2$	$A_2 = 0.5(Y_1 + Y_2)(P_2 - P_1)$	$A_1 + A_2$
P_n	μ_n	z_n	$^2p/_{(\mu z)_n} = Y_n$	$A_n = 0.5(Y_{n-1} + Y_n)(P_n - P_{n-1})$	$A_1 + A_2 + \ldots A_n$

Real gas potential

$$q = 1.988 \times 10^{-2} \frac{T_{sc}kh \ \left(m\langle P_e\rangle - m\langle P_{wf}\rangle\right)}{P_{sc}\ T\ \ln\frac{r_e}{r_w}} \quad (scf/day)$$

The real gas pseudo-pressure is defined as:

$$m(p) = \int_0^p \frac{2p}{\mu_g z} dp$$

Calculation of Real Gas Potential, m(p)

The m(p) can be calculated graphically or read from Tables. The graphical method requires that P, μ, z be given and m(p) calculated from the area under a curve (Table 1.3). This is illustrated below using the trapezoid method to calculate the area under the curve. Trapezoidal rule

$$Avg\ area = \frac{1}{2}(A_n + A_{n+1})h$$

Example 1.6

Calculate the gas flow rate using real gas potential method for FUPRE K56 gas well producing under a constant bottom hole flowing pressure of 2760 psi. The PVT result is given in the table below

P (psi)	Z	μ_g (cp)
0	1.0000	0.01275
600	0.9554	0.01438
1200	0.8792	0.01574
2760	0.86816	0.01939
2771	0.86828	0.01942
2817	0.86934	0.01958
2920	0.87186	0.01994
3000	0.87414	0.02022

Additional Information

Wellbore radius, r_w	*0.3296 ft*
Drainage radius, r_e	*928 ft*
Reservoir height, h	*22 ft*
Initial reservoir pressure, P_i	*3000 psi*
Formation absolute permeability, k	*68 mD*
Reservoir temperature, T	*170°C*
Gas gravity, γ_g	*0.67*
Skin, s	*0.67*

Solution

First, calculate the real gas potential at each pressure which represents the area under the curve for each value of pressure.

Recall:

$$m(p) = \int_0^p \frac{2p}{\mu z} dp$$

$$Y_n = \left(\frac{2p}{\mu z}\right)_n$$

$$A_n = 0.5(Y_{n-1} + Y_n)(P_n - P_{n-1})$$

At $P_1 = 600\,psi,\ \mu_1 = 0.9554,\quad z_1 = 0.01438$

$$Y_1 = \left(\frac{2p}{\mu z}\right)_1 = \frac{2*600}{0.9554*0.01438} = 87344.8137\,psi/cp$$

$$A_1 = 0.5(Y_0 + Y_1)(P_1 - P_0)$$

$$A_1 = 0.5(0 + 87344.8137)(600 - 0) = 26203444.11\,psi^2/cp$$

$$m(p) = \int_0^{600} \frac{2p}{\mu z} dp = A_1 = 26203444.11\ \ psi^2/cp$$

At $P_2 = 1200\,psi,\ \mu_2 = 0.8792,\quad z_2 = 0.01574$

$$Y_2 = \left(\frac{2p}{\mu z}\right)_2 = \frac{2*1200}{0.8792*0.01574} = 173427.8477\,psi/cp$$

$$A_2 = 0.5(87344.8137 + 173427.8477)(1200 - 600) = 78231798.42\,psi^2/cp$$

$$m(p) = \int_0^{1200} \frac{2p}{\mu z} dp = A_1 + A_2$$

$$= 26203444.11 + 78231798.42 = 104435242.5\,psi^2/cp$$

Applying the same procedure, the real gas potential for other pressures is presented in the table below

P	**z**	***μ***	$^{2p}/_{(\mu z)}$	**Area**	***m*(*p*)**
psi	**cp**	**–**	**(*psi*/*cp*)**	**(*psi*²/*cp*)**	**(*psi*²/*cp*)**
0	1	0.01275	0		
600	0.9554	0.01438	87344.8137	26203444.1225	26203444.1225
1200	0.8792	0.01574	173427.8477	78231798.4194	104435242.5418
2760	0.86816	0.01939	327915.1610	391047546.7747	495482789.3165
2771	0.86828	0.01942	328668.0577	3611207.7030	499093997.0195
2817	0.86934	0.01958	330989.7100	15172128.6567	514266125.6762
2920	0.87186	0.01994	335923.9280	34346052.3590	548612178.0353
3000	0.87414	0.02022	339460.3897	27015372.7096	575627550.7448

From the table, at:

$$P_{wf} = 2760\,psi,\;\; m\langle P_{wf}\rangle = 495482789.3165\;\left(psi^2/cp\right)$$

$$P_e = 3000\,psi,\;\; m\langle P_e\rangle = 575627550.7448\;\left(psi^2/cp\right)$$

$$T_{sc} = 60^{\circ}F = 520^{\circ}F \quad \& \quad P_{sc} = 14.7\,psi$$

$$q = 1.988 \times 10^{-2}\,\frac{T_{sc}kh\;\left(m\langle P_e\rangle - m\langle P_{wf}\rangle\right)}{P_{sc}\;T\;\ln\frac{r_e}{r_w}}$$

$$\therefore\ q = 1.988 \times 10^{-2}\ \frac{520{*}68{*}22{*}(575627550.7448 - 495482789.3165)}{14.7{*}630{*}\ln\left(\frac{928}{0.3296}\right)}$$

$$= 16849580.16\ scf/day = 16.8496\ MMscf/day$$

Example 1.7

Use the pressure-square approximation method to calculate the gas flow rate of Garon gas field well JT6 whose pay thickness is 54 ft, permeability is 200 mD and current bottom hole flowing pressure is 2000 psia. Given the following additional information:

Wellbore radius, r_w	*0.351 ft*
Drainage radius, r_e	*1180 ft*
Reservoir height, h	*33 ft*
Initial reservoir pressure, P_i	*3650 psi*
Reservoir temperature, T	*175 °C*
Gas gravity, γ_g	*0.67*
Pressure @ STC, P_{sc}	*14.7 psi*
Temperature @ STC, T_{sc}	*60 °C*

Solution

The first step is to calculate the gas viscosity and deviation factor at the average pressure expressed as:

$$\bar{P} = \sqrt{\frac{{P_{wf}}^2 + {P_e}^2}{2}}$$

$$\bar{P} = \sqrt{\frac{2000^2 + 3650^2}{2}} = 2943.0002\ psia$$

To calculate the compressibility factor, z

If yg < = 0.7 Then

$$T_c = 168 + \left(325{*}\gamma_g\right) - \left(12.5{*}{\gamma_g}^2\right)$$

$$T_c = 168 + (325{*}0.67) - \left(12.5{*}0.67^2\right) = 380.1388^{\circ}\mathrm{R}$$

$$P_c = 677 + \left(15 * \gamma_g\right) - \left(37.5 * {\gamma_g}^2\right)$$

$$P_c = 677 + (15 * 0.67) - \left(37.5 * 0.67^2\right) = 670.2163 \text{ psia}$$

$$T_r = \frac{(T + 460)}{T_c} = \frac{(175 + 460)}{380.1388} = \frac{635}{380.1388} = 1.67$$

$$P_r = \frac{P}{P_c} = \frac{2943.0002}{670.2163} = 4.39$$

Calculate z from Standing and Katz compressibility factors chart from Fig. 1.17

$$z_i(P_{ri}, T_r) = z_i(4.39, 1.67) = 0.845$$

The viscosity of the gas
Applying Lee Gonzalez

$$X = 3.448 + \frac{986.4}{(T + 460)} + \left(0.009 \times \gamma_g \times 28.96\right)$$

$$= 3.448 + \frac{986.4}{(175 + 460)} + (0.009 \times 0.67 \times 28.96) = 5.1760$$

$$Y = 2.47 - 0.224X$$

$$= 2.47 - 0.224(5.1760) = 1.3106$$

$$a = \left[9.379 + \left(0.01607 \times \gamma_g \times 28.96\right)\right](T + 460)^{1.5}$$

$$= [9.379 + (0.01607 \times 0.67 \times 28.96)](175 + 460)^{1.5} = 155067.4485$$

$$b = 209 + \left(19.26 \times \gamma_g \times 28.96\right) + (T + 460)$$

$$= 209 + (19.26 \times 0.67 \times 28.96) + (175 + 460) = 1217.7056$$

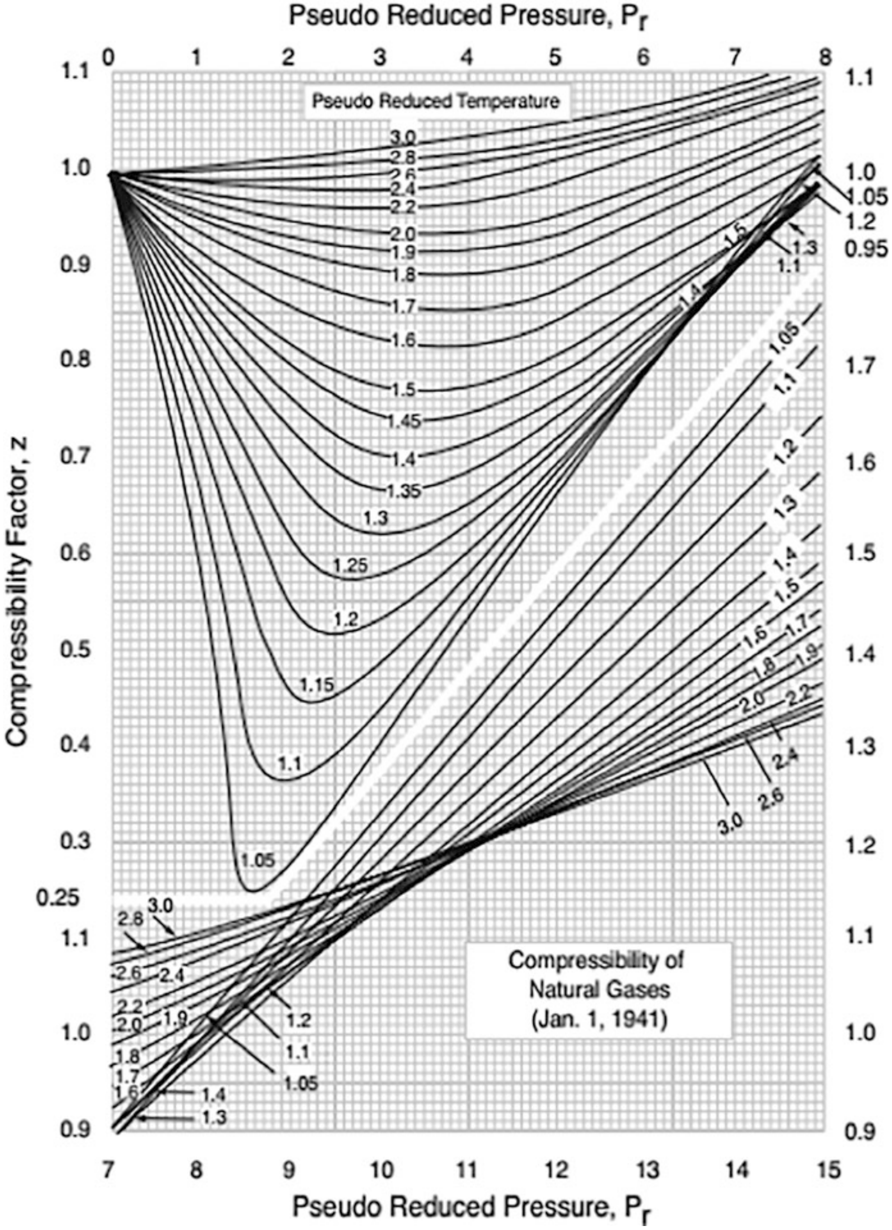


Fig. 1.17 Standing and Katz compressibility factors chart. (Courtesy of GPSA and GPA Engineering Data Book, EO Edition, 1987)

$$k = \frac{a}{b} = \frac{155067.4485}{1217.7056} = 127.3439$$

The density of the gas

$$\rho_g = \frac{28.96P\gamma_g}{zRT} = \frac{28.96 \times 2825 \times 0.67}{0.845 \times 10.73 \times (175 + 460)} = 9.5205\ lb/cuft$$

The gas viscosity is calculated as

$$\mu_g = 10^{-4}k * Exp\left(X\left\{\frac{\rho_g}{62.4}\right\}^Y\right)$$

$$= 10^{-4} \times 127.3439 \quad \times Exp\left(5.1760 \quad \left\{\frac{9.5205}{62.4}\right\}^{1.32346}\right) = 0.01957\ cp$$

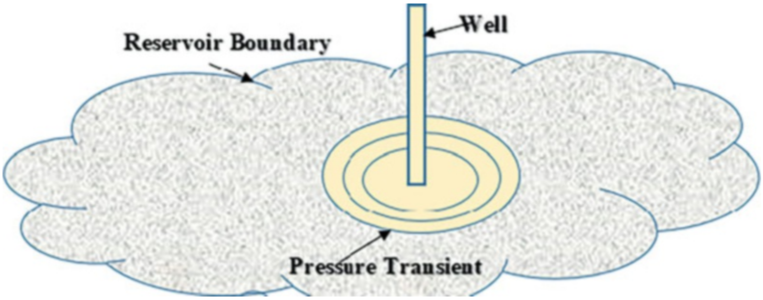
Therefore

$$q = 1.988 \times 10^{-2}\ \frac{T_{sc}kh\left(P_e{}^2 - P_{wf}{}^2\right)}{P_{sc}\ T\left(\mu_g z\right)_i\ \ln\frac{r_e}{r_w}} \quad (scf/day)$$

$$q = 1.988 \times 10^{-2}\ \frac{520*200*54*\left(3650^2 - 2000^2\right)}{14.7*(175 + 460)*(0.01957*0.845)*\ln\left(\frac{1180}{0.351}\right)}$$
$$= 830363932.2\ (scf/day) = 830.3639\ (MMscf/day)$$

1.5.3 Unsteady or Transient-State Flow

The state of fluid flow is termed unsteady-state flow, if the rate of change of pressure with respect to time at any position in the reservoir is not zero or constant. It is also called transient state whose behavior occurs when the boundary effect of the reservoir has not been felt and at this point, the reservoir is said to be infinite-acting. It can simply be defined as the flow regime where the distance/radius of pressure wave propagation from the wellbore has not reached any of the reservoir boundaries as shown in the figure below. Thus, at a short period of flow, the reservoir behaves as if it has no boundary, this will continue until the pressure transient gets to the boundary of the reservoir. Therefore, after the reservoir boundary has been contacted, the flow will either buildup to steady state or pseudo-steady state flow.



Mathematically, it can be represented as a function of time and space as:

$$\frac{\partial P}{\partial t} = f(i, t)$$

Equation of a transient state flow

$$P_i - P = 141.2\frac{q\beta\mu}{kh}\left[-\frac{1}{2}E_i\{-x\}\right] = -70.6\frac{q\beta\mu}{kh}\left[E_i\{-x\}\right]$$

Where Ei function or exponential integral is given as:

$$Ei(-x) = -\int_x^\infty \left(\frac{e^{-u}}{u}\right) du$$

Where

$$x = \frac{\theta\mu C_t r^2}{4(0.000264)\,kt} = \frac{\theta\mu C_t r^2}{0.001056\,kt}$$

$$P_i - P = -70.6\frac{q\beta\mu}{kh}E_i\left\{-\frac{\theta\mu C_t r^2}{0.001056\,kt}\right\}$$

$$\textbf{\textit{if x}} < 0.01,\ \ \textit{this implies that log approximation holds},\ \ \textit{then}$$

The Ei function can be approximated with negligible error by

$$Ei(-x) = \ln 1.781x$$

$$P_i - P = -70.6\frac{q\beta\mu}{kh}[\ln 1.781x] = -70.6\frac{q\beta\mu}{kh}[\ln 1.781 + \ln x]$$

$$P_i - P = -70.6\frac{q\beta\mu}{kh}[0.5772 + \ln x] = -70.6\frac{q\beta\mu}{kh}[0.5772 + 2.303\log x]$$

$$P_i - P = -70.6\frac{q\beta\mu}{kh}\left[0.5772 + 2.303\log\left\{\frac{\theta\mu C_t r^2}{4(0.000264)\,kt}\right\}\right]$$

$$P_i - P = 70.6\frac{q\beta\mu}{kh}\left[2.303\log\left\{\frac{4(0.000264)\,kt}{\theta\mu C_t r^2}\right\} - 0.5772\right]$$

$$P_i - P = (70.6*2.303)\frac{q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} + \log\{0.001056\} - 0.5772\right]$$

$$P_i - P = \frac{162.6q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23\right]$$

1.5.3.1 Pseudo-Steady with the Effect of Skin (Tables 1.4, 1.5 and 1.6a, b)

The pressure drop due to skin at the well is

$$\Delta P_s = \frac{141.2q\beta\mu}{kh}s$$

The total pressure drop is

$$P_i - P = \frac{162.6q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23\right] + \frac{141.2q\beta\mu}{kh}s$$

$$P_i - P = \frac{162.6q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23\right] + \frac{162.6q\beta\mu}{1.1513\,kh}s$$

Table 1.4 Values of exponential integral, $-\boldsymbol{Ei}(-\boldsymbol{y})$

***−Ei*(−*y*), 0.000 < 0.209, *interval* = 0.001**

y	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	+∞	6.332	5.639	5.235	4.948	4.726	4.545	4.392	4.259	4.142
0.01	4.038	3.944	3.858	3.779	3.705	3.637	3.574	3.514	3.458	3.405
0.02	3.355	3.307	3.261	3.218	3.176	3.137	3.098	3.062	3.026	2.992
0.03	2.959	2.927	2.897	2.867	2.838	2.810	2.783	2.756	2.731	2.706
0.04	2.681	2.658	2.634	2.612	2.590	2.568	2.547	2.527	2.507	2.487
0.05	2.468	2.449	2.431	2.413	2.395	2.377	2.360	2.344	2.327	2.311
0.06	2.295	2.279	2.264	2.249	2.235	2.22	2.206	2.192	2.178	2.164
0.07	2.151	2.138	2.125	2.112	2.099	2.087	2.074	2.062	2.050	2.039
0.08	2.027	2.015	2.004	1.993	1.982	1.971	1.960	1.950	1.939	1.929
0.09	1.919	1.909	1.899	1.889	1.879	1.869	1.860	1.850	1.841	1.832
0.1	1.823	1.814	1.805	1.796	1.788	1.779	1.770	1.762	1.754	1.745
0.11	1.737	1.729	1.721	1.713	1.705	1.697	1.689	1.682	1.674	1.667
0.12	1.660	1.652	1.645	1.638	1.631	1.623	1.616	1.609	1.603	1.596
0.13	1.589	1.582	1.576	1.569	1.562	1.556	1.549	1.543	1.537	1.530
0.14	1.524	1.518	1.512	1.506	1.500	1.494	1.488	1.482	1.476	1.470
0.15	1.464	1.459	1.453	1.447	1.442	1.436	1.431	1.425	1.420	1.415
0.16	1.409	1.404	1.399	1.393	1.388	1.383	1.378	1.373	1.368	1.363
0.17	1.358	1.353	1.348	1.343	1.338	1.333	1.329	1.324	1.319	1.314
0.18	1.310	1.305	1.301	1.296	1.291	1.287	1.282	1.278	1.274	1.269
0.19	1.265	1.261	1.256	1.252	1.248	1.243	1.239	1.235	1.231	1.227
0.2	1.233	1.219	1.215	1.210	1.206	1.202	1.198	1.195	1.191	1.187

***−Ei*(−*y*), 0.000 < 2.09, *interval* = 0.01**

y	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	+∞	4.038	3.335	2.959	2.681	2.468	2.295	2.151	2.027	1.919
0.1	1.823	1.737	1.660	1.589	1.524	1.464	1.409	1.358	1.309	1.265
0.2	1.223	1.183	1.145	1.110	1.076	1.044	1.014	0.985	0.957	0.931
0.3	0.906	0.882	0.858	0.836	0.815	0.794	0.774	0.755	0.737	0.719
0.4	0.702	0.686	0.670	0.655	0.640	0.625	0.611	0.598	0.585	0.572
0.5	0.560	0.548	0.536	0.525	0.514	0.503	0.493	0.483	0.473	0.464
0.6	0.454	0.445	0.437	0.428	0.420	0.412	0.404	0.396	0.388	0.381
0.7	0.374	0.367	0.360	0.353	0.347	0.340	0.334	0.328	0.322	0.316
0.8	0.311	0.305	0.300	0.295	0.289	0.284	0.279	0.274	0.269	0.265
0.9	0.260	0.256	0.251	0.247	0.243	0.239	0.235	0.231	0.227	0.223
1	0.219	0.216	0.212	0.209	0.205	0.202	0.198	0.195	0.192	0.189
1.1	0.186	0.183	0.180	0.177	0.174	0.172	0.169	0.166	0.164	0.161
1.2	0.158	0.156	0.153	0.151	0.149	0.146	0.144	0.142	0.140	0.138
1.3	0.135	0.133	0.131	0.129	0.127	0.125	0.124	0.122	0.120	0.118
1.4	0.116	0.114	0.113	0.111	0.109	0.108	0.106	0.105	0.103	0.102
1.5	0.100	0.099	0.097	0.096	0.094	0.093	0.092	0.090	0.089	0.088
1.6	0.086	0.085	0.084	0.083	0.081	0.080	0.079	0.078	0.077	0.076
1.7	0.075	0.074	0.073	0.072	0.071	0.070	0.069	0.068	0.067	0.066

(continued)

Table 1.4 (continued)

1.8	0.065	0.064	0.063	0.062	0.061	0.060	0.060	0.059	0.058	0.057
1.9	0.056	0.055	0.055	0.054	0.053	0.052	0.052	0.051	0.050	0.050
2	0.049	0.048	0.048	0.047	0.046	0.046	0.045	0.044	0.044	0.043

$-Ei(-y)$, 2.0 < 10.9, *interval* = 0.1

y	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
2	4.89E-02	4.26E-02	3.72E-02	3.25E-02	2.84E-02	2.49E-02	2.19E-02	1.92E-02	1.69E-02	1.48E-02
3	1.30E-02	1.15E-02	1.01E-02	8.94E-03	7.89E-03	6.87E-03	6.16E-03	5.45E-03	4.82E-03	4.27E-02
4	3.78E-03	3.35E-03	2.97E-03	2.64E-03	2.34E-03	2.07E-03	1.84E-03	1.64E-03	1.45E-03	1.29E-03
5	1.15E-03	1.02E-03	9.08E-04	8.09E-04	7.19E-04	6.41E-04	5.71E-04	5.09E-04	4.53E-04	4.04E-04
6	3.60E-03	3.21E-04	2.86E-04	2.55E-04	2.28E-04	2.03E-04	1.82E-04	1.62E-04	1.45E-04	1.29E-04
7	1.15E-04	1.03E-04	9.22E-05	8.24E-05	7.36E-05	6.58E-05	5.89E-05	5.26E-05	4.71E-05	4.21E-05
8	3.77E-05	3.37E-05	3.02E-05	2.70E-05	2.42E-05	2.16E-05	1.94E-05	1.73E-05	1.55E-05	1.39E-05
9	1.24E-05	1.11E-05	9.99E-05	8.95E-05	8.02E-05	7.18E-05	6.44E-05	5.77E-06	5.17E-06	4.64E-06
10	4.15E-06	3.73E-06	3.34E-06	3.00E-06	2.68E-06	2.41E-06	2.16E-06	1.94E-06	1.74E-06	1.56E-06

$$P_i - P = \frac{162.6 q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23 + \frac{s}{1.1513}\right]$$

$$P_i - P = \frac{162.6 q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23 + 0.8685\, s\right]$$

$$\boldsymbol{if\ x > 0.01},\ then$$

$$P_i - P = -70.6\frac{q\beta\mu}{kh}\left[E_i\{-x\}\right]$$

The E_i function can be read from Table 1.1
Casting the pressure drop equation in dimensionless form (P_D), we have:

$$P_i - P = 141.2\frac{q\beta\mu}{kh}P_D$$

Where

Table 1.5 P_D vs t_D – Infinite radial system, constant rate at inner boundary

t_D	P_D	t_D	P_D	t_D	P_D
0.0000	0.0000	0.20	0.4241	50.00	2.3884
0.0005	0.0250	0.30	0.5024	60.00	2.4758
0.001	0.0352	0.40	0.5645	70.00	2.5501
0.002	0.4950	0.50	0.6167	80.00	2.6147
0.003	0.0603	0.60	0.6622	90.00	2.6718
0.004	0.0694	0.70	0.7024	100.00	2.7233
0.005	0.0774	0.80	0.7387	150.00	2.9212
0.006	0.0845	0.90	0.7716	200.00	3.0636
0.007	0.0911	1.00	0.8019	250.00	3.1726
0.008	0.0971	1.20	0.8672	300.00	3.2630
0.009	0.1028	1.40	0.9160	350.00	3.3394
0.010	0.1081	2.00	1.0195	400.00	3.4057
0.015	0.1312	3.00	1.1665	450.00	3.4641
0.020	0.1503	4.00	1.2750	500.00	3.5164
0.025	0.1669	5.00	1.3625	550.00	3.5643
0.030	0.1818	6.00	1.4362	600.00	3.6076
0.040	0.2077	7.00	1.4997	650.00	3.6476
0.050	0.2301	8.00	1.5557	700.00	3.6842
0.060	0.2500	9.00	1.6057	750.00	3.7184
0.070	0.2680	10.00	1.6509	800.00	3.7505
0.080	0.2845	15.00	1.8294	850.00	3.7805
0.090	0.2999	20.00	1.9601	900.00	3.8088
0.100	0.3144	30.00	2.1470	950.00	3.8355
0.15	0.3750	40.00	2.2824	1000.00	3.8584

$$P_D = -\frac{1}{2} E_i\{-x\}$$

$$x = \frac{\theta \mu C_t r^2}{4(0.000264)\, kt} = \frac{\theta \mu C_t {r_w}^2}{0.000264\, kt} \cdot \frac{1}{4}\, \frac{r^2}{{r_w}^2}$$

$$t_D = \frac{0.000264\, kt}{\theta \mu C_t {r_w}^2} \qquad r_D = \frac{r}{r_w}$$

$$x = \frac{1}{4} \frac{{r_D}^2}{t_D}$$

Table 1.6a P_D vs t_D – Finite radial system with closed exterior, constant rate at inner boundary

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD} = 3.0$		$r_{eD} = 3.5$		$r_{eD} = 4.0$	
t_D	P_D	t_D	P_D	t_D	P_D	t_D	P_D	t_D	P_D	t_D	P_D
0.06	0.251	0.22	0.443	0.40	0.565	0.52	0.627	1.00	0.802	1.50	0.927
0.08	0.288	0.24	0.459	0.42	0.576	0.54	0.636	1.10	0.830	1.60	0.948
0.10	0.322	0.26	0.476	0.44	0.587	0.56	0.645	1.20	0.857	1.70	0.968
0.12	0.355	0.28	0.492	0.46	0.598	0.6	0.662	1.30	0.882	1.80	0.988
0.14	0.387	0.30	0.507	0.48	0.608	0.65	0.683	1.40	0.906	1.90	1.007
0.16	0.420	0.32	0.522	0.50	0.618	0.7	0.703	1.50	0.929	2.00	1.025
0.18	0.452	0.34	0.536	0.52	0.628	0.75	0.721	1.60	0.951	2.20	1.059
0.20	0.484	0.36	0.551	0.54	0.638	0.8	0.740	1.70	0.973	2.40	1.092
0.22	0.516	0.38	0.565	0.56	0.647	0.85	0.758	1.80	0.994	2.60	1.123
0.24	0.548	0.40	0.579	0.58	0.657	0.9	0.776	1.90	1.014	2.80	1.154
0.26	0.580	0.42	0.593	0.60	0.666	0.95	0.791	2.00	1.034	3.00	1.184
0.28	0.612	0.44	0.607	0.65	0.688	1	0.806	2.25	1.083	3.50	1.255
0.30	0.644	0.46	0.621	0.70	0.710	1.2	0.865	2.50	1.130	4.00	1.324
0.35	0.724	0.48	0.634	0.75	0.731	1.4	0.920	2.75	1.176	4.50	1.392
0.40	0.804	0.50	0.648	0.80	0.752	1.6	0.973	3.00	1.221	5.00	1.46
0.45	0.884	0.60	0.715	0.85	0.772	2	1.076	4.00	1.401	5.50	1.527
0.50	0.964	0.70	0.782	0.90	0.792	3	1.328	5.00	1.579	6.00	1.594
0.55	1.044	0.80	0.849	0.95	0.812	4	1.578	6.00	1.757	6.50	1.66
0.60	1.124	0.90	0.915	1.00	0.832	5	1.828			7.00	1.727
0.65	1.204	1.00	0.982	2.00	1.215					8.00	1.861
0.70	1.284	2.00	1.649	3.00	1.506					9.00	1.994
0.75	1.364	3.00	2.316	4.00	1.977					10.00	2.127
0.80	1.444	5.00	3.649	5.00	2.398						

$$P_D = -\frac{1}{2}E_i\left\{-\frac{1 r_D{}^2}{4\, t_D}\right\}$$

For log approximation to hold,

$$\frac{1}{4}\frac{r_D{}^2}{t_D} < 0.0025 \qquad OR \qquad \frac{r_D{}^2}{t_D} < 0.01$$

Or

$$\frac{t_D}{r_D{}^2} > 100$$

If log approximation holds, then

Table 1.6b P_D vs t_D – Finite radial system with closed exterior, constant rate at inner boundary

$r_{eD} = 4.5$		$r_{eD} = 5.0$		$r_{eD} = 6.0$		$r_{eD} = 7.0$		$r_{eD} = 8.0$		$r_{eD} = 9.0$		$r_{eD} = 10.0$	
t_D	P_D	t_D	P_D	t_D	P_D	t_D	P_D	t_D	P_D	t_D	P_D	t_D	P_D
2.0	1.023	3.0	1.167	4.0	1.275	6.0	1.436	8.0	1.556	10.0	1.651	12.0	1.732
2.1	1.040	3.1	1.180	4.5	1.322	6.5	1.470	8.5	1.582	10.5	1.673	12.5	1.750
2.2	1.056	3.2	1.192	5.0	1.364	7.0	1.501	9.0	1.607	11.0	1.693	13.0	1.768
2.3	1.702	3.3	1.204	5.5	1.404	7.5	1.531	9.5	1.631	11.5	1.713	13.5	1.784
2.4	1.087	3.4	1.215	6.0	1.441	8.0	1.559	10.0	1.653	12.0	1.732	14.0	1.801
2.5	1.102	3.5	1.227	6.5	1.477	8.5	1.586	10.5	1.675	12.5	1.750	14.5	1.817
2.6	1.116	3.6	1.238	7.0	1.511	9.0	1.613	11.0	1.697	13.0	1.768	15.0	1.832
2.7	1.130	3.7	1.249	7.5	1.544	9.5	1.638	11.5	1.717	13.5	1.786	15.5	1.847
2.8	1.144	3.8	1.259	8.0	1.576	10.0	1.663	12.0	1.737	14.0	1.803	16.0	1.862
2.9	1.158	3.9	1.270	8.5	1.607	11.0	1.711	12.5	1.757	14.5	1.819	17.0	1.890
3.0	1.171	4.0	1.281	9.0	1.638	12.0	1.757	13.0	1.776	15.0	1.835	18.0	1.917
3.2	1.197	4.2	1.301	9.5	1.668	13.0	1.810	13.5	1.795	15.5	1.851	19.0	1.943
3.4	1.222	4.4	1.321	10.0	1.698	14.0	1.845	14.0	1.813	16.0	1.867	20.0	1.968
3.6	1.246	4.6	1.340	11.0	1.757	15.0	1.888	14.5	1.831	17.0	1.897	22.0	2.017
3.8	1.269	4.8	1.360	12.0	1.815	16.0	1.931	15.0	1.849	18.0	1.926	24.0	2.063
4.0	1.292	5.0	1.378	13.0	1.873	17.0	1.974	17.0	1.919	19.0	1.955	26.0	2.108
4.5	1.349	5.5	1.424	14.0	1.931	18.0	2.016	19.0	1.986	20.0	1.983	28.0	2.151
5.0	1.403	6.0	1.469	15.0	1.988	19.0	2.058	21.0	2.051	22.0	2.037	30.0	2.194
5.5	1.457	6.5	1.513	16.0	2.045	20.0	2.100	23.0	2.116	24.0	2.096	32.0	2.236
6.0	1.510	7.0	1.556	17.0	2.103	22.0	2.184	25.0	2.180	26.0	2.142	34.0	2.278
7.0	1.615	7.5	1.598	18.0	2.160	24.0	2.267	30.0	2.340	28.0	2.193	36.0	2.319
8.0	1.719	8.0	1.641	19.0	2.217	26.0	2.351	35.0	2.449	30.0	2.244	38.0	2.360

(continued)

Table 1.6b (continued)

9.0	1.823	9.0	1.725	20.0	2.274	28.0	2.434	40.0	2.658	34.0	2.345	40.0	2.401
10.0	1.927	10.0	1.808	25.0	2.560	30.0	2.517	45.0	2.817	38.0	2.446	50.0	2.604
11.0	2.031	11.0	1.892	30.0	2.846					40.0	2.496	60.0	2.806
12.0	2.135	12.0	1.975							45.0	2.621	70.0	3.008
13.0	2.239	13.0	2.059							50.0	2.746	80.0	3.210
14.0	2.343	14.0	2.142							60.0	2.996	90.0	3.412
15.0	2.447	15.0	2.225							70.0	3.246	100.0	3.614

$$Ei(-x) = \ln 1.781x = \ln\left(1.781.\frac{1{r_D}^2}{4\ t_D}\right) = \ln\left(0.44525\frac{{r_D}^2}{t_D}\right)$$

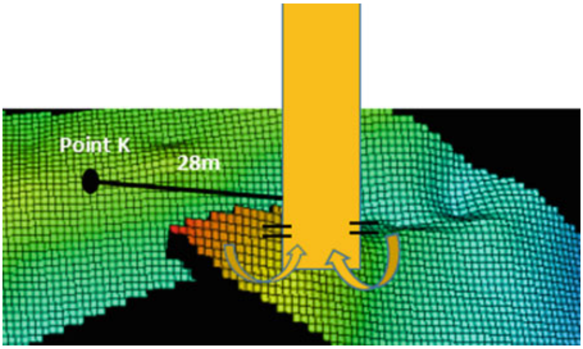
$$= \ln 0.44525 + \ln\left(\frac{{r_D}^2}{t_D}\right) = -080912 + \ln\left(\frac{{r_D}^2}{t_D}\right) = 080912 - \ln\left(\frac{{r_D}^2}{t_D}\right)$$

$$P_D = \frac{1}{2}\left[\ln\left(\frac{t_D}{{r_D}^2}\right) + 0.80912\right] = \frac{1}{2}\left[2.303\log\left(\frac{t_D}{{r_D}^2}\right) + 0.80912\right]$$

$$P_D = 1.151\log\left(\frac{t_D}{{r_D}^2}\right) + 0.4046$$

Example 1.8

Given the following data of a well as shown in the figure below in an infinite acting reservoir.



Wellbore radius, r_w	*0.425 ft*
Drainage radius, r_e	*540 ft*
Reservoir height, h	*35 ft*
Initial reservoir pressure, P_i	*2350 psi*
Porosity, ∅	*0.23*
Formation absolute permeability, k	*165 mD*
Oil viscosity, μ_o	*0.37 cp*
Oil formation volume factor, β_o	*1.218 rb/stb*
Total compressibility, C_t	*3.5 x 10^{-5} psi^{-1}*
Oil flow rate, q_o	*472 stb/day*

Calculate the pressure at point k after 6 h production.

Solution

We need to check if log approximation holds

$$x = \frac{\theta \mu C_t r^2}{4(0.000264)\ kt} = \frac{0.23*0.37*3.5 \times 10^{-5}*(28.5)^2}{4(0.000264)*165*6} = 0.00231$$

Since

$$\frac{\theta \mu C_t r^2}{4(0.000264)\ kt} = 0.00231 < 0.01$$

Therefore, log approximation holds

$$P_i - P(r,t) = \frac{162.6 q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23\right]$$

$$P(r,t) = P_i - \frac{162.6 q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23\right]$$

$$\begin{aligned} P(28.5\,ft, 6\ hrs) &= 2350 - \frac{162.6*472*1.218*0.37}{165*35} \\ &\quad \times \left[\log 6 + \log\left\{\frac{165}{0.23*0.37*3.5 \times 10^{-5}*(28.5)^2}\right\} - 3.23\right] \\ &= 2335.77\ psi \end{aligned}$$

Example 1.9

Use the information given in Example 1.7 to calculate the following:

- The pressure at a distance (radius) of 120 ft. after 6 h production
- The pressure at a distance (radius) of 210 ft. after 9 h production

Solution

We need to check if log approximation holds at 120 ft. and 6 h

$$x = \frac{\theta \mu C_t r^2}{4(0.000264)\ kt} = \frac{0.23*0.37*3.5 \times 10^{-5}*(120)^2}{4(0.000264)*165*6} = 0.041$$

Since

$$\frac{\theta\mu C_t r^2}{4(0.000264)\,kt} = 0.041 > 0.01$$

Therefore, log approximation does not hold, the Ei function is used

$$P(r,t) = P_i + 70.6\frac{q\beta\mu}{kh}E_i\left\{-\frac{\theta\mu C_t r^2}{0.001056\,kt}\right\}$$

$$P(120\,ft, 6\,h) = 2350 + 70.6\left[\frac{472*1.218*0.37}{165*35}\right]E_i\{-0.041\}$$

$$E_i\{-0.041\} = -2.658$$

$$P(120\,ft,\ \ 6\,h) = 2350 + 2.6004\{-2.658\} = 2343.088\,psi$$

The wellbore flowing pressure at a distance (radius) of 210 ft. after 9 h production
We need to check if log approximation holds at 210 ft. and 9 h

$$x = \frac{\theta\mu C_t r^2}{4(0.000264)\,kt} = \frac{0.23*0.37*3.5\times10^{-5}*(210)^2}{4(0.000264)*165*9} = 0.0838$$

Since

$$\frac{\theta\mu C_t r^2}{4(0.000264)\,kt} = 0.0838 > 0.01$$

Therefore, log approximation does not hold, the Ei function is used

$$P(r,t) = P_i + 70.6\frac{q\beta\mu}{kh}E_i\left\{-\frac{\theta\mu C_t r^2}{0.001056\,kt}\right\}$$

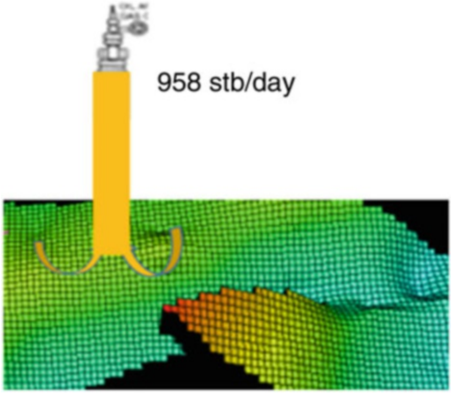
$$P(210\,ft, 9\,h) = 2350 + 70.6\left[\frac{472*1.218*0.37}{165*35}\right]E_i\{-0.0838\}$$

$$E_i\{-0.0838\} = -1.982$$

$$P(210\,ft, 9\,h) = 2350 + 2.6004\{-1.982\} = 2344.846\,psi$$

Example 1.10

Calculate the skin factor and pressure drop at a distance of 19.5 ft from the wellbore in the figure below after it has flown for 5 h.



Wellbore radius, r_w	*0.5 ft*
Drainage radius, r_e	*797 ft*
Reservoir height, h	*74 ft*
Initial reservoir pressure, P_i	*4050 psi*
Flowing bottomhole pressure	*3850 psi*
Porosity, ∅	*0.24*
Formation absolute permeability, k	*150 mD*
Oil viscosity, μ_o	*0.89 cp*
Oil formation volume factor, β_o	*1.238 rb/stb*
Total compressibility, C_t	*$7.8 \text{ x } 10^{-5}$ psi^{-1}*
Oil flow rate, q_o	*958 stb/day*

Solution

$$x = \frac{\theta \mu C_t r^2}{4(0.000264)\, kt} = \frac{0.24 * 0.85 * 7.8 \times 10^{-5} * (19.5)^2}{4(0.000264) * 150 * 5} = 0.00764$$

Since

$$\frac{\theta\mu C_t r^2}{4(0.000264)\,kt} = 0.00764 < 0.01$$

Therefore, log approximation holds

$$P_i - P = \frac{162.6q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23 + 0.8685\,s\right]$$

$$4050 - 3850 = \frac{162.6*958*1.238*0.85}{150*74}\left[\log 5 + \log\left\{\frac{150}{0.24*0.85*7.8*10^{-5}*(19.5)^2}\right\} -3.23 + 0.8685\,s\right]$$

$$200 = 14.7674[1.8633 + 0.8685\,s]$$

$$\frac{200}{14.7674} = 1.8633 + 0.8685\,s$$

$$13.5433 = 1.8633 + 0.8685\,s$$

$$s = \frac{13.5433 - 1.8633}{0.8685} = 13.4485$$

The pressure drop due to skin is

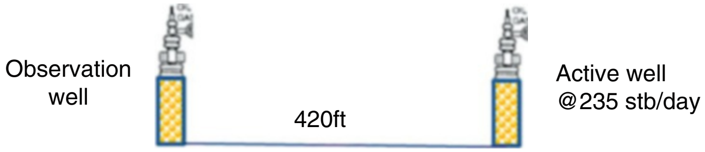
$$\Delta P_s = \frac{141.2q\beta\mu}{kh}s$$

$$\Delta P_s = \frac{141.2*958*1.238*0.85}{150*74} * 13.4485 = 172.4609\,psi$$

Example 1.11

The figure below shows an active and an observation well. The active well is currently producing at a flow rate of 235 stb/day after a period of 10 h. Calculate the pressure change in the observation well at:

- A distance of 420 ft
- The wellbore



Wellbore radius, r_w	*0.48 ft*
Total compressibility, C_t	*$18.9*10^{-6}$ psi^{-1}*
Reservoir height, h	*35 ft*
Initial pressure,P_i	*3860 psi*
Formation permeability, k	*997 mD*
Oil viscosity, μ_o	*0.86 cp*
Oil formation volume factor, β_o	*1.132 rb/stb*
Porosity, ∅	*20.7%*

Solution
Check if log approximation holds

$$x = \frac{\theta\mu C_t r^2}{4(0.000264)\,kt} = \frac{0.207*0.86*18.9\times10^{-6}*(420)^2}{4(0.000264)*997*10} = 0.0564$$

Since

$$\frac{\theta\mu C_t r^2}{4(0.000264)\,kt} = 0.0564 > 0.01$$

Therefore, log approximation will not hold, the Ei function is used

$$P(r,t) = P_i + 70.6\frac{q\beta\mu}{kh}E_i\left\{-\frac{\theta\mu C_t r^2}{0.001056\,kt}\right\}$$

$$P(420\,ft, 10\,h) = 3860 + 70.6\left[\frac{235*1.132*0.86}{997*45}\right]E_i\{-0.0564\}$$

$$E_i\{-0.0564\} = -2.360$$

$$P(420\,ft, 10\,h) = 3860 + 0.3600\{-2.360\} = 3859.1504\,psi$$

The pressure change is given as:

$$\varDelta P = P_i - P(r,t)$$

$$\varDelta P = 3860 - 3859.1504 = 0.8496\,psi$$

Pressure change at the wellbore

Check if log approximation holds

$$x = \frac{\theta \mu C_t r^2}{4(0.000264)\,kt} = \frac{0.207*0.86*18.9 \times 10^{-6}*(0.48)^2}{4(0.000264)*997*10} = 7.36*10^{-8}$$

Since

$$\frac{\theta \mu C_t r^2}{4(0.000264)\,kt} = 7.36*10^{-8} < 0.01$$

Therefore, log approximation holds

$$P_i - P = \frac{162.6 q\beta\mu}{kh}\left[\log t + \log\left\{\frac{k}{\theta\mu C_t r^2}\right\} - 3.23 + 0.8685\,s\right]$$

$$\begin{aligned}\varDelta P(r_w,t) &= \frac{162.6*235*1.132*0.86}{997*35} \\ &\quad \times \left[\log 10 + \log\left\{\frac{997}{0.207*0.86*18.9*10^{-6}(0.48)^2}\right\} - 3.23\right] \\ &= 7.3335\,psi\end{aligned}$$

1.5.3.2 Pseudo-Steady or Semi-State Flow

A reservoir attains pseudo-steady state (PSS), if the rate of change of pressure decline with time is constant. The pressure throughout the reservoir decreases at the same constant rate, this scenario cannot occur during build-up or falloff tests. In this state of flow, the boundary has been felt and static pressure at the boundary is declining uniformly throughout the reservoir. Mathematically, this definition states that the rate of change of pressure with respect to time at every position in the reservoir is constant, or a state where the mass rate of production is equal to the rate of mass depletion. This state can also be referred to as semi-steady state (SSS) or quasi-steady state.

$$\frac{\partial P}{\partial t} = constant$$

The radial flow equation for pseudo-steady state (bounded reservoir)

For oil

$$q = \frac{2\pi kh\left(P_e - P_{wf}\right)}{\mu\left[\ln\frac{r_e}{r_w} - \frac{3}{4}\right]}$$

Oil field units

$$q = \frac{7.08 \times 10^{-3} k(mD) h(ft)\left(P_e - P_{wf}\right)(psi)}{\mu\beta\left[\ln\frac{r_e}{r_w} - \frac{3}{4}\right]}$$

Effect of skin

$$q = \frac{2\pi kh\left(P_e - P_{wf}\right)}{\mu\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}$$

Oil field units

$$q = \frac{7.08 \times 10^{-3} k(mD) h(ft)\left(P_e - P_{wf}\right)(psi)}{\mu\beta\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}$$

For gas

$$q = 6.93 \times 10^{-4} \ \frac{kh\ \left({P_e}^2 - {P_{wf}}^2\right)}{T\left(\overline{\mu_g z}\right)\ \left[\ln\frac{r_e}{r_w} - \frac{3}{4}\right]}$$

Effect of skin

$$q = 6.93 \times 10^{-4} \ \frac{kh\ \left({P_e}^2 - {P_{wf}}^2\right)}{T\left(\overline{\mu_g z}\right)\ \left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}$$

Example 1.12
Given the following data below for a vertical well under pseudo steady state.

Wellbore radius, r_w	*5 in*
Drainage radius, r_e	*970 ft*
Reservoir height, h	*49 ft*
Pressure at the outer boundary, P_e	*5400 psi*
Wellbore flowing pressure, P_{wf}	*4800 psi*
Formation permeability, k	*45 mD*

(continued)

Oil viscosity, μ_o	*0.48 cp*
Oil formation volume factor, β_o	*1.3125 rb/stb*

(a) Calculate the flow rate of the well?

(b) Also, at what rate will the well be flowing if the bottom hole pressure is reduced to 3750 psi?

Solution

(a) Flow rate of the well

$$q = \frac{7.08 \times 10^{-3} k(mD) h(ft) \left(P_e - P_{wf}\right)(psi)}{\mu\beta\left[\ln\frac{r_e}{r_w} - \frac{3}{4}\right]}$$

Convert from inch to ft 5 in $= {}^5\!/_{12} = 0.4167\, ft$

$$q = \frac{7.08 \times 10^{-3} \times 45 \times 49 \times (5400 - 4800)(psi)}{0.48 \times 1.3125 \times \left[\ln\left(\frac{970}{0.4167}\right) - \frac{3}{4}\right]} = 2123.1857\, stb/d$$

(b) If the bottom hole pressure is reduced to 3750 psia, the rate is calculated as

$$q = \frac{7.08 \times 10^{-3} \times 45 \times 49 \times (5400 - 3750)(psi)}{0.48 \times 1.3125 \times \left[\ln\left(\frac{970}{0.4167}\right) - \frac{3}{4}\right]} = 5838.76\, stb/d$$

A substantial increase in production was observed when the bottom hole flowing pressure was reduced.

Example 1.13

Calculate the flowing bottomhole pressure for a vertical well in level J6 reservoir undergoing a pseudo steady state flow regime with the following data. Also, if the flow rate of the well increases by 28%, what will be the bottomhole flowing pressure?

Wellbore radius, r_w	*0.375 ft*
Drainage radius, r_e	*1150 ft*
Reservoir height, h	*53 ft*
Pressure at the outer boundary, P_e	*3857 psi*
Flow rate, q	*1835 stb/d*
Formation absolute permeability, k	*144 mD*
Oil viscosity, μ_o	*0.92 cp*

(continued)

Oil formation volume factor, β_o	*1.2643 rb/stb*
Skin, s	*4.16*

Solution

The flow equation for a pseudo steady state is given as

$$q = \frac{7.08 \times 10^{-3} k(mD) h(ft) \left(P_e - P_{wf}\right)(psi)}{\mu\beta\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}$$

Thus, the bottomhole flowing pressure is calculated as

$$P_{wf} = P_e - \frac{q\mu\beta\left[\ln\frac{r_e}{r_w} - \frac{3}{4} + s\right]}{7.08 \times 10^{-3} kh}$$

$$P_{wf} = 3857 - \frac{1835 * 0.92 * 1.2643 * \left[\ln\left(\frac{1150}{0.375}\right) - \frac{3}{4} + 4.16\right]}{7.08 \times 10^{-3} * 144 * 53} = 3405.18\, psi$$

If the rate increase by 28%, the new well flow rate becomes,

$$28\% of\ 1835 + 1835 = 0.28 * 1835 + 1835 = 2348.8\, stb/d$$

$$P_{wf} = 3857 - \frac{2348.8 * 0.92 * 1.2643 * \left[\ln\left(\frac{1150}{0.375}\right) - \frac{3}{4} + 4.16\right]}{7.08 \times 10^{-3} * 144 * 53} = 3278.67\, psi$$

1.6 Productivity Index (PI or j)

The productivity index is calculated mathematically as

$$PI = j = \frac{q}{P_e - P_{wf}} = \frac{q}{\Delta P}$$

$$PI = j = \frac{7.08 \times 10^{-3} k(mD) h(ft)}{\mu\beta \ln\frac{r_e}{r_w}}$$

Specific productivity index is given as:

$$j_s = \frac{j}{h} = \frac{7.08 \times 10^{-3} k(mD)}{\mu\beta \ln \frac{r_e}{r_w}}$$

1.6.1 Factors Affecting the Productivity Index

- Phase Behaviour of Fluids in the Reservoir
- Relative Permeability
- Oil Viscosity
- Oil Formation Volume Factor
- Skin

1.6.2 Phase Behaviour in Petroleum Reservoirs

As reservoir pressure drops below the bubble point, free gas begins to form and thus the oil relative permeability (k_{ro}) is reduced. If a well is produced at a flow rate that requires the wellbore flowing pressure (P_{wf}) to be less than the bubble point pressure (P_b), the oil relative permeability and the productivity index (PI) will be decreased around the wellbore.

1.6.3 Relative Permeability Behaviour

As free gas form in the pores of a reservoir rock, the ability of the liquid phase to flow is decreased. Even though the gas saturation may not be great enough to allow gas to flow, the space occupied by the gas reduces the effective flow area of the liquid. Conversely, in gas reservoir, the relative permeability to gas will be decreased if liquid saturation develops either as a result of retrograde condensation or water formation in the pores.

1.6.4 Oil Viscosity Behaviour

The viscosity of oil saturated with gas at constant temperature will decrease as pressure is decreased from an initial pressure to bubble point pressure (P_b). Below P_b, the viscosity will increase as gas comes out of solution leaving the heavier components of the hydrocarbon.

1.6.5 Oil Formation Volume Factor

As pressure is decreased in the reservoir, the hydrocarbon will expand and when the bubble point pressure is reached for an oil reservoir, gas starts coming out of solution which causes the oil to shrink thereby reducing the volume of the oil.

1.6.6 Skin

A well that is damaged results in low fluids flow potential. Thus, formation damage is an impairment of reservoir permeability around the wellbore, leading to low or no well production or injection. Or simply refers to the decrease in permeability that occurs in the near wellbore region of a reservoir. Formation damage is often quantified by “Skin” factor. Skin is strictly a measure of an excess pressure in the producing formation as fluids flow into a well. Skin alters the flow of fluid; that is an impairment to flow.

The excess pressure drop can occur from one or several of a wide variety of causes such as drilling mud, cement, completion fluid filtrate invasion, solids invasion, perforating damage, fines migration, formation compaction, swelling clays, asphaltene/paraffin deposition, scale precipitation, emulsions, reservoir compaction, relative permeability effects, effects of stimulation treatments, etc.

Example 1.14

A reservoir model developed for FUPRE field shows that the formation is made up of two stack reservoir with a vertical well producing oil at a steady state from a cylindrical region. The reservoir and well data is given as:

Wellbore radius, r_w	*4 in*
Drainage radius, r_e	*600 ft*
Reservoir height, h	*40 ft*
Pressure drop,ΔP	*200 psi*
Formation permeability, k	*120 mD*
Oil viscosity, μ_o	*1.86 cp*
Oil formation volume factor, β_o	*1.395 rb/stb*

Calculate the following:

I. The productivity index
II. The specific productivity index
III. The rate at which the well is flowing

Solution

$$j = \frac{q}{\Delta P} = \frac{7.08 \times 10^{-3} k(mD)h(ft)}{\mu\beta \ln \frac{r_e}{r_w}}$$

Convert from inch to ft 4 in = $4/12 = 0.3333\ ft$
Productivity index

$$j = \frac{q}{\Delta P} = \frac{7.08 \times 10^{-3} \times 120 \times 40}{1.86 \times 1.395 \times \ln\left(\frac{600}{0.3333}\right)} = 1.7473\ ^{bbl}/_{d-psi}$$

Specific productivity index

$$j_s = \frac{j}{h} = \frac{1.7473}{40} = 0.0437\ ^{bbl}/_{d-psi-ft}$$

Flow rate

$$q = j \times \Delta P$$

$$q = 1.7473 \times 200 = 349.46\ ^{bbl}/_{d}$$

1.7 Application of Dimensionless Parameters in Calculating Flow Rate and Bottom Flowing Pressure

Now, let us write the pressure drop in dimensionless pressure

$$P_i - P = 141.2 \frac{q\beta\mu}{kh} P_D(r_D, t_D)$$

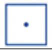
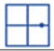
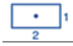
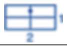
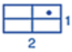
Also, dimensionless time base on drainage area (t_{AD}) is given as

$$t_{AD} = \frac{0.000264\ kt}{\varnothing h A C_t} \quad \text{for t in hrs and in oil field unit}$$

For infinite reservoir undergoing transient state condition, the dimensionless pressure is given as:

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln \frac{t_D}{{r_D}^2} + 0.80907\right]$$

Table 1.7 Shape Factors for Various Closed Single – Well Drainage Areas

In bounded reservoirs	C_A	Exact for $t_{DA}>$	Less than 1% error for $t_{DA}<$	Use infinite system solution with less than 1% error for $t_{DA}<$
			Apply for pseudo steady state flow	Apply for transient state flow
	31.62	0.1	0.06	0.1
	31.6	0.1	0.06	0.101
	27.6	0.2	0.07	0.09
	27.1	0.2	0.07	0.09
	21.9	0.4	0.12	0.08
	0.098	0.9	0.6	0.015
	30.8828	0.1	0.05	0.09
	12.9851	0.7	0.25	0.03
	4.5132	0.6	0.3	0.025
	3.3351	0.7	0.25	0.01
	21.8369	0.3	0.15	0.025
	10.8374	0.4	0.15	0.025
	4.5141	1.5	0.5	0.06
	2.0769	1.7	0.5	0.02
	3.1573	0.4	0.15	0.005

For a closed reservoir undergoing transient state condition, the dimensionless pressure based on area is given as:

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln t_{DA} + \ln \frac{A}{{r_w}^2} + 0.80907\right]$$

At the well, $r_D = 1$, i.e. $r = r_w$

For a pseudo-steady state flow condition, the dimensionless pressure based on area is given as:

$$P_D(r_D, t_D) = 2\pi t_{AD} + \frac{1}{2}\ln\frac{A}{{r_w}^2} + \frac{1}{2}\ln\frac{2.2458}{C_A} = 2\pi t_{AD} + \frac{1}{2}\ln\frac{2.2458A}{C_A {r_w}^2}$$

Substitute t_{AD} into the above equation gives

$$P_D = 2\pi\frac{0.000264kt}{\varnothing hC_tA} + \frac{1}{2}\ln\frac{A}{{r_w}^2} + \frac{1}{2}\ln\frac{2.2458A}{C_A}$$

$$P_D = \frac{0.001659kt}{\varnothing hC_tA} + \frac{1}{2}\ln\frac{2.2458A}{\mathrm{C}_A {r_w}^2}$$

Where C_A is the shape factor read from Table 1.7. The above equation applies to different systems when t_{AD} is greater than the time under the column with heading, "exact for t_{AD} >" given in Table 1.7. With a maximum of 1% error, the time when pseudo-state will start can be read from the column with the heading "less than 1 percent error for t_{AD} >." This makes it possible to use the pseudo-state equation to calculate pressure in a period that ideally is considered a transition period (transition from transient to pseudo-steady state). Figure 1.18 shows a comprehensive shape factors for various reservoir shapes and well locations.

Therefore, for a pseudo steady state flow; the equation becomes

$$P_i - P_{wf} = \frac{141.2q\mu B_o}{kh}\left[\frac{0.001659\,kt}{\varnothing hC_tA} + \frac{1}{2}\ln\frac{2.2458}{C_A {r_w}^2}\right]$$

Thus, for any time $t = 1, 2, 3\ldots$ n hours, we can get the corresponding bottom hole flowing pressure P_{wf} and these pressure obtained from the series of time generated to abandonment time will be used in the prediction stage.

This implies that;

$$P_{wf} = P_i - \frac{141.2q\mu B_o}{kh}\left[\frac{0.001659\,kt}{\varnothing hC_tA} + \frac{1}{2}\ln\frac{2.2458}{C_A {r_w}^2}\right]$$

Also the time to reach the bottom hole flowing pressure (P_{wf}) *is given as*:

$$t(hr) = \frac{\varnothing hC_tA}{0.001659\,k}\left[\frac{kh\left(P_i - P_{wf}\right)}{141.2q\mu B_o} - \frac{1}{2}\ln\frac{2.2458}{C_A {r_w}^2}\right]$$

- Check if log approximation holds

  It holds if

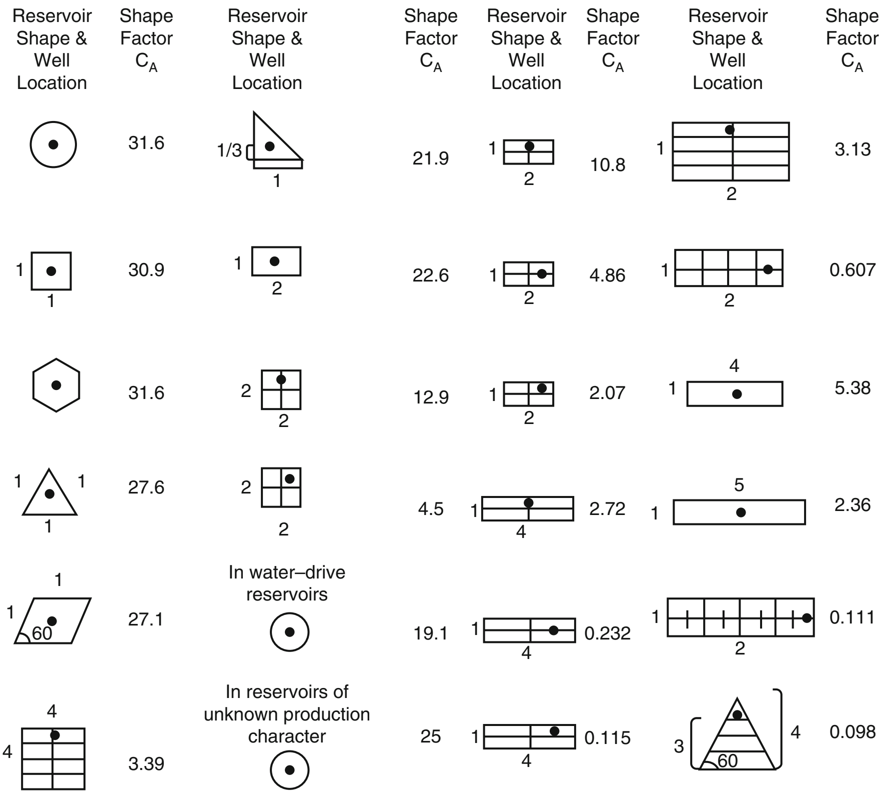


Fig. 1.18 Reservoir shape factors

$$t_{AD} = \frac{0.000264\,kt}{\varnothing hAC_t} > 100$$

Or

$$x = \frac{\theta\mu C_t r^2}{4(0.000264)\,kt} < 0.01$$

- Check for the flow regime at the given shape of the reservoir

Note, at any given time, the reservoir will behave like an infinite acting system, that is, the reservoir is still undergoing transient flow condition if

$$t_{DA(calculated)} < t_{DA(tabulated)}$$

Thus, P_D is calculated based on area as:

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln t_{DA} + \ln\frac{A}{{r_w}^2} + 0.80907\right]$$

For consistency in units, the area, A must be converted to feet (multiply by "43560")

Else, the reservoir has attained pseudo-steady state (bounded reservoir) if

$$t_{DA(calculated)} > t_{DA(tabulated)}$$

Then used

$$P_D(r_D, t_D) = 2\pi t_{AD} + \frac{1}{2}\ln\frac{A}{{r_w}^2} + \frac{1}{2}\ln\frac{2.2458}{C_A} = 2\pi t_{AD} + \frac{1}{2}\ln\frac{2.2458A}{C_A{r_w}^2}$$

Therefore,

$$P_i - P_{wf} = \frac{141.2q\mu B_o}{kh}\left[P_D(r_D, t_D) + s\right]$$

Example 1.15

Given a well located at the centre of a square reservoir producing at a rate of 150 stb/day of dry oil with the following reservoir and fluid properties:

Wellbore radius, r_w	*0.5 ft*
Area, A	*45 acres*
Reservoir height, h	*49 ft*
Initial reservoir pressure,, P_i	*3325 psi*
Porosity,∅	*18%*
Skin, s	*3.04*
Formation permeability, k	*100 mD*
Oil viscosity, μ_o	*4.18 cp*
Oil formation volume factor, β_o	*1.2125 rb/stb*
Total compressibility, C_t	*$24.9*10^{-6}$ psi^{-1}*

- Estimate the buttonhole flowing pressure at a period of 14 min of production
- Estimate the buttonhole flowing pressure at a period of 13 h of production
- Estimate the buttonhole flowing pressure at a period of 5 h of production at a dimensionless radius r_D of 4.5.

Solution

- Check log approximation applicability

$$t_D = \frac{0.000264\,kt}{\varnothing hAC_t} > 100$$

$$t_{DA} = \frac{0.000264*100*t}{0.18*49*45*43560*24.9*10^{-6}} > 0.0000613*t$$

To use the above equation, the time must be in hour. Thus, we have to convert the 14 min to hour. That is

$$t = \frac{14}{60} = 0.2333\,h$$

At t = 0.2333 h

$$t_{DA} = 0.0000613*t = 0.0000613*0.2333 = 0.0000143$$

Since

Therefore, log approximation will not hold

From Table 1.7, under column 5, $t_{DA(calculated)} = 0.09$

Check for the flow regime

Since $t_{DA(calculated)} < t_{DA(tabulated)}$

Therefore, the closed square with well at the centre is still undergoing transient state condition

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln t_{DA} + \ln\frac{A}{{r_w}^2} + 0.80907\right]$$

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln 0.0000143 + \ln\left(\frac{43560*45}{0.5^2}\right) + 0.80907\right] = 2.76445$$

$$P_i - P_{wf} = \frac{141.2q\mu B_o}{kh}\left[P_D(r_D, t_D) + s\right]$$

$$P_{wf} = P_i - \frac{141.2q\mu B_o}{kh}\left[P_D(r_D, t_D) + s\right]$$

$$P_{wf} = 3325 - \frac{141.2*150*4.18*1.2125}{100*49}[2.76445 + 3.04] = 3197.8404\,psi$$

At t = 13 h

$$t_{AD} = 0.0000613*t = 0.0000613*13 = 0.000797$$

Since $t_{DA(calculated)} < t_{DA(tabulated)}$, the closed square with well at the centre is still undergoing transient state condition

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln 0.000797 + \ln\left(\frac{43560*45}{0.5^2}\right) + 0.80907\right] = 4.7746$$

$$P_{wf} = 3325 - \frac{141.2*150*4.18*1.2125}{100*49}[4.7746 + 3.04] = 3153.8036\,psi$$

Example 1.16

A well located at the centre of a 2:1 rectangular reservoir as shown in the figure below, is producing at a steady rate of 500 stb/day of dry oil with the following reservoir and fluid properties:

Wellbore radius, r_w	*0.5 ft*
Area, A	*50 acres*
Reservoir height, h	*65 ft*
Initial reservoir pressure,, P_i	*4025 psi*
Porosity,∅	*28%*
Skin, s	*16.85*
Formation permeability, k	*580 Md*
Oil viscosity, μ_o	*2.2 cp*
Oil formation volume factor, β_o	*1.264 rb/stb*
Total compressibility, C_t	*15.0*10^{-6} psi^{-1}*

Perform the following calculations:

- At what time will log approximation hold
- What will be the pressure drop at the well after flowing for 27 min?
- What will be the pressure drop at the well after flowing for 5 months?

Solution

- At what time will log approximation hold

For log approximation to hold,

$$t_D = \frac{0.000264\,kt}{\varnothing h C_t r^2} > 100$$

At the wellbore

$$t = \frac{\varnothing h C_t {r_w}^2 t_D}{0.000264\,k} > 100$$

$$t = \frac{0.28*65*15.0*10^{-6}(0.5)^2 t_D}{0.000264*580} > 0.00151\ h$$

What will be the pressure drop at the well after flowing for 27 min?

Check for log approximation

The dimensionless time based on area is given as

$$t_{DA} = \frac{0.000264\,kt}{\varnothing h C_t A}$$

$$t = \frac{20}{60} = 0.3333$$

$$t_{DA} = \frac{0.000264*580*0.3333}{0.28*2.2*15.0*10^{-6}*50*43560} = 0.002536$$

Since t_{DA} < 100, therefore log approximation will not hold

Check for flow regime at the given shape of the reservoir

t_{DA} is read from Table 1.7 under column 5 at the centre of the 2:1 rectangle. Hence,

$$t_{DA(tabulated)} = 0.025$$

Since $t_{DA(calculated)}$ < $t_{DA(tabulated)}$. That is

$$0.003424 < 0.025$$

Therefore, the close 2:1 rectangle with well at the centre is undergoing transient state flow condition.

Thus, P_D is calculated based on area as:

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln t_{DA} + \ln\frac{A}{{r_w}^2} + 0.80907\right]$$

$$P_D(r_D, t_D) = \frac{1}{2}\left[\ln 0.002536 + \ln\left(\frac{43560*50}{0.5^2}\right) + 0.80907\right] = 5.4061$$

$$P_i - P_{wf} = \Delta P = \frac{141.2q\mu B_o}{kh}\left[P_D(r_D, t_D) + s\right]$$

$$\Delta P = \frac{141.2*500*2.2*1.264}{580*65}[5.4061 + 16.85] = 115.8997\,psi$$

What will be the pressure drop at the well after flowing for 5 months?

$$t = 5\,months = 5*30.4*24 = 3648\,h$$

Check for log approximation

The dimensionless time based on area is given as

$$t_{DA} = \frac{0.000264\,kt}{\varnothing h C_t A}$$

$$t_{DA} = \frac{0.000264*580*3648}{0.28*2.2*15.0*10^{-6}*50*43560} = 27.7560$$

Since $t_{DA} < 100$, therefore log approximation will not hold

Check for flow regime at the given shape of the reservoir

t_{DA} is read from Table 1.7 under column 5 at the centre of the 2:1 rectangle. Hence,

$$t_{DA(tabulated)} = 0.025$$

$$t_{DA(calculated)} > t_{DA(tabulated)}$$

Since $t_{DA(calculated)} > t_{DA(tabulated)}$, it implies that the system has attained pseudo-steady state of production. The shape factor is read from Table 1.7 under column 2 as:

$$C_A = 21.8369$$

Thus, P_D is calculated based on area as:

$$P_D(r_D, t_D) = 2\pi t_{AD} + \frac{1}{2}\ln\frac{A}{{r_w}^2} + \frac{1}{2}\ln\frac{2.2458}{C_A} = 2\pi t_{AD} + \frac{1}{2}\ln\frac{2.2458A}{C_A {r_w}^2}$$

$$\begin{aligned} P_D(r_D, t_D) &= 2{*}3.142{*}27.7560 \\ &\quad + \frac{1}{2}\ln\left(\frac{50{*}43560}{0.5^2}\right) + \frac{1}{2}\ln\left(\frac{2.2458}{21.8369}\right) = 181.2716 \end{aligned}$$

$$\varDelta P = \frac{141.2{*}500{*}2.2{*}1.264}{580{*}65}[181.2716 + 16.85] = 1031.7273\,psi$$

Exercises

1. Mention the three ways rock can be formed and which of them forms the largest share of the rocks on the earth's surface?.
2. List the key elements required to define a petroleum reservoir
3. Differentiate between a cap rock and a real
4. Rivers have been washing gravel, sand and mud down into Sydney harbour for thousands of years. Deep in the ancient deposits these materials have been packed down by the weight of overlying layers and pore spaces have been in-filled by cement-like carbonate minerals. Are the rocks that are forming Igneous, Sedimentary or Metamorphic? Justify your answer
5. The ore body at Broken Hill was found associated with layers of rock that frequently consisted of interbedded quartzites and garnet schists. Are they Igneous, Sedimentary or Metamorphic rocks and why?
6. Distinguish between drainage and imbibition process
7. In one phase envelop diagram, draw the following: black oil, volatile, condensate, wet and dry gas reservoir.
8. What is the term that defined the phase in the development of a petroleum system during which hydrocarbons migrate into the porous and permeable rock formation (the reservoir) and remain trapped
9. When the capillary pressure across the pore throats is greater than or equal to the buoyancy pressure of the migrating hydrocarbons. What term is this?
10. Which of these is not associated with fault: single, parallel, perpendicular, sealing and non-seal
11. Which of these is not a process that culminate into trap: formation of anticlines, folds, syncline and domes
12. A fluid flow process in which the saturation of the nonwetting phase increases.
13. Which concept defines a case when the mobility increases with saturation of the nonwetting phase?
14. A fluid flow process in which the saturation of the wetting phase increases.
15. When mobility increases with saturation of the wetting phase. The term is called

16. The classification of a hydrocarbon reservoir is basically dependent on the following except:
    - The composition of the hydrocarbon mixture in the reservoir,
    - The amount of the fluid in place,
    - The location of the initial pressure and temperature of the reservoir and
    - The condition of the surface (separator) production pressure and temperature.
17. A phase envelope or pressure-temperature (PT) phase diagram of a particular fluid system comprises of two major curves. These are
18. How many types of reservoir can be identified beyond the dew point curve? Name them.
19. The region where gas and liquid coexist in equilibrium is called
20. The region of quality lines identified in a pressure-temperature diagram is called
21. A reservoir whose fluid remains as a single phase liquid at the wellbore is called
22. What type of reservoir is identified when the pressure and temperature conditions existing in the separator indicate a high percentage of liquid around 85%
23. A black oil is often called
24. Which reservoir is characterized by a dark or deep color liquid having initial gas-oil ratios of 500 scf/stb or less, oil gravity of 30° API?
25. Which reservoir is characterized by a brown, orange, or green color liquid oil gravity of 40° API or higher and 65% of the reservoir is liquid at the separator condition
26. What is the range of a gas condensate reservoir's API oil gravity?
27. A gas reservoir whose production path passes through the two phase region is called
28. A fluid whose volume or density does not change with pressure is called
29. Which fluid experience large changes in volume as a function of pressure
30. When fluids move in a multi-direction within the reservoir towards the perforations at the wellbore creating an iso-potential lines. What type of flow system is this?
31. A system of mass flow rate, where there is no accumulation of mass within any component in the system is called
32. The flow of fluid across the boundaries of the reservoir
33. What type of flow is experienced in an unbounded reservoir?
34. Give an example of an incompressible fluid
35. Which parameter causes an additional pressure drop near the wellbore?
36. Skin is a reservoir phenomenon, true or false give a reason for your answer
37. Skin accounts for
38. A state where the mass rate of production is equal to the rate of mass depletion is termed
39. One of the conditions necessary for pseudo steady state to be attained is that reservoir outer boundary must be closed to flow. True or False, give reason for your answer
40. When a well that attains steady state is shut in, the pressure does not build up to average pressure. The pressure builds up to initial pressure because of

41. A reservoir attains pseudosteady state if the rate of pressure decline is constant. And the constant is related to

Ex 1.1 Given the following data:

Wellbore radius, r_w	*0.3728 ft*
Drainage radius, r_e	*1100 ft*
Reservoir height, h	*37 ft*
Initial pressure,P_i	*4200 psi*
Pressure at the outer boundary, P_e	*3640 psi*
Bottomhole flowing pressure, P_{wf}	*2800 psi*

Calculate

I. The reservoir pressure at a radius of 67 ft
II. The pressure gradient at 67 ft

Ex 1.2 An oil well is flowing at 230 stb/d from a uniform sand under steady state with the following data:

Total compressibility, S_{wc}	*23%*
Reservoir height, h	*32 ft*
Static Bottomhole pressure, P_{ws}	*2500 psi*
Formation permeability, k	*242 mD*
Oil viscosity, μ_o	*0.59 cp*
Oil formation volume factor, β_o	*1.342 rb/stb*
Porosity, ∅	*22%*

I. What is the pressure at 20 ft radius using a 560 ft drainage radius?
II. What is the pressure drop using the 560 ft drainage radius and wellbore radius of 5 inches?
III. Compare the pressure drop from 560 ft to 95 ft with that from 95 ft to 10 ft.
IV. What is the pressure gradient at 28 ft
V. What is the actual average radial velocity at 28 ft?
VI. How long will it take oil at 560 ft radius to reach the wellbore?

Ex 1.3 A gas well is producing under the following conditions:

Wellbore radius, r_w	*0.385 ft*
Drainage radius, r_e	*980 ft*
Reservoir height, h	*28 ft*
Reservoir temperature, T	*150 °F*
Initial pressure,P_i	*1700 psi*
Bottomhole flowing pressure, P_{wf}	*1450 psi*

(continued)

Formation permeability, k	*45 mD*
Gas gravity, γ_g	*0.62*
Skin factor, s	*1.06*

Calculate the gas flow rate using:

I. Pressure-square approximation
II. Real gas pseudo-pressure approach

Ex 1.4 An oil well producing at a constant rate of 350 stb/day under unsteady state flow conditions. The reservoir has the following rock and fluid properties:

Wellbore radius, r_w	*0.5 ft*
Total compressibility, C_t	$7.45*10^{-6}$ *psi*$^{-1}$
Reservoir height, h	*20 ft*
Initial pressure,P_i	*4200 psi*
Formation permeability, k	*80 mD*
Oil viscosity, μ_o	*1.48 cp*
Oil formation volume factor, β_o	*1.275 rb/stb*
Porosity, ∅	*18.5%*

Calculate the pressure at the following radius 0.67 ft, 6 ft, 12 ft and 115 ft after 2 h of production.

Ex 1.5 Given the following data of a well in an infinite acting reservoir.

Wellbore radius, r_w	*0.3512 ft*
Drainage radius, r_e	*850 ft*
Total compressibility, C_t	$3.6 * 10^{-6}$ *psi*$^{-1}$
Reservoir height, h	*46 ft*
Initial pressure,P_i	*3250 psi*
Formation permeability, k	*154 mD*
Oil viscosity, μ_o	*0.759 cp*
Oil formation volume factor, β_o	*1.3023 rb/stb*
Porosity, ∅	*25%*
Oil flow rate, q_o	*498 stb/day*
Time, t	*7 h*

- Calculate the wellbore flowing pressure at a distance (radius) of 60 ft after 7 h production.
- The wellbore flowing pressure at a distance (radius) of 118 ft after 7 h production
- The wellbore flowing pressure at a distance (radius) of 217 ft after 10 h production

Ex 1.6 Calculate the gas flow rate of a gas well with average reservoir pressure of 2100 psi and bottom hole flowing pressure of 1200 psi using pressure-square method.

Additional Data:

Wellbore radius, r_w	*0.451 ft*
Drainage radius, r_e	*945 ft*
Reservoir height, h	*33 ft*
Reservoir temperature, T	*175 °F*
Formation permeability, k	*238 mD*
Gas gravity, γ_g	*0.74*

Ex 1.7 Given the following data:

Wellbore radius, r_w	*0.45 ft*
Drainage radius, r_e	*810 ft*
Total compressibility, C_t	$3.23 * 10^{-6}\ psi^{-1}$
Reservoir height, h	*31 ft*
Pressure at the outer boundary, P_e	*3700 psi*
Bottomhole flowing pressure, P_{wf}	*2780 psi*
Formation permeability, k	*140 mD*
Oil viscosity, μ_o	*1.24 cp*
Oil formation volume factor, β_o	*1.148 rb/stb*
Skin factor, s	*4*

Calculate the oil flow rate assuming that the fluid is slightly compressible. Also compare the result with assuming the fluid is incompressible.

Ex 1.8 A producing well is located some 1500 ft away from an observation well, both near the center of a circular drainage area of radius, $r_e = 1000$ ft. If the producing well is flowing at the rate of 1200 stb/d, calculate:

I. The resulting pressure drop at the observation well
II. The pressure drop at the observation well if it produces at the rate of 800 stb/d
III. The total pressure drop if both wells produce at 1000 stb/d each.

Additional rock and fluid properties are:

$$r_w = 4\ inches,\ \ h = 77\ ft,\ \ k = 480\ mD,\ \ B_o = 1.462\ rb/stb,\ \ \mu_o = 0.68\ cp,$$

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Chapter 2
Resources and Reserves

Learning Objectives
At the end of this chapter, the students/readers should be able to:

- Know the reasons for estimating oil and gas reserves
- Identify the parties that make use of hydrocarbon reserves
- Understand hydrocarbon resources and its classification
- Understand hydrocarbon reserves and its classification
- Difference between resources and reserves
- Identify some uncertainties associated with hydrocarbon reserves estimation

2.1 Introduction

The development of oil and gas fields today depend solely on the amount of the recoverable hydrocarbon fluid (reserves) discovered in the subsurface formation (reservoir) and its economic viability. The estimation of these reserves are usually associated with some level of uncertainties and when these uncertainties are not factored into the prospect evaluation, the result is a wrong estimation of the reserves. This means that the value of reserves estimation is a key driver for exploration and production companies to decide whether to develop or abandon the prospect based on their set criteria. Therefore, in estimating oil and gas reserves, we rely on the integrity, skill and the judgment of the evaluator based on the amount of data available, the complexity of the formation geology and the degree of depletion of the reservoir (SPE, 1997).

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2.2 Parties that Use Oil and Gas Reserves

- Companies operating oil and gas field or own an interest in petroleum operations for in-house valuation
- Banks and other financial institutions involved in financing
- Stock markets around the world
- Regulatory bodies to protect the general public, to manage natural resources, and to promote uniformity
- Taxation agencies with authority over petroleum products
- Investors in petroleum companies
- Mineral rights owners
- Arbitration (negotiation, settlement, etc) parties. i.e. to work out a deal
- Government for energy policies and strategic planning

2.3 Reasons for Estimating Reserves

- To obtain approvals from relevant ministries and other regulatory bodies
- For exploration, development & production of oil and gas reservoir
- To negotiate property sales and acquisitions
- To determine the market value
- To design facilities
- To obtain financing
- Evaluation of profit/interest
- Government regulations & taxation
- Planning & development of national energy policies
- Investment in oil/gas sector
- Reconcile dispute or arbitration involving reserves

2.4 Resources and Reserves

Resources, sometimes referred to as accumulations, are the total assumed quantities of hydrocarbons found beneath the earth crust that could exist which may or may not be produced in the future.

Reserves are estimated remaining quantities of oil and natural gas and related substances anticipated to be recoverable from known accumulations, as of a given date, based on the following:

- Analysis of drilling, geological, geophysical, and engineering data;
- The use of established technology;
- Specified economic conditions, which are generally accepted as being reasonable, and shall be disclosed.

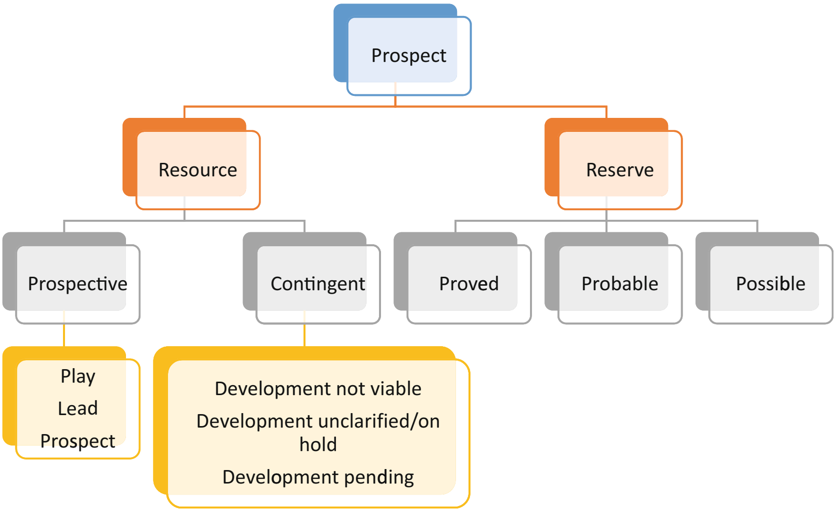

Fig. 2.1 Classification of hydrocarbon prospect

The classification of hydrocarbon resources and reserves is presented in Fig. 2.1.

2.4.1 Hydrocarbon Resources

Resources are the total estimated quantities of hydrocarbons found beneath the earth crust that could exist which may or may not be produced in the future. These are commonly referred to as "Accumulations". Resource is basically different from reserve whose hydrocarbon deposit is known to exist with reasonable certainty based on studies from geology and engineering. It encompasses all of the hydrocarbons that could exist, regardless of whether it is recoverable or known to exist. Therefore, a resource can either be discovered or undiscovered (unknown and cannot be estimated), economically recoverable or not economically recoverable. It includes portions of hydrocarbons that are assumed to be present but are not measured because they have not been explored or are located in inaccessible position.

The amount of naturally occurring accumulations of hydrocarbon estimated to be originally in place is known as original resources. Hence, if the prospect has been produced for a particular period of time, the original resources can also be defined on a given date as the sum of the estimated quantities of hydrocarbon remaining in the reservoir (naturally occurring accumulation) plus the quantities of the hydrocarbon already produced plus quantities in the accumulations yet to be discovered if any. Original resources can be classified as discovered or undiscovered and each of these is further classified in the flow chart below (Fig. 2.2).

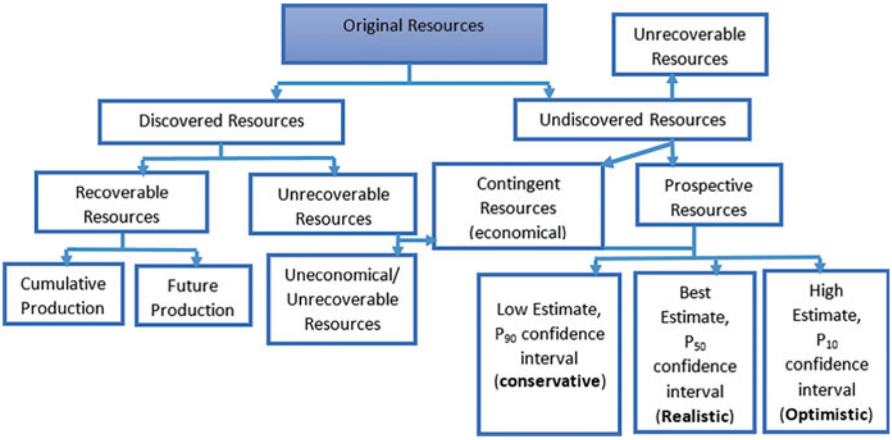

Fig. 2.2 Classification of original resources

Resources are broadly classified as contingent (chance or likely) and prospective (expected or soon-to-be) resources depending on whether it is discovered or undiscovered (McKelvey, 1972).

2.4.1.1 Contingent Resources

Contingent Resources are those potentially recoverable estimated quantities of hydrocarbon from discovered accumulations on a given date, whose prospect or project is not currently viable commercially or mature enough and are uneconomical for development due to one or more uncertainties or contingencies. Some of these contingencies may be that there is no current viable market(s) for the hydrocarbon, or if commercial recovery of the hydrocarbon content is clinging on technology under development, or evaluation of the accumulation is insufficient to clearly assess commerciality.

Furthermore, the fact that contingent resource is not commercially viable does not mean it cannot be seen as a reserve (that is, the movement from contingent resources into reserves category) but if the key contingencies preventing commercial development are adequately addressed or removed, then it can be called a hydrocarbon reserve.

Classification of Contingent Resources

Development Not Viable

A discovered accumulation of hydrocarbons where viable processes of recovering the hydrocarbon content have not yet been developed or a scenario where there are no current plans to develop or to acquire additional data at the said time due to limited production potential.

Development Unclarified or on Hold

A discovered accumulation of hydrocarbons of significant size where activities of the project are not cleared or are on hold and/or where justification as a commercial development may be subject to significant delay such as political, environmental, technical or the dwindling market conditions.

Development Pending

This is an accumulation of discovered hydrocarbons where further data acquisitions are required to confirm commerciality. In this case, the activities of the prospect are presently happening or ongoing to provide an acceptable explanation of commercial development in the anticipated or foreseeable future.

2.4.1.2 Prospective Resources

On the other hand, prospective resources which can be referred to as expected or soon-to-be resources; are defined as the estimated volumes associated with undiscovered accumulations or as estimated quantities of hydrocarbon as of a given date to be potentially recoverable and are analyzed on the basis of indirect evidence but have not yet been drilled. They are technically viable and economical to produce but they present a higher risk than contingent resources since the risk of discovery is also added.

Furthermore, we should note that while the engineers and geoscientists take into consideration the possibility of hydrocarbons discovery and development when determining the quantities of prospective resources, they also make some assumptions which include a range of uncertainty whether the hydrocarbons will be found. The prospective resources are further classified as low, best and high estimate as shown in Fig. 2.2.

Also, there can be a movement from prospective resources to contingent resources, only if hydrocarbons are discovered and the accumulated discovery must be further evaluated to determine an estimated quantity that would be recoverable under appropriate development projects.

Classification of Prospective Resources

According to the guidelines for the evaluation of petroleum reserves and resources by Society of Petroleum Engineers (2001), prospective resources can be classified as:

Play

A project associated with a prospective trend of potential prospects, but requires more data acquisition and/or evaluation to define specific leads or prospects. This is a concept of exploration that includes a specific source rocks, reservoir rocks, migration path and the type of trap to allow the discovery of recoverable quantity of hydrocarbon (Norwegian Petroleum Directorate,1997).

Lead

A project associated with a potential accumulation that is currently poorly defined and requires more data acquisition and/or evaluation to be classified as a prospect. This implies that the data available is not enough to fully classify it as a prospect for development.

Prospect

A project associated with a potential accumulation that is sufficiently well defined to represent a viable drilling target. It implies a trap that has been identified and adequately mapped but yet to be drilled. At this stage, there are some questions asked to fully evaluate the play or prospect. These are:

- If we are certain of the hydrocarbon source, what then is the content (oil or/and gas)?
- Can the content in the source rock migrate to the reservoir rock where it is accumulated and how much of it?
- Does the reservoir have a storage capacity?
- What are the characteristics of the reservoir?
- Are there trapping mechanisms to help prevent the content of the reservoir from further migration?
- If there is a trap, how efficient is it (seal or non-sealing)?

2.4.2 Hydrocarbon Reserves

Reserves are seen as the heart of the oil and gas business. These can be defined as the estimated quantities of hydrocarbon such as crude oil, condensate, natural gas (associated or non-associated gas) that are anticipated to be commercially recoverable with the use of established technology on a known hydrocarbon accumulations from a given date forward under existing economic conditions, established operating conditions and current government regulations with a legal right to produce and a production & transportation facilities to deliver the products to the market. Also, the interpretations of reliable geologic, geophysics, drilling and engineering data available at the time of estimation are key factors that support the reserves definition. Reserves estimates are generally revised as additional geologic or engineering data becomes available or as economic conditions change.

In the previous statement, we established that contingent resources can be moved to reserves. Therefore, based on development project(s), hydrocarbon reserves must satisfy four criteria, and these are: discovered, recoverable, commercial, and remaining quantity as at the date of evaluation (PRMS, 2017). Also, there must be a reasonable expectation that all required internal and external approvals will be forthcoming and evidence of company's intention to proceed with the development within a reasonable time frame; say 10 years to the international oil companies. Generally, if they cannot develop it within this time frame, they might be mandated by the regulatory body to farm-out to marginal field operators.

2.4.2.1 Hydrocarbon Reserves Classification

Classification by Development Operations

Oil and gas reserves can be classified to be on production, which implies that the prospect is currently producing and the product delivered to the market for consumption. It can be classified as being under development, which means that every expedient approval has been obtained and the project development is in progress. Furthermore, having satisfied all criteria for reserves, It can be classified has been scheduled or outlined for development with substantial desire to develop but all mandatory approvals have not be finalized or complete detailed development plan have not been made.

Reserves are further classified according to the degree of certainty associated with the estimation (Ross, 2001). These are: proved and unproved (probable and possible reserves).

Classification by Degree of Uncertainty of Estimation

Proved Reserves

Proved reserves are those quantities of hydrocarbon reserves based on analysis of geological and engineering data that can be estimated with a reasonably high degree of certainty to be commercially recoverable from a given date forward from known reservoirs and under current economic conditions, operating methods, and government regulations. It is likely that the actual remaining quantities recovered will exceed the estimated proved reserves. In general, reserves are considered proved if the commercial producibility of the reservoir is supported by actual production or formation tests. In its method of estimation, if probabilistic methods are used, there should be at least a 90% probability that the quantities actually recovered will equal or exceed the estimate and if deterministic methods are used, the term with reasonable certainty is intended to express a high degree of confidence that the quantities will be recovered.

Probable Reserves

Are those quantities of hydrocarbon based on geologic and/or engineering data similar to that used in the estimation of proved reserves; but technical, contractual, economic, or regulatory uncertainties deter such reserves from being classified as proved. In this context, when probabilistic methods are used, there should be at least a 50% probability that the quantities actually recovered will equal or exceed the sum of estimated proved plus probable reserves.

Possible Reserves

Possible reserves are those additional reserves that are less certain to be recovered than probable reserves. It is unlikely that the actual remaining quantities recovered will exceed the sum of the estimated proved plus probable plus possible reserves. They can also be defined as those unproved reserves which analysis of geological and engineering data suggests that they are less likely to be recoverable than probable reserves. In this context, when probabilistic methods are used, there should be at least a 10% probability that the quantities actually recovered will equal or exceed the sum of estimated proved plus probable plus possible reserves.

2.5 Identification of Uncertainty in Reserves Estimation

Numerous uncertainties exist in estimating reserves and remaining recoverable resources of conventional oil held by countries. These uncertainties include: geologic, production performance, product market and uncertainties in oil price forecast, the use of ambiguous definitions and inclusion of different subcategories of conventional oil by reporting sources, the inclusion of politics in reserves estimation, the inconsistent and unclear effects of aggregation of reserve data to country and regional estimation, the anticipated volume of undiscovered oil, and the nature and extent of reserve growth and its allocation to individual countries.

2.5.1 Uncertainty in Geologic data

Uncertainties arising from geological data include errors in getting the exact locations of the geologic structure, the field size, pay thickness, porosity and permeability variation, reservoir and aquifer sizes, reservoir continuity, fault position, petrofacies determination, and insufficient knowledge of the depositional environment. A number of techniques are available for the quantification of geologic uncertainties. One of the widely used techniques is to quantify the uncertainty in the geological model with a geostatistical tool. Geostatistics involves synthesizing geological data using statistical properties such as a variogram (Bennett and Graf 2002). This process enables the geologists to generate multiple realizations of the geological models (Stochastic) which allows quantification and minimization of uncertainties associated with the geological information.

2.5.2 Uncertainty in Seismic Predictions

- The quality of the seismic data (bandwidth, frequency content, signal-to-noise ratio, acquisition and processing parameters, overburden effects, etc.)
- The uncertainty in the rock and fluid properties and the quality of the reservoir model used to tie subsurface control to the 3D seismic volume

2.5.3 Uncertainty in Volumetric Estimate

The uncertainties in reservoir volume estimate will arise from several properties and characteristics of the reservoir.

2.5.3.1 Gross Rock Volume (GRV) of a Trap

- The incorrect positioning of structural elements during the processing of the seismic and lack of definition of reservoir limits from seismic data
- Incorrect interpretation
- Errors in the time to depth conversion
- Dips of the top of the formation
- Existence and position of faults
- Whether the faults are sealing to prevent further lateral migration of the hydrocarbon

2.5.3.2 Rock Properties: Net-to-Gross and Porosity

The uncertainty associated with the properties of the reservoir rock originates from the variability in the rock. It is determined through petrophysical evaluation, core measurements, seismic response, and their interpretation. Most times, the core samples are not properly handled carefully in the process of transporting it from the field to the laboratory for analysis. Also in the laboratory, artificial properties are induced during the core preparation and analysis. While petrophysical logs and measurements in the laboratory may not be quite accurate, the samples collected may be representative only for limited portions of the formations under analysis. Thus, there are some risks associated with the petrophysical parameters estimation such as depth matching, operational risks, log interpretation and reservoir heterogeneities.

2.5.3.3 Fluid Properties

For fluid properties, a few well-chosen samples may provide a representative selection of the fluids. The processes of convection and diffusion over geologic times have generally ensured a measure of chemical equilibrium and homogeneity within the reservoir, although sometimes gradients in the fluid composition are observed. Sampling and analysis may be a significant source of uncertainty. PVT or fluid properties vary with pressure, temperature, and chemical composition from one region to another. As a result of this regional trend, correlations developed from regional samples that are predominantly paraffinic in nature may not provide acceptable results when applied to other regional crude oil systems that are dominant in naphthenic or aromatic compounds.

The effective use of PVT correlations depends on the knowledge of their developments and limitations. In addition, samplings of these properties are not always readily available due to cost and time. Thus, the engineers in view of achieving their goals resort to the use of empirically derived correlations in estimating these properties. However, a significant error is usually associated with the estimation of these fluid properties which in turn propagates additional errors in all petroleum engineering calculations.

2.5.3.4 Fluid Contacts

One of the parameters required for the estimation of hydrocarbon reserve is the gross rock volume (bulk volume of the rock) whose accuracy is dependent on the fluid contacts (gas-oil and/or water-oil contact). Therefore, if the contacts are not adequately determined, it will lead to either over or under estimation of the bulk volume. Thus, affecting the overall value of the estimated reserve.

2.5.3.5 Recovery Factor (RF)

Recovery is based on the execution of a project and it is affected by the shape and the internal geology of the reservoir, its properties and fluid contents, and the development strategy. If a reservoir is poorly defined, material balance calculations or analog methods may be used to arrive at an estimate of the range of RFs. Uncertainty ranges in the RF can often be based on a sensitivity analysis. Besides, the reservoir drive mechanism and the problem of reservoir monitoring or management of some level of uncertainties.

2.5.4 Economic Significant of Reservoir Uncertainty Quantification

During the life of a reservoir, the pre-reservoir and post-reservoir performance evaluations are generally not equal. This is due to inadequate quantification of uncertainties associated with the reservoir model input parameters and the resulting composite uncertainty associated with the pre-reservoir performance prediction. The decision to develop a reservoir is based on the prediction of production performance following history-matching process. Likewise, in some instances, the decision to obtain additional reservoir measurement data is taken when the uncertainty of the forecast is great.

Hence, acquisition of further data is the reason for accurate quantification of uncertainty associated with reservoir performance forecast so that projected recovery will be accurately estimated for economic decisions. These vital reasons underline the economic importance of increasing interest to properly quantify the uncertainties associated with reservoir performance simulation.

2.6 Reservoir Characterization

An accurate description of reservoir rock, fluid contents, rock-fluid systems, fluid description and flow performance are required to provide a sound basis for reservoir engineering studies. Hence, proper reservoir characterization is important to analyze the effects of heterogeneity on reserve estimation and reservoir performance due to primary, secondary, and/or enhanced oil recovery operations. Porosity and permeability are important flow properties; an accurate reservoir characterization requires accurate porosity and permeability description as a function of space.

Reservoir characterization is a process carried out to reduce geological uncertainties by quantitatively predicting the properties of a reservoir and define reservoir structural changeability or variability. It is a process ranging from the discovery phase to the management phase of a reservoir. Prior to performing a reservoir simulation, accurate characterization is the first key step to undertake which helps to identify uncertainty range inherent in reservoirs. Here we try to assess the range of reservoir performance from an understanding of the subsurface uncertainties. This concept is a limitation and it is not considered in the material balance method presented in Chap. 5 of this book.

At this point, we need not border ourselves with a thorough review of literature in reservoir rock characterization which would not be practically possible because of the wide nature of this discipline and it is not incorporated in this present book. However, the process combines the technical disciplines of geology, geophysics, reservoir engineering, production engineering, petrophysics, economics, and data management with key objectives on modeling each reservoir unit, understanding and predicting well behavior, understanding past reservoir performance, and forecasting future reservoir performance. Hence, it is used to assert a strong impact on plans for the development and performance of a field.

Exercises

1. Distinguish between resources and reserves
2. What prompt the development of oil and gas fields today?
3. Mention five parties that make use of hydrocarbons reserves
4. Name the two basic types of resources
5. What makes a contingent resources to become a reserves?
6. Distinguish between contingent and prospective resources
7. State the criteria that hydrocarbon pool must satisfy to be termed reserves:
8. Explain exhaustively how uncertainties affect oil and gas reserves estimation
9. What is a process that reduces geological uncertainties by quantitatively predicting the properties of a reservoir and define reservoir structural changeability called?

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Chapter 3
Volumetric Reserves Estimation

Chapter Learning Objectives

Upon completion of this chapter, students/readers should be able to:

- List the methods available for estimating hydrocarbon reserves
- Identify the factors that affect the volumetric reserves estimation
- Understand the input parameters required to perform volumetric reserve estimation
- Know the various sources where the input parameters can be obtained
- Learn the step by step approach to calculate hydrocarbon reserves
- Determine bulk volume from Isopach map
- Determine hydrocarbon initially in place and other volumetric calculations
- Understand the deterministic and probabilities methods of reserves estimation
- Perform condensate reserve calculations

Nomenclature

Parameter	Symbol	Unit
Porosity	$\varnothing$	*– or %*
Water saturation	S_w	*– or %*
Oil saturation	S_o	*– or %*
Gas saturation	S_g	*– or %*
Water resistivity	R_w	*Ω*
True resistivity	R_t	*Ω*
Rock resistivity	R_o	*Ω*
Reservoir thickness	h	*ft*
Area	A	*acre*
Matrix density	ρ_{ma}	*g/cm^3*

(continued)

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Parameter	Symbol	Unit
Fluid density	ρ_f	*g/cm*^{3}
Bulk density	ρ_b	*g/cm*^{3}
Saturation exponent	n	–
Cementation factor	m	–
Log, rock matrix & fluid transit time	Δt_{log}, Δt_{ma}, Δt_{fl}	*μsec*
Initial oil & gas formation volume factor	β_{oi} & β_{gi}	*rb/stb &* *scf/stb*
Net-to-gross ratio	*N/G*	–
Recovery factor	*RF*	*– or %*
Stock tank oil initially in place	*STOIIP* (N)	*stb*
Free gas initially in place	*FGIIP*	*scf*
Ultimate recovery	*UR*	*stb or scf*
Bulk volume	V_b	*acre-ft*
Cumulative oil & gas produced	N_p & G_p	*stb & scf*
Gas deviation or compressibility factor	z	–
Critical temperature & pressure	T_c & P_c	^0R *& psi*
Reduce temperature & pressure	T_r & P_r	^0R *& psi*

3.1 Overview of Reserve Estimation

The estimation of hydrocarbon reserves for a producing field is a process that continues throughout the entire life of the field. This process is usually associated with some level of uncertainties in calculating the reserves. These reserves estimation methods are affected by the reservoir type, sources of reservoir energy (drive mechanism), quantity and quality of the geologic, engineering and geophysical data, the assumptions adopted when making the estimation, available technology, the experience and knowledge of the evaluator(s). The oil and gas reserves estimation methods can be grouped into the following categories: analogy, volumetric, decline analysis, material balance calculations for oil and gas reservoirs, and reservoir simulation.

The selection of appropriate method to estimate reserves and resources, and the accuracy of the estimation, depend largely on the following factors: The type, quantity, and quality of geoscience, engineering, and economic data available for technical and commercial analyses, the complexity of the formation geology, the recovery mechanism, the stage of development, and the maturity or degree of depletion. More importantly, reserves and resources assessment rely on the integrity, skill and judgment of the experienced professional evaluators (PRMS Guideline 2011).

In the early stages of development, reserves estimations are restricted to the analogy and volumetric calculations. The analogy method is applied to reserves estimation by comparing factors for the analogous and current fields or wells. This implies that in analogy method, the reserves are estimated on the basis of a relationship of resemblance or equivalence between two fields. This method directly compares a poorly or newly discovered reservoir to a known reservoir that has similar geologic and petrophysical properties such as lithology of the formation, depth, porosity to mention a few. Hence, the accuracy with this method is the least among other methods of reserve estimation.

Furthermore, a close-to-abandonment analogous field is taken as an approximation to the current field. This method is the most useful technique when running the economics on the current field; which is supposed to be an exploratory field (Petrobjects 2003).

3.2 Volumetric Method

The volumetric method is probably the easiest method used by engineers to estimate reserves. It requires a limited amount of data for the estimation, this implies that immediately after discovery of the hydrocarbon accumulations, during initial delineation and development of a field, the volumetric method is the key to hydrocarbon volume estimation. Reserves estimation is often high with this method, because it does not consider the heterogeneity of the reservoir and it includes the undrained compartments that do not account to flow and are included in making up the bulk rock volume of the reservoir or accumulation. At this stage, the level of inherent error can be reduced if the reservoir is accurately described or characterized.

3.2.1 Errors in Volumetric Method

Volumetric method is subject to considerable error because it is often used to evaluate reserves when little data are available; it requires the estimation of the reservoir rock and fluid properties and the reservoir volume from spot measurements of the properties that are then applied to the entire reservoir. The porosity and saturation are measured either from core samples or logs that are measured from a small portion of the reservoir and under best circumstances, it only approximates the condition in the reservoir. The areal extent of the reservoir is rarely known until many wells are drilled while the volume is estimated using zone thickness measured at one or more points in the reservoir. The volumetric method is only seen as a gross estimate of oil or gas in place.

3.2.2 Application of Volumetric Method

- The volumetric result is useful in reserves estimation of the initial oil and gas in place.
- The volumetric result is useful in reserves estimation of oil and gas in place at any time of depletion.
- Volumetric estimation is useful during the development period before reservoirs limit have been defined.
- Later in the life of the reservoir, when reservoir volume is defined and performance data are available, volumetric estimation provide valuable checks on oil and gas in place estimates obtained from material balance and reservoir simulation methods.

The volumetric method is a straightforward approach which requires determination of the areal extent of the reservoir or bulk volume (calculated as area times pay thickness), the rock pore volume, and the fluid content within the pore volume to calculate the amount of hydrocarbons-in-place. The ultimate recovery can thus be estimated by applying an appropriate recovery factor. Each of the variables used in the volumetric reserves calculation above has inherent uncertainties, and when combined; cause significant uncertainties in the reserves estimate (Petrobjects 2003). Therefore, the following steps consist the volumetric method of reserves estimation:

Step 1: Determination of hydrocarbon rock bulk volume (hydrocarbon saturated portion) from area and thickness (isopach map). Explanation of this method is presented in the next page.

Step 2: Determination of average porosity either from core analysis or well logs.

From core analysis

$$\varnothing = \frac{\sum \varnothing_j A_j}{\sum A_j} = \frac{\sum \varnothing_j A_j h}{\sum A_j h_1}$$

From density log as

$$\varnothing_D = \frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f}$$

From sonic log as

$$\varnothing_S = \frac{\Delta t_{log} - \Delta t_{tma}}{\Delta t_{fl} - \Delta t_{tma}}\left(\frac{1}{C_p}\right)$$

From resistivity log as

$$\varnothing = \left[\frac{aR_w}{R_o}\right]^{1/m}$$

Step 3: Determination of water saturation or indirectly, the hydrocarbon saturation

Water saturation

$$S_w = \frac{\sum_{i=1}^{n} \varnothing_i h_i s_{wi}}{\sum_{i=1}^{n} \varnothing_i h_i}$$

Oil saturation

$$S_o = \frac{\sum_{i=1}^{n} \varnothing_i h_i s_{oi}}{\sum_{i=1}^{n} \varnothing_i h_i}$$

Gas saturation

$$S_g = \frac{\sum_{i=1}^{n} \varnothing_i h_i s_{gi}}{\sum_{i=1}^{n} \varnothing_i h_i}$$

From resistivity log

$$S_w = \sqrt[n]{\frac{aR_w}{\varnothing^m R_t}}$$

Step 4: Obtain the net-to-gross ration either from core or log, this is evaluated as the thickness of sand only (net) divided by the entire thickness of the reservoir (gross). This means the fraction of the reservoir that consists of porous rock such as sand or carbonate, excluding shale.

Step 5: Obtain the formation volume factor from the PVT laboratory analysis on the fluid sample from the reservoir.

Step 6: Practically, take a cut-off value which varies with companies to account for the pore spaces not connected (contributing to flow) in the reservoir rock or error during the determination of the petrophysical data.

Step 7: Volume correction of hydrocarbon at atmospheric pressure and temperature at stock tank value or surface volume.

Mathematically, the volumetric method is express as:
Stock tank oil initially in place

$$\boldsymbol{STOIIP = N = \frac{7758Ah\varnothing(1 - S_w)^{N/G}}{B_{oi}}}$$

Free gas initially in place

$$FGIIP = G = \frac{43560 A h \varnothing (1 - S_w)^{N/G}}{B_{gi}}$$

$$ultimate\ Recovery = STOIIP * RF$$

For a volumetric saturated oil reservoir, the gas cap size is calculated as:

$$m = \frac{bulk\ volume\ of\ gas}{bulk\ volume\ of\ oil} = \frac{(Ah)_{gc}}{(Ah)_{oz}} = \frac{GB_{gi}}{NB_{oi}}$$

3.2.3 Sources of the Volumetric Input Data

Parameter	Source(s)	By
Reservoir thickness, h	From cores or logs	Geophysicists and Geologists
Area of the reservoir, A	Mainly from surface seismic	Geologists and Petrophysicists
Net-to-gross ratio, h	From cores or logs	Geologists and Petrophysicists
Porosity, $\varnothing$	From cores or logs	Petrophysicist
Saturation, S_w	From cores or logs	Petrophysicist
Oil & gas formation volume factor, B_{oi} & B_{gi}	From PVT laboratory or correlations	PVT laboratory technologist

3.2.4 Calculation of Reservoir Bulk Volume (Table 3.1)

The volumetric method of reserves estimation largely depends on the bulk volume, calculated as follows:

(a) Prepare a structure map with contours from top to bottom of the reservoir, in subsea depths

(b) Mark out a small square on the map e.g. (10 cm × 10 cm). Use the scale on the map to determine the area of the square in acres. Planimeter the square and determine the area in planimeter units. Then determine the planimeter constant in acres/planimeter unit by dividing the actual area in acres by the area into planimeter units. Use the planimeter constant to covert the areas of the map from planimeter units to acres.

Table 3.1 Estimation of bulk volume

Contour interval	Area	Ratio	Rule	Average Area	Bulk volume
$\boldsymbol{C_1}$	A_1				
$\boldsymbol{h_1 = C_2 - C_1}$		$^{A_1}/_{A_2}$	$^{A_1}/_{A_2} > 0.5$	$A_{g1} = \frac{1}{2}(A_1 + A_2)$	$A_{g1} * h_1$
$\boldsymbol{C_2}$	A_2				
$\boldsymbol{h_2 = C_3 - C_2}$		$^{A_2}/_{A_3}$	$^{A_1}/_{A_2} < 0.5$	$A_{g2} = \frac{1}{3}(A_n + A_{n+1} + \sqrt{A_n * A_{n+1}})$	$A_{g2} * h_2$
$\boldsymbol{C_3}$	A_3				
$\boldsymbol{h_3 = C_4 - C_3}$		$^{A_3}/_{A_4}$	$^{A_1}/_{A_2} > 0.5$	$A_{g3} = \frac{1}{2}(A_1 + A_2)$	$A_{g3} * h_3$
$\boldsymbol{C_4}$	A_4				

(c) Planimeter the area (acres) enclosed by each contour on the reservoir map and draw an acre-ft diagram.

(d) Calculate the net rock volume, Ah applying any of these numerical methods below:

Trapezoidal Rule

$$\Delta V_b = \frac{h}{2}(A_n + A_{n+1}) = \sum_{i=1}^{n} \Delta V_{bi}$$

Pyramidal Rule

$$\Delta V_b = \frac{h}{3}\left(A_n + A_{n+1} + \sqrt{A_n * A_{n+1}}\right) = \sum_{i=1}^{n} \Delta V_{bi}$$

Where,

ΔV = bulk volume in acre-feet

A_n = the area enclosed by the upper Isopach line in areas

A_{n+1} = the area enclosed by the lower Isopach line in acres

h = Vertical height between two successive Isopach lines or the net sand thickness

Rules for applying any of the above methods:

Use Trapezoidal rule if

$$\frac{A_n}{A_{n+1}} > 0.5$$

Use Pyramidal rule if

$$\frac{A_n}{A_{n+1}} < 0.5$$

To calculate the bulk volume of the reservoir from Isopach or contour map, there is need to understand the concept of contouring which can be defined as the process of tracing contour lines on the surface of the earth. This is not only applicable to petroleum engineers but contour survey is also carried out at the beginning of any engineering project such as a road, a railway, a canal, a dam, a building etc.

3.3 What is a Contour?

Contour is an imaginary line on the ground surface joining points of equal elevation or a line on which every point is at the same level above or below a chosen reference surface. In most maps, the reference surface is sea level. This line on the map represents a contour and is called contour line.

Therefore, a map showing contour lines is known as Contour map. Contour maps are one of the most effective means of displaying information about the geologic structure (i.e. the degree of buckling and faulting of the layers) of an area. A contour map gives an idea of the altitudes of the surface features as well as their relative positions in the plan. A map showing structure contours for a certain rock layer throughout an area is called a *structure contour map* (Fig. 3.1). Such maps are used to illustrate the size, shape and location of geologic structures.

Contour lines are drawn as fine and smooth freehand curved lines. Sometimes they are represented by broken lines. They are inked in either in black or brown colour. A drawing pen gives a better line than a writing pen and French curves should be used as much as possible. Every fifth contour is made *thicker* than the rest.

The elevation of contours must be written in a uniform manner, either on the higher side or in a gap left in the line. When the contour lines are very long, their elevations are written at two or three places along the contour. In the case of small-scale maps, it is sufficient to figure every fifth contour. Therefore, the constant vertical distance between two consecutive contours is called the contour interval. The contour interval is constant between the consecutive contours.

3.3.1 Methods of Contouring

There are basically two main methods of locating contours; these are the Direct Method and Indirect Method.

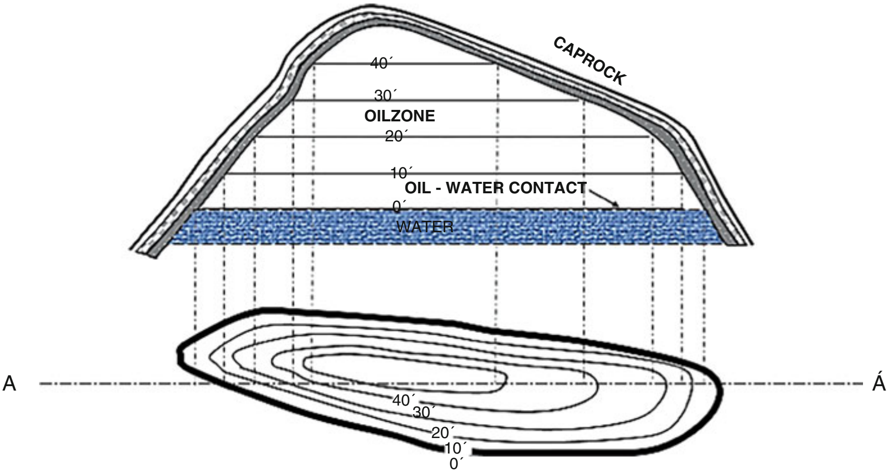


Fig. 3.1 Cross section of reservoir at A - Á

3.3.1.1 Direct Method

This method requires a lot of time to be invested in searching for points of the same elevation on the ground surface. This implies that it is very slow and tedious but it is the most accurate method of contouring, thus suitable for small area and where great accuracy is required. In this method, the contours to be located are directly traced out in the field by locating and marking a number of points on each contour. These points are then surveyed and plotted on plan and the contours drawn through them (Fig. 3.2).

For a radial line, temporary benchmarks are first established at the centre and near the ends of the radial lines. The contour points are then located and marked on these lines and their positions are determined by measuring their distances along the radial lines. They are then plotted on the plan and the contours drawn by joining all the corresponding points with the help of a plane table instrument (Fig. 3.3).

3.3.1.2 Indirect Method

In this method, the points located and surveyed are not necessarily on the contour lines but the spot levels are taken along the series of lines laid out over the area. The spot levels of the several representative points representing hills, depressions, ridge and valley lines and the changes in the slope all over the area to be contoured are also observed. Their positions are then plotted on the plan and the contours drawn by interpolation. This method of contouring is also known as contouring by spot levels.

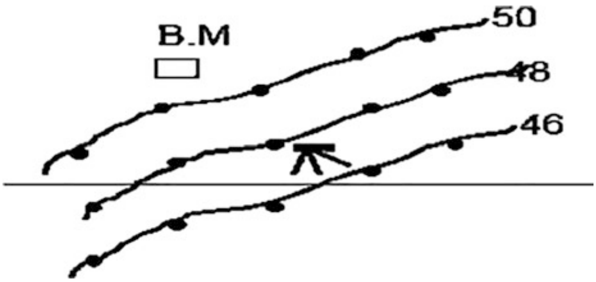


Fig. 3.2 Direct method of contouring

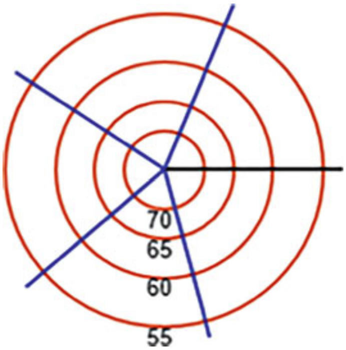


Fig. 3.3 Direct method of radial line of contouring

3.3.1.3 Conversion from Planimeter Unit to Field Unit

For a map scale of 1:10,000

$$
\begin{aligned}
&10\ \text{cm} \times 10\ \text{cm is equivalent to } 100,000 \times 100,000\ \text{cm}^2 \\
&= 1\ \text{km}^2 \\
&= 247.1\ \text{acres}
\end{aligned}
$$

Planimetered area of 10 cm × 10 cm = 230 P.U.

$$\text{Planimeter constant} = \frac{247.1}{230}\left(\frac{acres}{P.U}\right)$$

Example 3.1

Table 3.2 shows the conversion of planimeter readings to field unit (acres)

From Table 3.1, column 4 is calculated as column 2 plus column 3 divided by 2

Column 5 is calculated as column 4 multiplied by (247.1/230)

Example 3.2

An oil block was acquired by your company and the areas obtained by planimetering 8 Isopach contours of an oil reservoir are given as follows (Table 3.3):

Table 3.2 Planimeter reading conversion to acres

	Planimeter Reading		Average	Area
Contour	1st	2nd	(PU)	(ac)
3000	37	36	36.5	39.21
2500	29	29.5	29.25	31.42
2000	24	26	25	26.86
1500	31	30	30.5	32.77
1000	20	20	20	21.49
500	13	15	14	15.04
0	9	8	8.5	9.13

Given that the average porosity is 0.27, connate water saturation as 0.24, formation volume factor is 1.324. From the lithology log, it was determined by the Petrophysicist that the net reservoir sand thickness 220 ft and the gross sand thickness is 298 ft. As a geologist, you are required to calculate the stock tank oil initially in place from volumetric techniques.

Solution

Contour interval	Area (acres)	Ratio	Rule	Ave Area (ac)	Bulk Volume (ac-ft)
50	23.9				
		0.267338	Pyramidal	53.1747	1488.892
78	89.4				
		0.756345	Trapezoidal	103.8	1141.8
89	118.2				
		0.859636	Trapezoidal	127.85	3963.35
120	137.5				
		0.815056	Trapezoidal	153.1	8420.5
175	168.7				
		0.890237	Trapezoidal	179.1	7164
215	189.5				
		0.857854	Trapezoidal	205.2	5130
240	220.9				
		0.883247	Trapezoidal	235.5	7065
270	250.1				**34373.54**

From Table 3.2, column 1 is calculated as the upper interval minus the lower interval

Column 3 is the ratio

$$\frac{A_n}{A_{n+1.}}$$

Column 4 is application of the rules above

Column 5 is calculated as follows

Trapezoidal rule

Table 3.3 Depth-Area data from contour map

Contour interval	Area (acres)
1000–1050	23.9
1050–1128	89.4
1128–1217	118.2
1217–1337	137.5
1337–1512	168.7
1512–1727	189.5
1727–1967	220.9
1967–2237	250.1

$$Avg\ area = \frac{1}{2}(A_n + A_{n+1})$$

Pyramidal Rule

$$Avg\ area = \frac{1}{3}\left(A_n + A_{n+1} + \sqrt{A_n * A_{n+1}}\right)$$

$$Avg\ area\ 1 = \frac{1}{3}\left(23.9 + 89.4 + \sqrt{23.9 * 89.4}\right) = 53.1747$$

$$Avg\ area\ 2 = \frac{1}{2}(89.4 + 118.2) = 103.8$$

$$Avg\ area\ 3 = \frac{1}{2}(118.2 + 137.5) = 127.85$$

$$Avg\ area\ 4 = \frac{1}{2}(137.5 + 168.7) = 153.1$$

$$Avg\ area\ 5 = \frac{1}{2}(168.7 + 189.5) = 179.1$$

$$Avg\ area\ 6 = \frac{1}{2}(189.5 + 220.9) = 205.2$$

$$Avg\ area\ 7 = \frac{1}{2}(220.9 + 250.1) = 235.5$$

Column 6 is calculated as column 5 multiplied by column 1 respectively. Therefore,
The cumulative bulk volume = 34373.54 ac-ft

$$STOIIP = N = \frac{7758 * A * h * \varnothing * (1 - S_{wc})}{B_{oi}}$$

$$STOIIP = N = \frac{7758 * 34373.54 * 0.27 * (1 - 0.24)}{1.324} * \frac{220}{298} = 30511940.17$$
$$= 30.51\ MMStb$$

Example 3.3
A hydrocarbon reservoir is mapped out in areas recorded at corresponding depths as given in Table 3.4

Additional information

Crest of the reservoir	*8800 ftss*
Base of the reservoir	*9300 ftss*
Oil-water contact, OWC	*11,700 ftss*
Gas-oil contact, GOC	*10,400 ftss*
Gas expansion factor, E_i	*234.5 scf/cuf*
Initial oil formation volume factor, B_{oi}	*1.235 rb/stb*
Porosity, ∅	*20%*
Connate water saturation, S_{wc}	*23%*

(a) Using interpolation method, determine the cumulative bulk at GOC and down to OWC
(b) Calculate the gas initially in place
(c) Calculate the oil initially in place assuming the formation is all sand
(d) Calculate the oil initially in place assuming the formation has 2/3^{rds}productive limestone with Φ =12% and Swc =25%.
(e) Calculate the gas cap size, m
(f) Determine the centroid depth of the reservoir

Note: (1 ac-ft = 7758.4bbls)

Table 3.4 Depth versus cumulative bulk volume (CBV)

Depth (ft)	CBV (ac-ft)
9800	0
10,000	6087.3
10,300	34249.8
10,400	43704.8
10,500	53049.3
10,700	72035.3
11,400	140148.8
11,500	150162.8
12,000	204032.8

Solution

(a) CBV@GOC = 43704.8 ac-ft

CBV@OWC

Depth	CBV
11,500	150162.8
11,700	CBV@OWC
12,000	204032.8

$$CBV = g(x) = f(x_o)\frac{(x_1 - x)}{(x_1 - x_o)} + f(x_1)\frac{(x - x_o)}{(x_1 - x_o)}$$

$$f(x_o) = 150162.8, \quad f(x_1) = 204032.8, \quad x = 11700, \; x_o = 11500, \quad x_1 = 12000$$

Therefore,

$$CBV = g(x) = 150162.8\left[\frac{(12000 - 11700)}{(12000 - 11500)}\right] + 204032.8\left[\frac{(11700 - 11500)}{(12000 - 11500)}\right]$$

$$Therefore\ CBV@OWC = 171710.8\ ac - ft$$

(b) $GIIP = \frac{43560Ah\varnothing(1 - S_{wc})}{B_{gi}}$

$$B_{gi} = \frac{1}{5.615E_i} = \frac{1}{234.5 * 5.615} = 0.00075946\ rb/Scf$$

Or

$$B_{gi} = \frac{1}{E_i} = \frac{1}{234.5} = 0.004264\ cuft/Scf$$

$$GIIP = \frac{43560*43704.8*0.2*(1-0.23)}{0.004264} = 6875757213\ SCF$$
$$= 6.8758\ MMMScf$$

$$GIIP = \frac{43560*43704.8*0.2*(1-0.23)}{0.00075946} = 3.8604*10^{11} STB$$

$$STOIIP = \frac{7758Ah\varnothing(1-S_{wc})}{B_{Oi}}$$

The cumulative bulk volume of the oil is calculated as

$$Ah = CBV@OWC - CBV@GOC = 171710.8 - 43704.8 = 128006\ ac-ft$$

$$STOIIP = \frac{7758*128006*0.2*(1-0.23)}{1.235} = 123832278.9\ STB$$
$$= 123.83\ MMSTB$$

$$STOIIP = \frac{7758*128006*0.12*(1-0.25)*\frac{2}{3}}{1.235} = 48246342.41\ STB$$
$$= 48.24\ MMSTB$$

$$m = \frac{volume\ of\ gas}{volume\ of\ oil} = \frac{43704.8}{128006} = 0.3414$$

(c) The centroid depth of the reservoir

The total bulk volume of the reservoir is the sum of the bulk volume of the gas plus that of the oil column. Thus,

Total hydrocarbon bulk volume = 43704.8 + 128006 = 171710.8 ac − ft.

The centroid of the reservoir lies on half of the total hydrocarbon bulk volume given as

$$half\ the\ bulk\ volume = \frac{171710.8}{2} = 85855.4\ ac-ft$$

The value 85855.4 ac-ft falls between 10,700 ft and 11,400 ft. Hence by interpolation, the centroid depth is calculated as:

$$
\begin{aligned}
Centroid\ depth &= 10700 * \frac{(140148.8 - 85855.4)}{(140148.8 - 72035.5)} \\
&\quad + 11400 * \frac{(85855.4 - 72035.5)}{(140148.8 - 72035.5)} \\
&= 10842.03\, ft
\end{aligned}
$$

Example 3.4

A hydrocarbon reservoir is mapped out in area recorded at corresponding depths as given in the table below (Table 3.5)

I. Determine the cumulative bulk volume (CBV) down to the OWC
II. Calculate the hydrocarbon pore volume in MMbbls down to the OWC for the following three sets of petrophysical data:

- All sand with porosity of 21% and connate water saturation of 20%
- 50% productive limestone with porosity 17% and connate water saturation of 30%
- 5/8 ths sand as above; 3/8 ths limestone as above (a & b)

Additional Data:

Crest of top reservoir	*8800 ftss*
Crest of base of reservoir	*9300 ftss*
Oil-water contact (OWC)	*10,700 ftss*

The bulk volume is calculated using the rule above. Thus, we use

Table 3.5 Depth-Area data

Depth	Top Area (acres)	Base Area (acres)
8800	0	
8800–9000	39.852	
9000–9300	109.572	0
9300–9400	135.168	19.440
9400–9500	150.072	38.880
9500–9700	177.876	64.800
9700–10,000	210.636	95.904
10,000–10,400	258.744	139.968
10,400–10,500	274.488	152.928
10,500–11,000	356.784	219.672

Trapezoidal rule if

$$\frac{A_n}{A_{n+1}} > 0.5$$

$$\Delta V_b = \frac{h}{2}\left(A_n + A_{n+1}\right) = \sum_{i=1}^{n} \Delta V_{bi}$$

Pyramidal rule if

$$\frac{A_n}{A_{n+1}} < 0.5$$

$$\Delta V_b = \frac{h}{3}\left(A_n + A_{n+1} + \sqrt{A_n * A_{n+1}}\right) = \sum_{i=1}^{n} \Delta V_{bi}$$

Depth interval	Area Diff	Ratio	Rule	Calculation of Bulk volume
8800	0			
8800–9000	39.852		Trapezoidal	0.5*(0 + 39.852)*200
9000–9300	109.572	0.36371	Pyramidal	0.5*[(39.852 + 109.572)*(39.852 + 109.572)]*300
9300–9400	115.728	0.81064	Trapezoidal	0.5*(109.57 + 115.728)*100
9400–9500	111.192	0.90069	Trapezoidal	0.5*(115.728 + 111.192)*100
9500–9700	113.076	0.84369	Trapezoidal	0.5*(111.192 + 113.076)*200
9700–10,000	114.732	0.84447	Trapezoidal	0.5*(113.076 + 114.732)*300
10,000–10,400	118.776	0.81407	Trapezoidal	0.5*(114.732 + 118.776)*400
10,400–10,500	121.56	0.94264	Trapezoidal	0.5*(118.776 + 121.560)*100
10,500–11,000	137.112	0.76934	Trapezoidal	0.5*(121.560 + 137.112)*500

Depth	Top Area (acres)	Base Area (acres)	Area diff	BV	CBV
8800	0		0	–	0
8800–9000	39.852		39.852	3985.200	3985.200
9000–9300	109.572	0	109.572	32325.710	36310.910
9300–9400	135.168	19.440	115.728	11265.000	47575.910
9400–9500	150.072	38.880	111.192	11346.000	58921.910
9500–9700	177.876	64.800	113.076	22426.800	81348.710
9700–10,000	210.636	95.904	114.732	34171.200	115519.910
10,000–10,400	258.744	139.968	118.776	46701.600	162221.510
10,400–10,500	274.488	152.928	121.560	12016.800	174238.310
10,500–11,000	356.784	219.672	137.112	64668.000	238906.310

Since the OWC (10,700 ft) did not fall on any of the depth in the table, we use the interpolation techniques to find the bulk volume at OWC.

CBV@OWC

Depth	CBV
10,500	174238.31
10,700	CBV@OWC
11,000	238906.31

By interpolation

$$\frac{(11000-10500)}{(10700-10500)}=\frac{(238906.31-174238.31)}{(CBV@OWC-174238.31)}$$

$$\frac{(500)}{(200)}=\frac{(64668)}{(CBV@OWC-174238.31)}$$

$$CBV@OWC=174238.31+\frac{200*64668}{500}=200105.51\ acre-ft$$

Therefore, the bulk volume of the oil is 200105.51 acre-ft

(a) All sand with porosity of 21% and connate water saturation of 20%

$$STOIIP=\frac{7758Ah\varnothing(1-S_{wc})}{B_{Oi}}*\frac{N}{G}$$

$$STOIIP=\frac{7758*200105.51*0.21*(1-0.20)}{1.2831}=203262657.52\ STB$$
$$=203.2627\ MMSTB$$

(b) 50% productive limestone with porosity 17% and connate water saturation of 30%

$$STOIIP=\frac{7758*200105.51*0.17*(1-0.30)}{1.2831}*0.5=71988857.87\ STB$$
$$=71.9888\ MMSTB$$

(c) 5/8 ths sand as above; 3/8 ths limestone as above (a & b)

$$STOIIP=\left\{\frac{7758Ah\varnothing(1-S_{wc})}{B_{Oi}}*\frac{N}{G}\right\}_{sand}+\left\{\frac{7758Ah\varnothing(1-S_{wc})}{B_{Oi}}*\frac{N}{G}\right\}_{limestone}$$

$$\left\{\frac{7758Ah\varnothing(1-S_{wc})}{B_{Oi}}*\frac{N}{G}\right\}_{sand}=\frac{7758*200105.51*0.21*(1-0.20)}{1.2831}*\frac{5}{8}$$
$$=127039160.9\ stb$$

$$\left\{\frac{7758Ah\varnothing(1-S_{wc})}{B_{Oi}}*\frac{N}{G}\right\}_{limestone}=\frac{7758*200105.51*0.17*(1-0.30)}{1.2831}*\frac{3}{8}$$
$$=53991643.40\ stb$$

Therefore, the STOIIP for the formation of 5/8 ^{ths} sand and 3/8 ^{ths} limestone is

$$STOIIP=127039160.9+53991643.40=181030804.3\ stb=181.0308\ MMstb$$

Example 3.5

A volumetric gas reservoir located offshore Niger Delta with the following data in the table below, determine the following:

Initial reservoir pressure, P_i	*3800 psia*
Cumulative gas production, G_p	*24.6 MMMscf*
Reservoir thickness, h	*25 ft*
Wellbore flowing pressure, P_{wf}	*2750 psia*
Connate water saturation, S_{wc}	*23%*
Gas gravity, γ_g	*0.68*
Porosity, ∅	*18%*
Reservoir temperature	170^0F

- Area extent of the reservoir,
- The gas reserve at 2750 psia.
- The recovery factor at 2750 psia.

Solution

The amount of gas produced at 2750 psia is calculated mathematically as

$$G_{p@2750\ psia}=G-G_{reserve@2750\ psia}$$

$$G_{p@2750\ psia}=G-G_{reserve@2750\ psia}$$

Recall:

$$G=\frac{43560Ah\varnothing(1-S_{wc})}{B_{gi}}$$

$$G_{reserve}=\frac{43560Ah\varnothing(1-S_{wc})}{B_g}$$

$$G_{p@2750\ psia} = 43560Ah\varnothing(1 - S_{wc})\left[\frac{1}{B_{gi}} - \frac{1}{B_g}\right]$$

$$B_g = \frac{0.0283zT}{P}$$

To calculate the compressibility factor, z
If yg < = 0.7 Then

$$T_c = 168 + \left(325*\gamma_g\right) - \left(12.5*{\gamma_g}^2\right)$$

$$T_c = 168 + (325*0.68) - \left(12.5*0.68^2\right) = 383.22^0\text{R}$$

$$P_c = 677 + \left(15*\gamma_g\right) - \left(37.5*{\gamma_g}^2\right)$$

$$P_c = 677 + (15*0.68) - \left(37.5*0.68^2\right) = 669.86\text{ psia}$$

$$T_r = \frac{(T + 460)}{T_c} = \frac{(170 + 460)}{383.22} = \frac{630}{383.22} = 1.64$$

$$P_{ri} = \frac{P_i}{P_c} = \frac{3800}{669.86} = 5.67$$

Calculate z from Fig. 3.4

$$z_i(P_{ri}, T_r) = z_i(5.67, 1.64) = 0.89$$

$$P_r = \frac{P}{P_c} = \frac{2750}{669.86} = 4.11$$

$$z(P_r, T_r) = z(4.11, 1.64) = 0.83$$

Therefore,

$$B_{gi} = \frac{0.0283z_iT}{P_i} = \frac{0.0283*0.89*(170 + 460)}{3800} = 0.004176\ cuft/scf$$

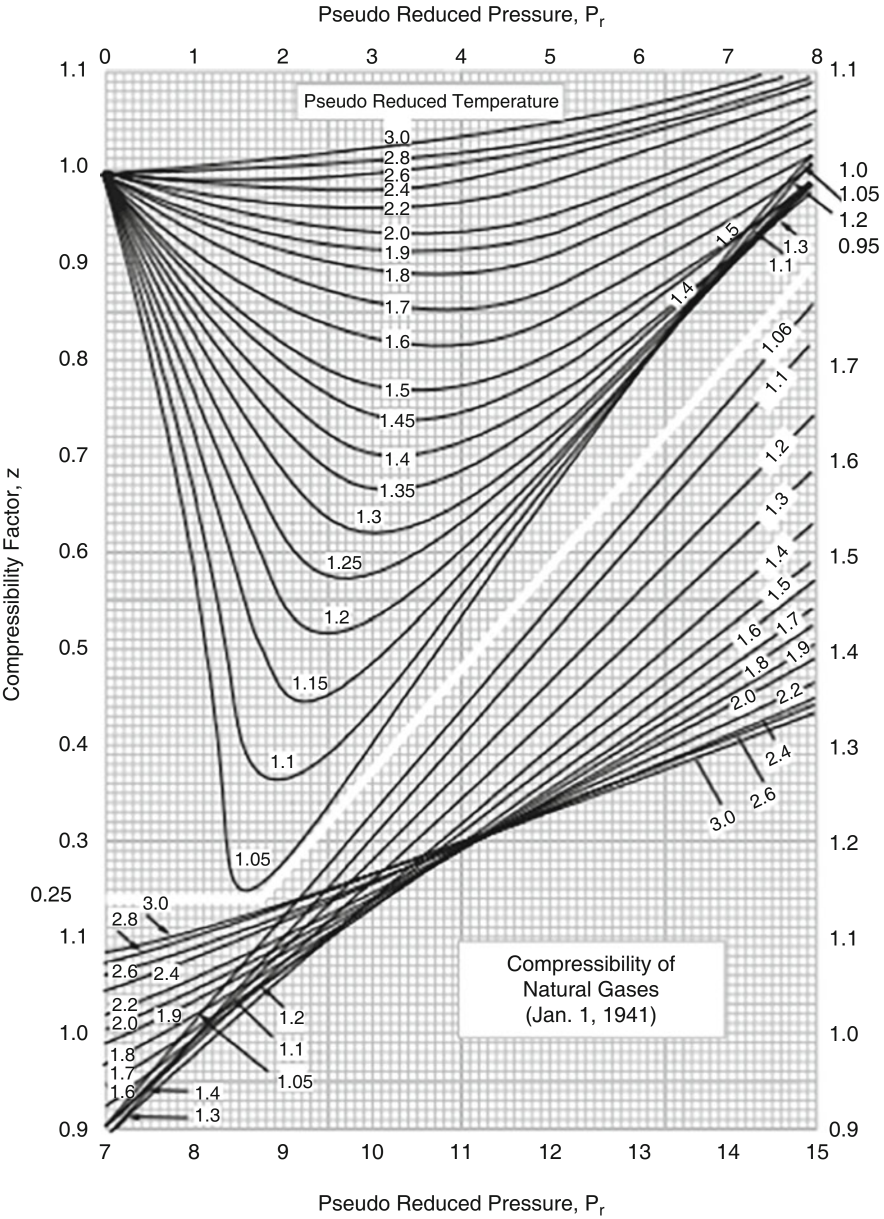


Fig. 3.4 Standing and Katz compressibility factors chart. (Courtesy of GPSA and GPA Engineering Data Book, EO Edition, 1987)

$$B_g = \frac{0.0283zT}{P} = \frac{0.0283*0.83*(170+460)}{2750} = 0.005381\ cuft/scf$$

$$G_p = 43560Ah\varnothing(1-S_{wc})\left[\frac{1}{B_{gi}} - \frac{1}{B_g}\right]$$

The area extent is

$$A = \frac{G_{p@2750\ psia}}{43560h\varnothing(1-S_{wc})}\left[\frac{B_{gi}B_g}{B_g - B_{gi}}\right]$$

$$A = \frac{24.6*10^9}{43560*25*0.18*(1-0.23)}\left[\frac{0.005381*0.004176}{0.005381-0.004176}\right] = 3039.348\ acres$$

To calculate the gas reserve at 2750 psia

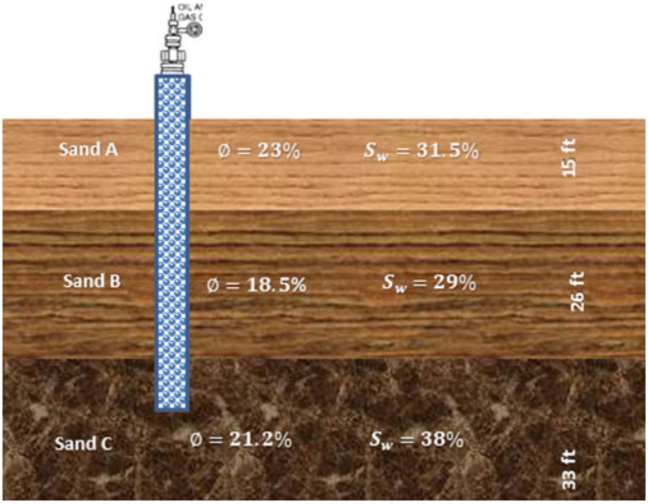
$$G_{reserve} = \frac{43560Ah\varnothing(1-S_{wc})}{B_g} = \frac{43560*3039.348*25*0.18*(1-0.23)}{0.005381}$$
$$= 8.52527*10^{10}scf = 85.253\ \ MMMscf$$

$$G = \frac{43560Ah\varnothing(1-S_{wc})}{B_{gi}}$$

$$G = \frac{43560*3039.348*25*0.18*(1-0.23)}{0.004176} = 109,852,779,243.1030\ scf$$
$$= 109.8528\ MMMscf$$

$$RF = \frac{G_p}{G} = \frac{24.6*10^9}{109852779243.1030} = 0.2239 = 22.39\%$$

Example 3.5
Given a stacked reservoir with three sand of different reservoir pay thickness in the figure below. The reservoir structure is 710 acres. The initial oil formation volume factor from PVT laboratory analysis is given as 1.3251 rb/stb. The petrophysical properties are stated on each sand in the figure. Calculate the stock tank oil initially in place for this stacked reservoir.



Solution

The average porosity is given as

$$\varnothing = \frac{\sum \varnothing_j h_i}{\sum h_i}$$

$$\varnothing = \frac{\varnothing_A h_A + \varnothing_B h_B + \varnothing_C h_C}{h_A + h_B + h_C}$$

$$\varnothing = \frac{(0.23*15) + (0.185*26) + (0.212*33)}{15 + 26 + 33} = 0.2062 = 20.62\%$$

Average water saturation

$$S_w = \frac{\sum_{i=1}^{n} \varnothing_i h_i s_{wi}}{\sum_{i=1}^{n} \varnothing_i h_i}$$

$$S_w = \frac{\varnothing_A h_A s_{wA} + \varnothing_B h_B s_{wB} + \varnothing_C h_C s_{wC}}{\varnothing_A h_A + \varnothing_B h_B + \varnothing_C h_C}$$

$$S_w = \frac{(0.23*15*0.315) + (0.185*26*0.29) + (0.212*33*0.38)}{(0.23*15) + (0.185*26) + (0.212*33)} = 0.3369$$

$$h = \sum_{i=1}^{n} h_i = h_A + h_B + h_C = 15 + 26 + 33 = 74\,ft$$

$$STOIIP = N = \frac{7758Ah\varnothing(1 - S_w)}{B_{oi}}$$

$$STOIIP = \frac{7758*710*74*0.2062*(1 - 0.3369)}{1.3251} = 42058993.8\ stb$$
$$= 42.059\ MMstb$$

Example 3.6

The result of a gas chromatographic test performed at FUPRE Laboratory on a gas sample collected from a gas field in the Niger Delta region is shown in Table 3.6.

Additional information:

Initial reservoir pressure, P_i	*3200 psia*
Reservoir thickness, h	*40 ft*
Connate water saturation, S_{wc}	*22%*
Area, A	*80 acres*
Porosity, $\varnothing$	*17.5%*
Reservoir temperature	*200 °F*

Calculate the gas initially in place

Table 3.6 Gas composition

Components	Mole Percent (x_i)
Nitrogen	0.15
Carbon dioxide	0.94
Methane	88.43
Ethane	3.41
Propane	3.64
i-butane	0.65
n-butane	1.50
i-pentane	0.44
n-pentane	0.39
Hexane	0.30
Heptane Plus	0.15

Solution

Components	Mole Percent (x_i)	M_i	X_iM_i
Nitrogen	0.15	28.0135	0.04202
Carbon dioxide	0.94	44.01	0.413694
Methane	88.43	16.042	14.18594
Ethane	3.41	30.069	1.025353
Propane	3.64	44.096	1.605094
i-butane	0.65	58.122	0.377793
n-butane	1.5	58.122	0.87183
i-pentane	0.44	72.149	0.317456
n-pentane	0.39	72.149	0.281381
Hexane	0.3	86.175	0.258525
Heptane Plus	0.15	106	0.159
			19.53809

(a) The apparent molecular weight = 19.5381
(b) The gas specific gravity

$$\gamma_g = \frac{M_a}{M_{air}} = \frac{M_a}{28.96} = \frac{19.5381}{28.96} = 0.675$$

(c) The critical temperature and pressure

If yg < = 0.7 Then

$$T_c = 168 + \left(325 * \gamma_g\right) - \left(12.5 * {\gamma_g}^2\right)$$

$$P_c = 677 + \left(15 * \gamma_g\right) - \left(37.5 * {\gamma_g}^2\right)$$

ElseIf yg > 0.7 Then

$$T_c = 187 + \left(330 * \gamma_g\right) - \left(75.5 * {\gamma_g}^2\right)$$

$$P_c = 706 - \left(51.7 * \gamma_g\right) - \left(11.1 * {\gamma_g}^2\right)$$

Since yg < = 0.7 Then

$$T_c = 168 + (325 * 0.675) - \left(12.5 * 0.675^2\right) = 381.68^0 R$$

$$P_c = 677 + (15 * 0.675) - \left(37.5 * 0.675^2\right) = 670.04\, psia$$

(d) The gas deviation factor of the gas at 3200 psia and 200 °F

$$T_r = \frac{T}{T_c} = \frac{(200 + 460)}{381.68} = 1.729$$

$$P_r = \frac{3200}{670.04} = 4.776$$

At this point, we can apply any of the correlations for gas deviation factor or read directly from the chart as a function of reduced temperature and pressure.

From the chart (Fig. 3.4), z = 0.88

Using Papay's Correlation

$$z = 1 - \left[\frac{3.52P_r}{10^{0.9813T_r}}\right] + \left[\frac{0.274{P_r}^2}{10^{0.8157T_r}}\right]$$

$$z = 1 - \left[\frac{3.52(4.775834)}{10^{0.9813(1.729197)}}\right] + \left[\frac{0.274(4.775834)^2}{10^{0.8157(1.729197)}}\right] = 0.9049$$

(e) The density of the gas

$$\rho_g = \frac{28.96P\gamma_g}{zRT} = \frac{28.96 \times 3200 \times 0.675}{0.88 \times 10.73 \times (200 + 460)} = 10.03751\ lb/cuft$$

(f) The gas formation volume factor

$$B_g = \frac{0\ 02827 \times 0.88 \times (200 + 460)}{3200} = 0.005131\ cuft/scf$$

$$GIIP = G = \frac{43560Ah\varnothing(1 - S_{wc})}{B_{gi}}$$

$$GIIP = G = \frac{43560{*}80{*}40{*}0.175{*}(1 - 0.22)}{0.005131} = 3708245566.16644\ scf$$

$$G = 3.708245\ MMMscf$$

Example 3.7

A 15600 acres oil reservoir owned by ABC Company located at FUPRE field was characterized as undersaturated reservoir with the following information:

Initial reservoir pressure, P_i	*3800 psia*
Abandonment pressure, P_a	*400 psia*
Reservoir thickness, h	*80 ft*
Connate water saturation, S_{wc}	*38%*
Initial oil FVF, β_{oi}	*1.458 rb/stb*
Oil FVF, β_{oa}	*1.126 rb/stb*
Gas saturation at abandonment, S_{ga}	*30%*
Residual oil saturation after water invasion, S_{or}	*18%*
Porosity, ∅	*23%*
Reservoir temperature	*200 °F*

Calculate the following:

1. Initial oil in place
2. Oil in place after volumetric depletion to abandonment pressure
3. Oil in place after water invasion at initial pressure
4. Oil produced by volumetric depletion to abandonment pressure and recovery factor
5. Oil produced by full water drive and recovery factor

Solution

1. Initial oil in place is given by:

$$N = \frac{7758Ah\varnothing(1 - S_{wc})}{B_{oi}}$$

$$N = \frac{7758*15600*80*0.23*(1-0.38)}{1.458} = 946948503.7\ STB = 946.85\ MMSTB$$

2. Oil in place after volumetric depletion to abandonment pressure is given by:

Naturally, nature avoids vacuum and a volumetric reservoir with no water influx; as the saturation of the oil reduces, the gas replaces the void space created by the oil. Thus, the above equation is adjusted as:

$$N_a = \frac{7758Ah\varnothing\left(1 - S_{wc} - S_{ga}\right)}{B_{oa}}$$

$$N_a = \frac{7758*15600*80*0.23*(1-0.38-0.30)}{1.126} = 632854371.6\ STB$$

$$= 632.85\ MMSTB$$

3. Oil in place after water invasion at initial pressure

$$N_i = \frac{7758Ah\emptyset S_{or}}{B_{oi}}$$

$$N_i = \frac{7758 * 15600 * 80 * 0.23 * 0.18}{1.458} = 274920533.3\ STB = 274.92\ MMSTB$$

4. Oil produced by volumetric depletion to abandonment pressure and recovery factor

$$N_{pa} = N - N_a$$

$$946948503.7 - 632854371.6 = 314094132.1\ STB = 314.09\ MMSTB$$

Recovery factor

$$RF = \frac{N - N_a}{N} = \frac{(946948503.7 - 632854371.6)}{946948503.7} = 0.3317 = 33.17\%$$

5. Oil produced by full water drive and recovery factor

$$N_{pi} = N - N_i$$

$$946948503.7 - 274920533.3 = 672027970.4\ STB = 672.03\ MMSTB$$

Recovery factor

$$RF = \frac{N - N_i}{N} = \frac{(946948503.7 - 274920533.3)}{946948503.7} = 0.7097 = 70.97\%$$

Example 3.8

As a Petroleum Engineer working in Sly Tech Dynamic currently operating a gas field located at Ugbomro. You are asked by the management team to evaluate the following to aid their decision with the data given in the Table 3.7:

Table 3.7 Ugbomro gas field data

Parameter	Value
Initial reservoir pressure	4870 psia
Abandonment pressure	475 psia
Reservoir area	210 acres
Pay zone (thickness)	55 ft
Connate water saturation	0.28 (28%)
Porosity	0.25 (25%)
Gas FVF @ 4870 psia	0.006273 ft^3/SCF
Gas FVF @ 3000 psia	0.008521 ft^3/SCF
Gas FVF @ 475 psia	0.048279 ft^3/SCF
Residual gas saturation after water invasion	0.37 (37%)

1. The gas initially in place
2. Gas in place after volumetric depletion to a pressure of 3000 psia
3. Gas in place after volumetric depletion to abandonment pressure
4. Gas in place after water invasion at initial pressure
5. Gas in place after water invasion at a pressure of 3000 psia
6. Gas in place after water invasion at a pressure of 475 psia
7. Gas produced by full water drive at initial pressure
8. Recovery factor after full water drive at initial pressure
9. Gas produced by partial water drive at 3000 psia
10. Recovery factor after partial water drive at 3000 psia
11. Gas produced by volumetric to abandonment pressure
12. Recovery factor at abandonment pressure

1. Initial gas in place is calculated as:

$$GIIP = G = \frac{43560 Ah\varnothing(1 - S_{wc})}{B_{gi}}$$

$$G = \frac{43560*210*55*0.25*(1 - 0.28)}{0.006273} = 1.4437 \times 10^{10}\ SCF$$
$$= 14.437\ MMMSCF$$

2. Gas in place after volumetric depletion to 3000 psia

$$G_{@3000\ psia} = \frac{43560 Ah\varnothing(1 - S_{wc})}{B_{g@3000}}$$

$$G_{@3000\ psia} = \frac{43560*210*55*0.25*(1-0.28)}{0.008521} = 1.0628 \times 10^{10}\ SCF$$
$$= 10.628\ MMMSCF$$

3. Gas in place after volumetric depletion to abandonment pressure

$$G_a = \frac{43560Ah\varnothing(1-S_{wc})}{B_{ga}} = \frac{43560Ah\varnothing(1-S_{wc})}{B_{g@475}}$$

$$G_a = \frac{43560*210*55*0.25*(1-0.28)}{0.048279} = 1.8758 \times 10^{9}\ SCF$$
$$= 1.876\ MMMSCF$$

4. Gas in place after water invasion at initial pressure

$$G_{inv@P_i} = G_{inv@4870psia} = \frac{43560Ah\varnothing S_{gr}}{B_{gi}}$$

$$G_{inv@4870psia} = \frac{43560*210*55*0.25*0.37}{0.006273} = 7.4188 \times 10^{9}\ SCF$$
$$= 7.419\ MMMSCF$$

5. Gas in place after water invasion at a pressure of 3000 psia

$$G_{inv@P} = \frac{43560Ah\varnothing S_{gr}}{B_g}$$

$$G_{inv@3000\ psia} = \frac{43560*210*55*0.25*0.37}{0.008521} = 5.462 \times 10^{9}\ SCF$$
$$= 5.462\ MMMSCF$$

6. Gas in place after water invasion at a pressure of 475 psia

$$G_{inv@P_a} = G_{inv@475\ psia} = \frac{43560Ah\varnothing S_{gr}}{B_g}$$

$$G_{inv@475\ psia} = \frac{43560*210*55*0.25*0.37}{0.048279} = 0.964 \times 10^9\ SCF$$
$$= 0.964\ MMMSCF$$

7. Gas produced by full water drive at initial pressure

$$G_{p@P_i} = G - G_{inv@P_i} = G - G_{inv@4870psia}$$

$$G_{p@4870} = 1.4437 \times 10^{10} - 7.4188 \times 10^9 = 7.018 \times 10^9 SCF = 7.018 MMMSCF$$

8. Recovery factor after full water drive at initial pressure

$$RF@P_i = \frac{G_{p@P_i}}{G} = \frac{7.018 \times 10^9}{1.4437 \times 10^{10}} = 0.4861 = 48.61\%$$

9. Gas produced by partial water drive at 3000 psia

$$G_{p@P} = G - G_{inv@P} = G - G_{inv@3000psia}$$

$$G_{p@3000} = 1.4437 \times 10^{10} - 5.462 \times 10^9 = 8.975 \times 10^9 SCF = 8.975 MMMSCF$$

10. Recovery factor after partial water drive at 3000 psia

$$RF@P = RF@3000\ psia$$
$$= \frac{G_{p@P}}{G} = \frac{G_{p@3000\ psia}}{G} = \frac{8.975 \times 10^9}{1.4437 \times 10^{10}} = 0.6217 = 62.17\%$$

11. Gas produced by volumetric to abandonment pressure

$$G_{p@P_a} = G - G_{inv@P_a} = G - G_{inv@475psia}$$

$$G_{r@475\ psia} = 1.4437 \times 10^{10} - 0.964 \times 10^9 = 1.3473 \times 10^{10} SCF$$
$$= 13.473 MMMSCF$$

12. Recovery factor at abandonment pressure

$$RF@P_a = RF@475\,psia$$

$$= \frac{G_{p@P_a}}{G} = \frac{G_{p@475\ psia}}{G} = \frac{1.3473 \times 10^{10}}{1.4437 \times 10^{10}} = 0.9332 = 93.32\%$$

As production and pressure data from a field become available, decline analysis and material balance calculations, become the predominant methods of calculating reserves. These methods greatly reduce the uncertainty in reserves estimation; however, during the early depletion, caution should be exercised in using them. Decline curve relationships are empirical and rely on a uniform, lengthy production periods. These are explained in subsequent chapters of this book.

3.4 Deterministic Versus Probabilistic Volumetric Reserves Estimation

The aspect of uncertainty in hydrocarbon reserves estimation cannot be overemphasized since the estimation of reserves is done under conditions of uncertainties. There are basically two methods of returning the results of reserves estimation for any of the techniques such as volumetric, material balance, decline curve etc. employed for reserves estimation. These methods are the deterministic and probabilistic methods. Thus, if a single best estimate of reserves is made based on known geological, engineering and economic data, the method is called deterministic whose procedure is to select a single value for each parameter to input into an appropriate equation (volumetric, material balance, decline curve etc.), to obtain a single answer. In volumetric method, all input parameters are exactly known and variability is sometimes ignored.

On the other hand, when the known geological, engineering, and economic data are used to generate a range of estimates and their associated probabilities; the method of estimation is called probabilistic. This method is more rigorous and less commonly used; it utilizes a distribution curve for each input parameter and through the use of Monte Carlo Simulation. In this method, all input parameters are not exactly known and variability cannot be ignored.

Since the oil and gas business is associated with some inherent uncertainties, it implies that a quality control and assurance should be made before making any decision to develop the hydrocarbon prospect because a wrong evaluation of the hydrocarbon initial in place leads to a wrong decision which in turn leads to an entire failure of the field development. However, a comparison of the deterministic and probabilistic methods can provide quality assurance for estimating hydrocarbon reserves. This means that when the values of the reserves calculated deterministically and probabilistically agree with minimal deviation or tolerance of error, then confidence on the calculated reserves is increased. On the contrary, when there is a significant difference in value, then the assumptions made need to be reexamined.

A Monte-Carlo technique is employed to evaluate hydrocarbons in place where each input parameter required for the reserves estimation are represented by statistical distributions. Monte-Carlo methods are mainly used in three distinct problem classes, such as optimization, numerical integration and generating draws from a probability distribution. There are basically five types of statistical distribution used with this method. These are:

3.4.1 Fixed Value

$$\boldsymbol{value = constant}$$

3.4.2 Uniform Distribution

It is defined by a minimum and maximum value with an equal probability for all values between these two extremes (Fig. 3.5).

$$\boldsymbol{value = minimum + (minimum - maximum) * probability}$$

3.4.3 Triangular Distribution

This distribution is defined by a minimum, maximum and mode value given as (Fig. 3.6):

$$\boldsymbol{P_{mode} = \frac{(Mode - Minimum)}{(Maximum - Minimum)}}$$

$$P < P_{mode}, \quad then \tag{If}$$

$$\boldsymbol{Value = Minimum + (Mode - Minimum) * \sqrt{\frac{P}{P_{mode}}}}$$

$$P > P_{mode}, \quad then \tag{ElseIf}$$

$$\boldsymbol{Value = Maximum - (Maximum - Mode) * \sqrt{\frac{(1 - P)}{(1 - P_{mode})}}}$$

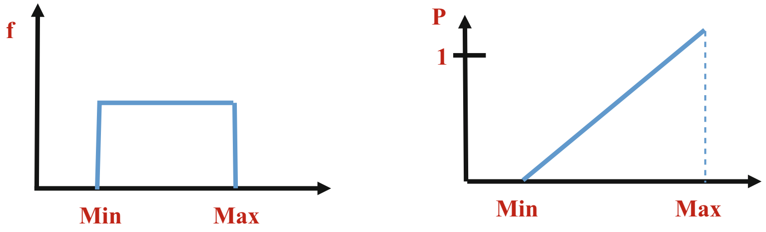


Fig. 3.5 Uniform distrubution curves

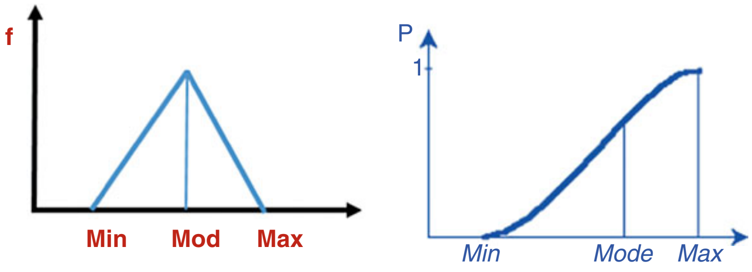


Fig. 3.6 Traingular distrubution curves

3.4.4 Normal Distribution

This is defined by an average and standard deviation with (Fig. 3.7):

$$\boldsymbol{Value} = \boldsymbol{Average} + \left(\boldsymbol{Std} * \left\{ \sqrt{\ln\left(\frac{\mathbf{1}}{\boldsymbol{P}^2}\right)} \right\} \right)$$

3.4.5 Log Normal Distribution

This is defined by an average and standard deviation with (Fig. 3.8):

$$\boldsymbol{Value} = \exp[\log(\boldsymbol{Average})] + \left[\log\left(\mathbf{1} + \frac{\boldsymbol{Std}}{\boldsymbol{Average}}\right) * \left\{ \sqrt{\ln\left(\frac{\mathbf{1}}{\boldsymbol{P}^2}\right)} \right\} \right]$$

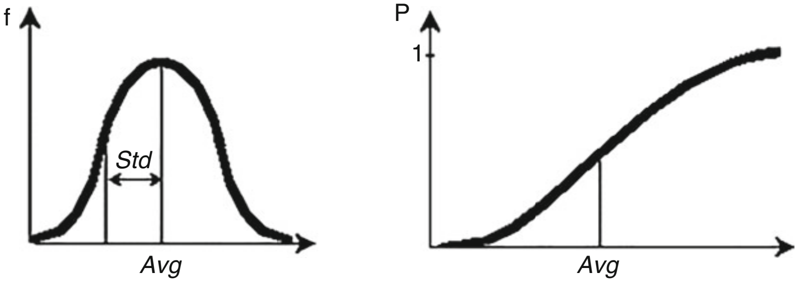


Fig. 3.7 Normal distrubution curves

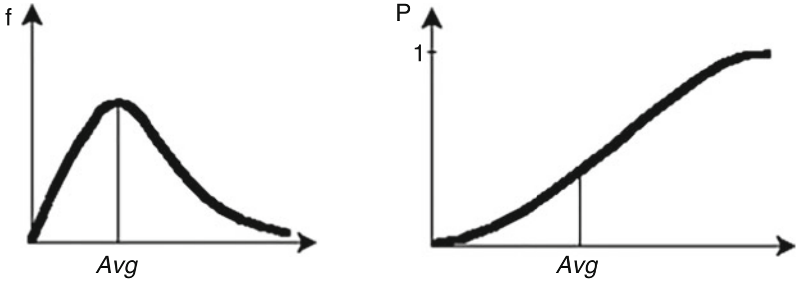


Fig. 3.8 Log normal distrubution curves

3.5 Condensate Reservoir Calculation

The example applies here for calculating condensate in place was written by Engr. Ogbarode Napoleon Ogbon in his Lecture note on Natural Gas Engineering II.

3.5.1 Applications of Gas and Condensate Inplace Value

- Determination of economic value of gas and condensate in place to make a decision on project economic viability

3.5.2 Major Points for Consideration

- As the gas-condensate reservoir fluid pressure drops below the dew point, liquid hydrocarbon (condensate) will begin to drop.
- It is necessary to recombine the condensate with the gas in a proper ratio to calculate the original volume of gas-in-place in the reservoir

3.5.3 Data Required to Allow Estimates of the Gas-in-Place Volume Are

- The geologic data
- The reservoir data
- The production data
- The geologic and reservoir data are used to provide plots of gas compressibility, etc.
- This method uses standard charts and simple equations to calculate hydrocarbon-in-place volumes in gas-condensate reservoirs.

3.5.4 Method Basic Requirements

- It is based on correlations established by Rzasa and Katz (2011) and provides a means to calculate the gas-in-place volume in a gas-condensate reservoir Based on
  - The amount of produced gas
  - The amount of produced associated condensate.
- Plots of correlations based on this method are readily available for use.
- However, it requires a clear understanding of the behaviour of oil and gas under various reservoir and surface operating conditions including:
  - Reservoir pressure and temperature, or depth to calculate the required parameters,
  - Compositions of oil and gas or their gravities and molecular weights,
  - Gravities and production rates of separator condensate and gas,
  - Rock porosity,
  - Gas or interstitial water saturation
  - Area-thickness, in the absence of which calculations are based on one acre of reservoir volume.

Example 3.9

Determine the gas condensate-in-place for a reservoir with the following parameters:

Reservoir pressure, P_e	*3000 psia*
Tank gas production, q_{ptank}	*200 Mcf/d*
Primary trap gas production, q_{ptank}	*4000 Mcf/d*
Tank condensate production, q_c	*400 bbl/d*
Tank condensate gravity,	*50° API*
Connate water saturation, S_{wc}	*12%*
Tank gas gravity, $\gamma_{gtank}(\text{air} = 1)$	*1.25*

(continued)

Primary trap gas gravity, γ_{gtrap}(air = 1)	*0.65*
Porosity, ∅	*30%*
Reservoir temperature	*240 °F*

Solution

Basic Steps → Rzasa and Katz (2011) correlations as illustrated by Standing (1977).

Step 1: Calculate average separator gas gravity (if not known) based on One acre-foot of reservoir by averaging the gas gravities from various stage separators using gas flow rates from individual stage separators for weighting.

$$\gamma_{avg}=\frac{\left(q_{gtrap}\times\gamma_{gtrap}\right)+\left(q_{gtank}\times\gamma_{gtank}\right)}{\left(q_{gtrap}+q_{gtank}\right)}$$

Where;

$$\gamma_{avg},\gamma_{gtrap},\gamma_{gtank} = \textit{average separator, primary gas trap\&tank gas gravity respectively},$$

$$q_{gtrap}\&q_{gtank}\left({}^{ft^3}/_{day}\right) = \textit{primary gas trap\&tank gas production respectively}$$

Step 2: Calculate the produced condensate-to-gas ratio in bbl/mm.ft^3

By dividing the daily condensate production (in barrels) by the total gas production (in millions of cubic feet).

$$CGR = \frac{q_c}{q_{gtrap}+q_{gtank}}\left({}^{bbl}/_{ft^3}\right)\times 1000000 \rightarrow {}^{bbl}/_{mm.ft^3}$$

q_C = Condensate production – bbl/day
CGR = Condensate-Gas-Ratio – bbl/mm.ft^3

Step 3: Determine Reservoir Fluid Gravity

From Fig. 3.9, Using the

- Condensate API
- Condensate-to-gas ratio on the x-axis
- Determine the Well- reservoir-fluid gravity to separator-gas gravity ratio on the y-axis to determine the
- Then calculate the Well- reservoir-fluid gravity, based on the known values of condensate gravity and separator-gas gravity (step 1).

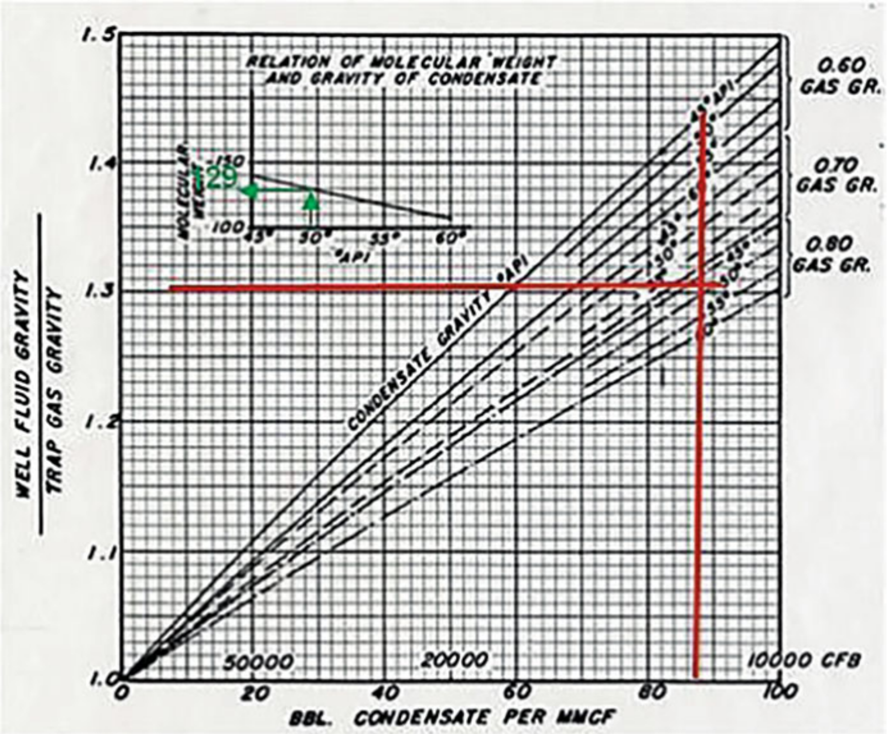


Fig. 3.9 Well fluid gravity determination

Step 4: Determine Reservoir Fluid Pseudo-critical press & Temperature

From Fig. 3.10 showing the correlation between pseudo-critical pressure and temperature and the well-reservoir-fluid gravity, determine the two pseudo-critical parameters for a given reservoir-fluid gravity from step 3.

Step 5: Determine the Pseudo-reduced Pressure and Temperature

Calculate the pseudo-reduced pressure and temperature, based on known pseudo-critical pressure and temperature (step 4) and the reservoir temperature and pressure.

$$T_{pr} = \frac{T_r + 460}{T_{pc}} = \frac{240 + 460}{445} = 1.57$$

$$P_{pr} = \frac{P_r}{P_{pc}} = \frac{3000}{640} = 4.7$$

Step 6: Determine Gas Compressibility Factor Using and Standing & Katz

Use pseudo-reduced parameters to determine the gas compressibility factor (Z) from the study by Standing and Katz (1942). → Fig. 3.4.

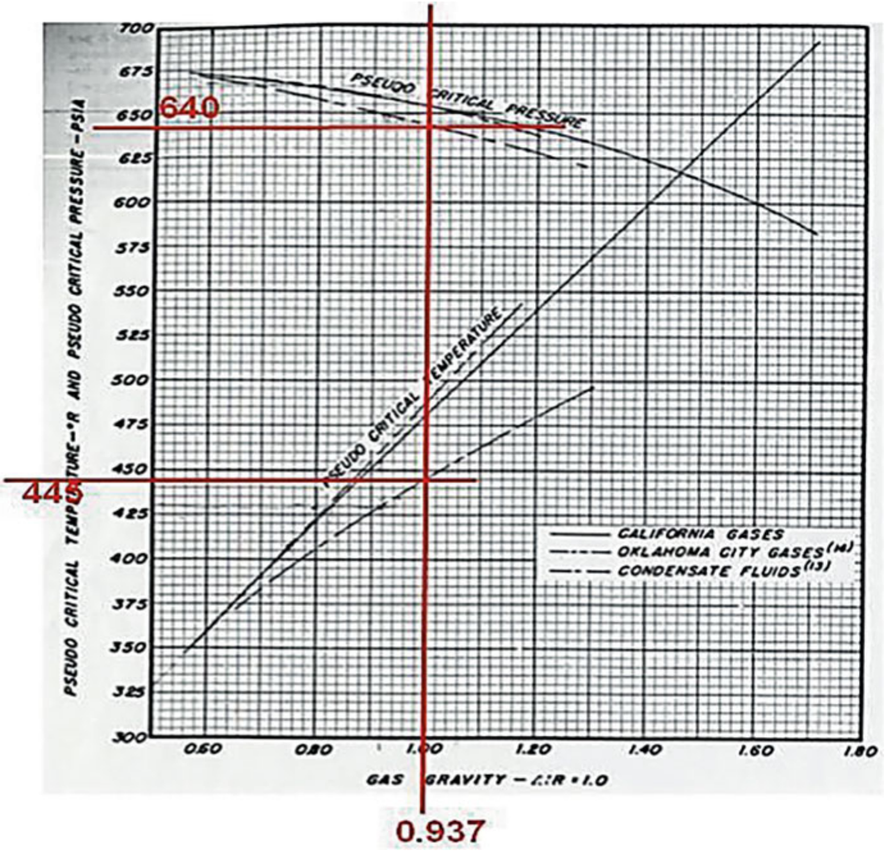


Fig. 3.10 Pseudo critical properties well-reservoir fluid

Z = 0.82

Step 7: Calculate the hydrocarbon volume of 1 acre-foot of reservoir using porosity and gas (or water) saturation.

$$V_h = 43560\varnothing S_h = 43560 * 0.3 * (1 - 0.12) = 11499.84\ \mathit{cubic\ feeet}$$
$$\approx 11500\ \mathit{cubic\ feet}$$

Step 8: Calculate the Total Number of moles in an acre-foot of reservoir rock using:

- General gas- law equation
- The known reservoir hydrocarbon pore volume,
- The reservoir pressure
- The reservoir temperature,

- The gas compressibility factor (Z),

$$N = \frac{PV_h}{ZRT}\ (lb - moles)$$

Where

N is pound-moles, P is pressure in psia (pounds per square inch absolute), V_h is volume in cubic feet, Z is compressibility factor, R is gas constant (10.73), T is temperature in degrees Rankine (temperature in degrees Fahrenheit +460).

$$N = \frac{3000*11499.84}{0.82*10.73*700} = 5601.46\,pound\ moles$$

Step 9: Determining Produced Gas Volume at Surface without Surface Condensate Production

If all the hydrocarbons are to be produced as gas at the surface, the gas volume can be calculated by multiplying the number of moles by 379 (one pound-mole of any gas occupies 379 cubic feet at the standard conditions of 14.7 psia and 60 °F).

$$V_s = 379*N = 379*5601.46 = 2122953.34\,SCF$$

Therefore, the reservoir gas-in-place = 2122953.34 SCF

Step 10: Determining Produced Gas Volume at Surface with Surface Condensate Production

On the other hand, if the condensate is produced with gas at the surface, the following equations can help calculate the number of pound-moles in one barrel of condensate:

$$N_B = \frac{5.615*62.4*\gamma_C}{MW_C}$$

Where;

N_B = Pound-moles in one barrel of condensate, γ_C = condensate specific gravity condensate specific gravity, MW_C = condensate molecular weight, 5.615 = the cubic feet equivalent of one barrel & 62.4 = water density in pounds per cubic feet.

Condensate density = pounds per cubic feet.

Condensate specific gravity is calculated from its gravity in API

$$\gamma_C = \frac{141.5}{131.5 + API_C} = \frac{141.5}{131.5 + 50} = 0.7796$$

From Fig. 3.7, with 50° API condensate, $MW_C = 129$

$$\therefore\ N_B = \frac{5.615*62.4*\gamma_C}{MW_C} = \frac{5.615*62.4*0.7796}{129} = 2.1175\frac{moles}{bbl.cond}$$

On the right side of the equation for $N_B$calculation, the numerator is the condensate mass in pounds in one barrel of condensate, and the denominator is its molecular weight, thereby giving the number of moles.

Step 11: Calculate the number of pound-moles in gas per condensate barrel

$$N_{g/C} = \frac{Gas-condensate\ ratio}{379} = \frac{ft^3\ of\ gas\ per\ bbl\ of\ condensate}{379} . {}^{lb}.moles/_{bbl}$$

$$N_{g/C} = \frac{(4000+200)\times 10^3}{400} = 10500\ cubic\ feet\ per\ barrel\ of\ condensate$$

∴.Gas mole per barrel of condensate

$$= \frac{10500}{379} = 27.7045$$

Step 12: Based on known moles of gas and condensate, determine the fraction of produced gas and condensate in the reservoir fluid stream.

$$N = N_B + N_{g/C} = 2.1175 + 27.7045 = 29.822\ moles$$

Gas mole fraction

$$\frac{N_{g/C}}{N} = \frac{27.7045}{29.82} = 0.9291 \approx 0.93$$

Condensate fraction

$$\frac{N_B}{N} = \frac{2.1175}{29.822} = 0.071$$

Step 13: Calculate the produced gas volume per acre-foot of reservoir rock

Multiply the gas mole fraction (from step 12) by total number of moles (from step 9) and 379 to determine the cubic feet of gas in-place

Gas-in-place = 2,122,953.34 x 0.9291 = 1972435.948 ft^3

Step 14: Calculate total volume of Condensate

Multiply the condensate mole fraction (from step 12) by total number of moles (from step 9) to get the number of condensate moles in 1 acre-foot of rock and then divide it by number of moles per barrel of condensate (from step 10) to obtain volume in barrels.

Condensate-in-place

$$= \frac{0.071 * 5601.46}{2.1175} = 187.8175 \; barrels\; per\; acre - foot\; of\; rock$$

Exercises

Ex 3.1

I. State the methods available for reserves estimation and explain when each of the is used
II. List and explain the factors affecting reserves estimate
III. Why is STOIIP determined from material balance method different from volumetric method?
IV. Give reasons while reserves estimation is often high with volumetric method
V. The calculation of STOIIP from volumetric method involves several parameters, state these parameters and the sources where they can be obtained
VI. What is the practical implication of taking cut-off in petrophysical data?
VII. What is the factor used to covert acre-ft to stb?
VIII. What is the implication of net-to-gross ratio in volumetric estimate of STOIIP?

Ex 3.2 The hydrocarbon contents of a reservoir were determined from the data of cumulative bulk volume (CBV) at the indicated depths on the table below.

Depth (ft)	CBV (ac-ft)	Depth (ft)	CBV (ac-ft)
9400	0.00	11,200	58866.34
9600	3159.46	11,400	64419.35
9800	11623.16	11,600	70282.13
10,000	23833.84	11,800	76.469.17
10,200	33547.92	12,000	82988.40
10,400	38822.03	12,200	89843.60
10,600	43697.74	12,400	97035.93
10,800	48568.69	12,600	104564.78
11,000	53597.63		

The following petrophysical and PVT parameters apply: Gas-Oil contact (GOC) = 10,400 ftss; Oil-Water contact (OWC) = 12,200 ftss; oil formation volume factor = 1.3279 rb/stb; Gas expansion factor (E_i) = 185.1 scf/cuft; sand/shale factor (F) = 88%; porosity = 18%; connate water saturation = 13%.

I. Indicate on a depth vs CBV plot, the depth at which the oil volume is acting
II. Calculate the volume of free-gas initially in place (in MMMscf)
III. Calculate the volume of the stock tank oil initially in place (in MMstb)

IV. Calculate the gas cap size, m
V. Find the centroid depth of the reservoir

Ex 3.3 A hydrocarbon reservoir is mapped out in area recorded at corresponding depths as given in the table.

Depth (ftss)	Top Area (acres)	Base Area (acres)
8800	0.000	
9000	33.21	
9300	91.31	0.00
9400	112.64	16.20
9500	125.06	32.40
9700	148.23	54.00
10,000	175.55	79.92
10,400	215.62	116.64
10,500	228.74	127.44
11,000	297.32	183.06

I. Using numerical approximation method, determine (in Mac-ft) the cumulative bulk volume (CBV) down to the OWC
II. Find the centroid depth of the reservoir
III. Calculate the hydrocarbon pore volume in MMbbls down to the OWC for the following three sets of petrophysical data:

(a) All sand with porosity 21% and connate water saturation of 20%
(b) 50% productive limestone with porosity 17% and connate water saturation of 30% 50% non-productive
(c) 5/8 $^{\text{ths}}$ sand as above; 3/8$^{\text{rds}}$ limestone as above

Additional Data:

Crest of top reservoir = 8800 ftss; crest of base of reservoir = 9400 ftss; OWC = 10,700 ftss; 1 acres = 7758.4 bbls.

Ex 3.4 Calculate the following with the data given in Table 3.8:

Table 3.8 Ugbomro gas field data

Parameter	Value
Initial reservoir pressure	5000 psia
Abandonment pressure	555 psia
Reservoir area	300 acres
Pay zone (thickness)	70 ft
Connate water saturation	0.25 (25%)
Porosity	0.23 (23%)
Gas FVF @ 5000 psia	0.007473 ft^3/SCF
Gas FVF @ 3540 psia	0.009831 ft^3/SCF
Gas FVF @ 555 psia	0.05379 ft^3/SCF
Residual gas saturation after water invasion	0.29 (29%)

I. The gas initially in place
II. Gas in place after volumetric depletion to a pressure of 3540 psia
III. Gas in place after volumetric depletion to abandonment pressure
IV. Gas in place after water invasion at initial pressure
V. Gas in place after water invasion at a pressure of 3540 psia
VI. Gas in place after water invasion at a pressure of 555 psia
VII. Gas produced by full water drive at initial pressure
VIII. Recovery factor after full water drive at initial pressure
IX. Gas produced by partial water drive at 3540 psia
X. Recovery factor after partial water drive at 3540 psia
XI. Gas produced by volumetric to abandonment pressure
XII. Recovery factor at abandonment pressure

Ex 3.5 Use the Monte-Carlo techniques to calculate the porosity value from the set of data assuming a uniform probability of 0.89:

0.23, 0.21, 0.20, 0.24, 0.21, 0.205, 0.22, 0.21, 0.22, 0.22, 0.24, 0.235, 0.215, 0.21, 0.23, 0.21, 0.23, 0.23, 0.21, 0.20, 0.21, 0.23, 0.21, 0.20, 0.21, 0.21, 0.23, 0.20, 0.20, 0.21, 0.21, 0.20, 0.20, 0.21, 0.23, 0.23,

For:

I. Fixed Value
II. Uniform Distribution
III. Triangular Distribution
IV. Normal Distribution
V. Log Normal Distribution

Ex 3.6 Calculate the gas reserve in a gas field of 2300 acres, with 40 ft. sand thickness, 23% porosity, 17% water saturation, initial pressure of 3200 psi and temperature of 200 °F. the composition of the gas and their weight fractions are as follows: 93.63% methane; 3.54% ethane; 1.46% propane; 0.38% isobutene, 0.36% pentane and 0.17% hexane plus.

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Chapter 4
Water Influx

Chapter Learning Objectives
Upon completion of this chapter, students/readers should be able to:

- Understand the concept of water influx or encroachment into a reservoir
- Understand the various types of aquifer models
- Identify when to apply each of the aquifer models
- Perform calculations on each of the aquifer models

Nomenclature

Parameter	Symbol	Unit
Initial reservoir pressure	P_i	*psi*
Current reservoir pressure	P	*psi*
Cumulative water influx	W_e	*Bbl*
Initial volume of water in the aquifer	W_i	*bbl*
Aquifer porosity	$\varnothing$	*- or %*
Aquifer water & rock compressibility	c_w & c_f	psi^{-1}
Total compressibility	c_t	psi^{-1}
Aquifer thickness	h	*ft*
Reservoir radius	r_e	*ft*
Aquifer radius	r_a	*ft*
Aquifer/water influx constant	C_s or C	*bbl/d/psi*
Time	t	*hours, months, year*
Dimensionless water influx	W_D	–
Water viscosity	μ_w	*cp*
Aquifer permeability	k	*mD*
Dimensionless pressure	P_D	–
Dimensionless pressure derivative	P'_D	–

S. Okotie, B. Ikporo, *Reservoir Engineering*,
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4.1 Introduction

Water influx can also be referred to as water encroachment or aquifer influx. It can be defined as an underground layer of water-bearing porous rock which flows out into any available space in the reservoir rock. In this context, an aquifer is referred to as a large pool of water body underlying a hydrocarbon accumulation in the reservoir structure that is made up of more than one fluid arranged according to density differences. Prior to hydrocarbon accumulation, the original system was occupied or filled with water and during the drainage process; the migrated hydrocarbons from the source rock displaced some of the water out of the pore space in the reservoir. This means that majority of hydrocarbon pools discovered globally have an associated aquifer which could be a key source of energy (primary recovery) for the hydrocarbon production once a well is drilled.

4.1.1 Classification of Aquifer Influx

Aquifer influx can be classified based on pressure maintenance, outer boundary conditions, flow regime, flow geometry as shown in Fig. 4.1.

The classification of aquifer system as shown in Fig. 4.1, is key to understanding and evaluation of hydrocarbon reservoirs performance. As hydrocarbon is produced from the reservoir, the pressure of the reservoir declines (changes) and the aquifer responds to offset the pressure decline due to fluids production, which is dependent on the strength of the aquifer. Besides, if there is a strong support from the aquifer,

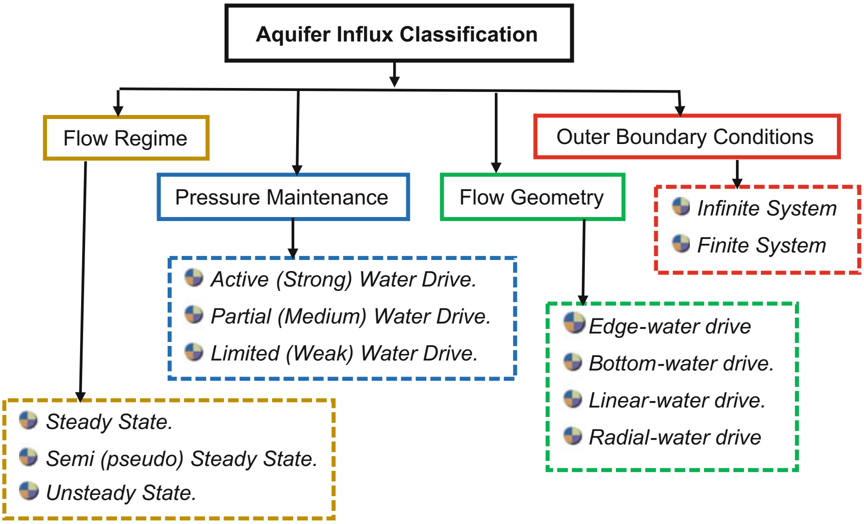


Fig. 4.1 Classification of aquifer influx

there will be a gradual decline in the reservoir pressure leading to a good hydrocarbon recovery. Also, there will be fairly steady gas-oil ratio during the life of the reservoir with excessive water production in shallow wells.

Consequently, in evaluating the performance of hydrocarbon reservoirs, we need to accurately determine the amount of water encroaching into the reservoir whose value is dependent on the water viscosity, the permeability of the rock in the aquifer and the cross-sectional area between the water zone and the region where the hydrocarbon is accumulated.

4.2 Aquifer Models

There are several analytical aquifer models presented in the past to estimate the amount of water encroaching into hydrocarbon reservoirs and some of these models are briefly presented below. The aquifer analytical models make use of simplified assumptions that do not consider the heterogeneous nature of the reservoir but a relatively homogeneous reservoir which has deterred the ideal comparison that is adopted in the analytical solutions. But when the equations are accurately discretization, they are relatively easy to program in computer spreadsheets with the exception of the Van Everdingen & Hurst, whose model does not demand much computer power.

4.2.1 Pot Aquifer Model

This method is one the simplest model for estimating the amount of water encroaching into hydrocarbon reservoirs. Mathematically, it is given as:

$$\boldsymbol{W_e = (c_w + c_f) f W_i (P_i - P)}$$

Where

$$W_i = \frac{\pi(r_a{}^2 - r_e{}^2)h\theta}{5.615}, f = \frac{\theta}{360}$$

Example 4.1
The data of a reservoir whose encroachment angle is 120° is given below. Calculate the volume of water initially in the aquifer and the cumulative water influx; the aquifer can deliver to the reservoir at 3750 psi.

Aquifer permeability, K_w	280 mD
Initial reservoir pressure, P_i	*4000 psi*
Current reservoir pressure, P	*3750 psi*
Reservoir thickness, h	*31 ft*
Aquifer radius, r_a	*12,000 ft*
Reservoir radius, r_e	*2800 ft*
Reservoir compressibility, C_f	$3.4 * 10^{-6}$ *psi*$^{-1}$
Aquifer compressibility, C_w	$4.8 * 10^{-6}$ *psi*$^{-1}$
Porosity, $\varnothing$	*16%*

Solution

$$f = \frac{\varnothing}{360} = \frac{120}{360} = 0.3333$$

The volume of water initially in the aquifer

$$W_i = \frac{\pi({r_a}^2 - {r_e}^2)h\varnothing}{5.615} = \frac{3.1416*\left(12000^2 - 2800^2\right)*31*0.16}{5.615}$$
$$= 377861241.3\ bbl$$

Cumulative water influx at 3750 psi is

$$W_e = \left(c_w + c_f\right)fW_i(P_i - P)$$
$$= \left(3.4 \times 10^{-6} + 4.8 \times 10^{-6}\right)*0.3333*377861241.3*(4000 - 3750)$$
$$= 258179.36\ bbl$$

4.2.2 Schilthuis Model

Schilthuis (1936) was the first to develop useful expressions for calculating water influx in a hydrocarbon reservoir. His steady-state expression is given by:

$$W_e = C_s \int_0^t (p_i - p)dt$$

In a differential form, the rate of water influx is given as:

$$\frac{dW_e}{dt} = C_s(p_i - p)$$

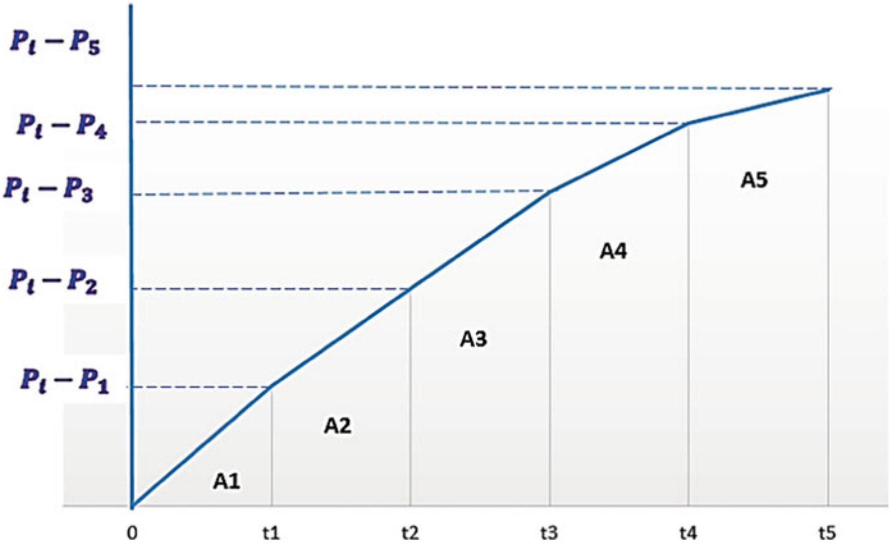


Fig. 4.2 Pressure drops as a function of time

Furthermore, as the reservoir pressure drops from initial pressure at time (t = 0) to any time (t). The cumulative water influx is a contribution of the cumulative pressure drop from initial reservoir pressure. This is illustrated in Fig. 4.2.

The area under the curve can be solved using any of the numerical integration techniques. Here we apply trapezoidal rule to solve for the cumulative water influx after each time step.

$$W_e = C_s \left\{ \frac{(t_{n+1} - t_n)}{2} [P_i - P_n] \right\}$$

Here are the steps to calculate the cumulative water influx with Schilthuis steady state model:

Step 1: Calculate the pressure drop at each time, t
Step 2: Calculate the water influx at each time, t
Step 3: Calculate the cumulative water influx at each time, t

This can be represented in a tabular form (Table 4.1) as:

Example 4.2

An oil reservoir whose predominant drive mechanism is powered by water drive, has an estimated aquifer constant of 148 bbl/year/psi. Table 4.2 shows the pressure history. Calculate the cumulative steady state water influx at each time step.

Table 4.1 Schilthuis aquifer model calculation

Time (days)	Pressure (psi)	Pressure Drop (psi)	Water Influx (bbl)	Cumulative Water Influx (bbl)
0	P_i	0	0	0
t_1	P_1	$\Delta P_1 = P_i - P_1$	$W_{e1} = C_s\left\{\frac{(\Delta P_1+0)}{2}(t_1 - 0)\right\}$	W_{e1}
t_2	P_2	$\Delta P_2 = P_i - P_2$	$W_{e2} = C_s\left\{\frac{(\Delta P_1+\Delta P_2)}{2}(t_2 - t_1)\right\}$	$W_{e1} + W_{e2}$
t_3	P_3	$\Delta P_3 = P_i - P_3$	$W_{e3} = C_s\left\{\frac{(\Delta P_2+\Delta P_3)}{2}(t_3 - t_2)\right\}$	$W_{e1} + W_{e2} + W_{e3}$
t_4	P_4	$\Delta P_4 = P_i - P_4$	$W_{e4} = C_s\left\{\frac{(\Delta P_3+\Delta P_4)}{2}(t_4 - t_3)\right\}$	$W_{e1} + W_{e2} + W_{e3} + W_{e4}$
t_5	P_4	$\Delta P_5 = P_i - P_5$	$W_{e5} = C_s\left\{\frac{(\Delta P_4+\Delta P_5)}{2}(t_5 - t_4)\right\}$	$W_{e1} + W_{e2} + W_{e3} + W_{e4}$ $+W_{e5}$

Table 4.2 Pressure history of Example 4.2

Time (years)	Pressure (psi)
0	5200
0.5	5120
1	5040
1.5	5005
2	4950
2.5	4910

Solution

Calculation of pressure drops

$$\Delta P_1 = 5200 - 5120 = 80\,psi$$

$$\Delta P_2 = 5200 - 5040 = 160\,psi$$

$$\Delta P_3 = 5200 - 5005 = 195\,psi$$

$$\Delta P_4 = 5200 - 4950 = 250\,psi$$

$$\Delta P_5 = 5200 - 4910 = 290\,psi$$

At the end of 0.5 years (6 months), the cumulative water influx is

$$W_e = C_s\left\{\frac{(\Delta P_1 + 0)}{2}(t_1 - 0)\right\} = 148\left\{\frac{(80 + 0)}{2}(0.5 - 0)\right\} = 2960\,bbl$$

At the end of 1 year, the cumulative water influx is

$$W_e = W_{e1} + W_{e2} = C_s\left\{\frac{(\Delta P_1 + 0)}{2}(t_1 - 0) + \frac{(\Delta P_1 + \Delta P_2)}{2}(t_2 - t_1)\right\}$$

$$= 2960 + 148\left\{\frac{(80 + 160)}{2}(1 - 0.5)\right\} = 2960 + 8880 = 11840\ bbl$$

At the end of 1.5 year, the cumulative water influx is

$$W_e = W_{e1} + W_{e2} + W_{e3}$$

$$= C_s\left\{\frac{(\Delta P_1 + 0)}{2}(t_1 - 0) + \frac{(\Delta P_1 + \Delta P_2)}{2}(t_2 - t_1) + \left\{\frac{(\Delta P_2 + \Delta P_3)}{2}(t_3 - t_2)\right\}\right\}$$

$$= 2960 + 8880 + 148\left\{\frac{(160 + 195)}{2}(1.5 - 1)\right\} = 2960 + 8880 + 13135$$

$$= 24975\ bbl$$

The cumulative water influx at the end of 2 and 2.5 years are given in the table below

Time (days)	Pressure (psi)	Pressure Drop (psi)	Water Influx (bbl)	Cumulative Water Influx (bbl)
0	5200	0	0	0
0.5	5120	80	2960	2960
1	5040	160	8880	11,840
1.5	5005	195	13,135	24,975
2	4950	250	16,465	41,440
2.5	4910	290	19,980	61,420

4.2.3 Hurst Modified Steady-State Model

Analysis of water expansion into a hydrocarbon reservoir indicates that water influx should often be an unsteady-state process. Hence, the Hurst modified steady-state eq. (1958) should give better results. The equation is:

$$\boldsymbol{W_e = C_h \int_0^t \frac{(p_i - p)}{\log \alpha t} dt = C_h \sum_0^t \left(\frac{\Delta P}{\ln \alpha t}\right) \Delta t}$$

In a differential form, the rate of water influx is expressed as:

$$\frac{dW_e}{dt} = C_h \frac{(p_i - p)}{\log \alpha t}$$

Where C_h is the water influx constant in barrels per day per pound per square inch, α is a time conversion constant which depends upon the units of time, t.

4.2.4 Van Everdingen & Hurst Model

Van Everdingen & Hurst method of calculating water influx requires the principle of superposition which is a tedious exercise, but it provides an exact solution to the radial diffusivity equation and can be applied at the early stage. To abate the intricacy of water influx calculations, Carter and Tracy (1960) proposed a direct water influx calculation technique that does not require superposition. The primary difference between Carter-Tracy and Van Everdingen & Hurst techniques is that the former assumes constant water influx rates over each finite time interval. Hence, the cumulative water influx at any time "t_n" can be calculated directly from previous values obtained at t_{n-1}.

$$W_e = C\Delta p W_{eD}(t_D)$$

C is the water influx constant given in oil field units as:

For radial system:

$$C = 1.119 f \varnothing h c_{tw} {r_e}^2 \ (bbl/psi)$$

$$f = \frac{\theta}{360}$$

$$t_D = \frac{0.000264kt}{\mu_w \varnothing_w c_{tw} {r_e}^2}\ (t\ in\ hours) = \frac{0.006336kt}{\mu_w \varnothing_w c_{tw} {r_e}^2}\ (t\ in\ days)$$
$$= \frac{2.309kt}{\mu_w \varnothing_w c_{tw} {r_e}^2}\ (t\ in\ years)$$

For linear system:

$$C = 0.1781 wL\varnothing h c_{tw}\ (bbl/psi)$$

$$t_D = \frac{0.000264kt}{\mu_w \varnothing_w c_{tw} L}\ (t\ in\ hours) = \frac{0.006336kt}{\mu_w \varnothing_w c_{tw} L}\ (t\ in\ days) = \frac{2.309kt}{\mu_w \varnothing_w c_{tw} L}\ (t\ in\ years)$$

Using superposition, the water influx at time t_D, is

$$\boldsymbol{W_e(T_D) = C[\Delta p_0 W_{eD}(T_D) + \Delta p_1 W_{eD}(T_D - t_{D1}) + \Delta p_2 W_{eD}(T_D - t_{D2})]}$$

The pressure drop is illustrated in the Fig. 4.3.

According to Van Everdingen (1953)

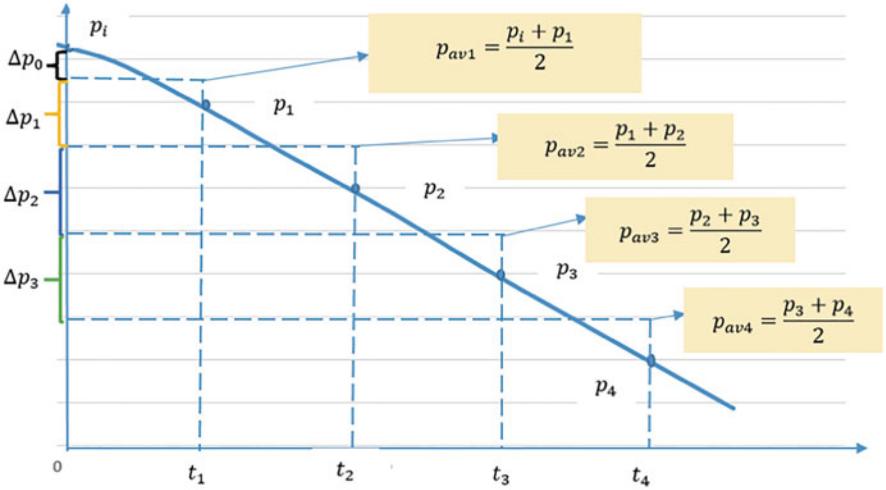


Fig. 4.3 Determination of pressure drop

$$\Delta p_j = \frac{p_i - p_{j+1}}{2}, j \leq 1\, for\, j = 0, 1$$

$$\Delta p_j = \frac{p_{j-1} - p_{j+1}}{2}, j > 1\, for\, j = 2 \ldots\ldots\ldots n\ \boldsymbol{i.e\ from\ P_2}$$

$$\Delta p_0 = p_i - p_{av1} = p_i - \frac{p_i + p_1}{2} = \frac{p_i - p_1}{2}$$

$$\Delta p_1 = p_{av1} - p_{av2} = \frac{p_i + p_1}{2} - \frac{p_1 + p_2}{2} = \frac{p_i - p_2}{2}$$

$$\Delta p_2 = p_{av2} - p_{av3} = \frac{p_1 + p_2}{2} - \frac{p_2 + p_3}{2} = \frac{p_1 - p_3}{2}$$

$$\Delta p_3 = p_{av3} - p_{av4} = \frac{p_2 + p_3}{2} - \frac{p_3 + p_4}{2} = \frac{p_2 - p_4}{2}$$

The dimensionless water influx can be read from Tables 4.3, 4.4, and 4.5 as a function of dimensionless time for infinite and finite aquifer.

The values of the dimensionless water influx can also be calculated using the following polynomial correlations developed by Edwardson and coworkers far back in 1962 as a function of time for infinite-acting aquifers. The result with this correlation is close to the values in the tables above.

$$W_{eD}(t_D) = 2\sqrt{\frac{t_D}{\pi}}\ t_D \leq 0.01$$

Table 4.3 Dimensionless Water Influx W_{eD} for Infinite Aquifer (Van Everdingen and Hurst W_{eD})

t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
0	0	79	35.697	455	150.25	1190	340.84	3250	816.09	35	6,780,247
0.01	0.112	80	36.058	460	151.64	1200	341308	3300	827.088	40	7650.096
0.05	0.278	81	36.418	465	153.03	1210	345.77	3350	838.067	50	9363.099
0.1	0.404	82	36.777	470	154.42	1220	348.23	3400	849.028	60	11,047,299
0.15	0.52	83	37.136	475	155.8	1225	349.46	3450	859.974	70	12,708.36
0.2	0.606	84	37.494	480	157.18	1230	350.69	3500	870.903	75	13,531,457
0.25	0.689	85	37.851	485	158.57	1240	353.14	3550	881.816	80	14,350.12
0.3	0.758	86	38.207	490	159.95	1250	355.6	3600	892.712	90	15,975.39
0.4	0.898	87	38.563	495	161.32	1260	358.05	3650	903.594	100	17,586.28
0.5	1.02	88	38.919	500	162.7	1270	360.5	3700	914.459	125	21560.732
0.6	1.14	89	39.272	510	165.44	1275	361.72	3750	925.309	1.5(10)^5	2.538(10)^4
0.7	1.251	90	39.626	520	168.18	1280	362.94	3800	936.144	2.0"	3.308'
0.8	1.359	91	39.979	525	169.55	1290	365.39	3850	946.966	2.5"	4.066'
0.9	1.469	91	40.331	530	170.91	1300	367.83	3900	957.773	10"	4817"
1	1.569	93	40.684	540	173.64	1310	370.27	3950	968.566	4.0"	6.26 T'
2	2447	94	41.034	550	176.36	1320	372.7	4000	979.344	5.0"	7699"
3	3.202	95	41.385	560	179.07	1325	373.92	4050	990.108	6.0"	9.113'
4	3.893	96	41.735	570	181.77	1330	375.14	4100	1000.86	7.0"	1.051(10)^5
5	4.539	97	42.084	575	183.12	1340	377.57	4150	1011.6	8.0"	1.189"
6	5.153	98	42.433	580	184.47	1350	380	4200	1022.32	9.0"	1326'
7	5.743	99	42.781	590	187.17	1360	382.43	4250	1033.03	1.0(10)^6	1.462"
8	6.314	100	43.129	600	189.85	1370	384.86	4300	1043.72	1.5"	2.126"
9	6.869	105	44.858	610	192.53	1375	386.07	4350	1054.41	2.0"	2.781'
10	7.411	110	46.574	620	195.208	1380	387.283	4400	1065.082	2.5"	3.427"
11	7.94	115	48.277	625	196.544	1390	389.705	4450	1075.743	3.0"	4.064"
12	8.457	120	49.968	630	197.878	1400	392.125	4500	1086.39	4.0"	5.313"

13	8.964	125	51.648	640	200.542	1410	394.543	4550	1097.024	5.0"	6.544"
14	9.461	130	53.317	650	203.201	1420	396.959	4600	1107.646	6.0"	7.761"
15	9.949	135	54.976	660	205.854	1425	398.167	4650	1118.257	7.0"	8.965"
16	10.434	140	56.625	670	208.502	1430	399.373	4700	1128.854	8.0"	$1.016(10)^6$
17	10.913	145	58.265	675	209.825	1440	401.786	4750	1139.439	9.0"	1.134"
18	11.386	150	59.895	680	211.145	1450	404.197	4800	1150.012	1.0(10)7	1.252"
19	11.855	155	61.517	690	213.784	1460	406.606	4850	1160.574	1.5"	1.828"
20	12.319	160	63.131	700	216.417	1470	409.013	4900	1171.125	2.0"	2.398"
21	12.778	165	64.737	710	219.046	1475	410.214	4950	1181.666	2.5"	2.961"
22	13.233	170	66.336	720	221.67	1480	411.418	5000	1192.198	3.0"	3.517"
23	13.684	175	67.928	725	222.98	1490	413.82	5100	1213.222	4.0"	4.610"
24	14.131	180	69.512	730	224.289	1500	416.22	5200	1234.203	5.0"	5.689"
25	14.573	185	71.09	740	226.904	1525	422.214	5300	1255.141	6,0"	6.758"
26	15.013	190	72.661	750	229.514	1550	428.196	5400	1276.037	7.0"	7.816"
27	15.45	195	74.226	760	232.12	1575	434.168	5500	1296.893	8.0"	8.866"
28	15.883	200	75.785	770	234.721	1600	110.128	5600	1317.709	9.0"	9.911"
29	16.313	205	77.338	775	236.02	1625	116.077	5700	1338.486	$1.0(10)^8$	$1.095(10)^7$
30	16.742	210	78.886	780	237.318	1650	452.016	5800	1359.225	1.5"	1.604"
31	17.167	215	80.428	790	239.912	1675	457.945	5900	1379.927	2.0"	2.108"
32	17.59	220	81.965	800	242.501	1700	463.863	6000	1400.593	2.5"	2.607"
33	18.011	225	83.497	810	245.086	1725	469.771	6100	1421.224	3.0"	3.100"
34	18.429	230	85.023	820	247.668	1750	475.669	6200	1441.82	4.0"	4.071"
35	18.845	235	86.545	825	248.957	1775	481.558	6300	1462.383	5.0"	5.032"
36	19.259	240	88.062	830	250.245	1800	487.437	6400	1482.912	6.0"	5.984"
37	19.671	245	89.575	840	252.819	1825	493.307	6500	1503.408	7.0"	6.928"
38	20.08	250	91.084	850	255.388	1850	499.167	6600	1523.872	8.0"	7.865"
39	20.488	255	92.589	860	257.953	1875	505.019	6700	1544.305	9.0”	8.797"

(continued)

Table 4.3 (continued)

t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
40	20.894	260	94.09	870	260.515	1900	510.861	6800	1564.706	$1.0(10)^9$	9.725"
41	21.298	265	95.588	875	261.795	192.5	516.695	6900	1585.077	1.5"	$1.429(10)^8$
42	21.701	270	97.081	880	261.073	1950	522.52	7000	1605.418	2.0"	1.880"
43	22.101	275	98.571	890	265.629	1975	528.337	7100	1625.729	2.5"	2.328"
44	22.5	280	100.057	900	268.181	2000	534.145	7200	1646.011	3.0"	2.771"
45	22.897	285	101.54	910	270.729	2025	539.945	7300	1666.265	4.0"	3.645"
46	23.291	290	101.019	920	273.274	2050	545.737	7400	1686.49	5.0"	4.510"
47	23.684	295	104.495	925	274.545	2075	551.522	7500	1706.688	6.0"	5.368"
48	24.076	300	105.968	930	275.815	2100	557.299	7600	1726.859	7.0"	6.220"
49	24.466	305	107.437	940	278.353	2125	561.068	7700	1747.002	8.0"	7.066"
50	24.855	310	108.904	950	280.888	2150	568.83	7800	1767.12	9.0"	7.909"
51	25.244	315	110.367	960	283.42	2175	574.585	7900	1787.212	1.0(10)10	8.747"
52	25.633	320	111.827	970	285.948	2200	580.332	8000	1807.278	1.5"	1.288"(10)9
53	26.02	325	113.284	975	287.211	2225	586.072	8100	1827.319	2.0"	1.697"
54	26.406	330	114.738	980	288.473	2250	591.806	8200	1847.336	2.5"	2.103"
55	26.791	335	116.189	990	290.995	2275	597.532	8300	1867.329	3.0"	2.505"
56	27.174	340	117.638	1000	293.514	2300	601.252	8400	1887.298	4.0"	3.299"
57	27.555	345	119.083	1010	296.030	2325	608.965	8500	1907.243	5.0"	4.087"
58	27.935	350	120.526	1020	298.543	2350	614.672	8600	1927.166	6.0"	4.868"
59	28.314	355	121.966	1025	299.799	2375	620.372	8700	1947.065	7.0"	5.643"
60	28.691	360	123.403	1030	301.053	2400	626.066	8800	1966.942	8.0"	6.414"
61	29.068	365	124.838	1040	303.560	2425	631.755	8900	1986.796	9.0"	7.183"
62	294.43	370	126.720	1050	306.065	2450	637.437	9000	2006.628	1.0(10)11	7.948'
63	29.818	375	127.699	1060	308.567	2475	643.113	9100	2026.438	1.5"	$1.17(10)^{10}$
64	30.192	380	129.126	1070	311.066	2500	648.781	9200	2046.227	2.0"	1.55"

65	30.565	385	130.550	1075	312.314	2550	660.093	9300	2065.996	2.5"	1.92"
66	30.937	390	131.972	1080	313.562	2600	671.379	9400	2085.744	3.0"	2.92"
67	31.308	395	133.391	1090	316.055	2650	682.640	9500	2105.473	4.0"	3.02"
68	31.679	400	134.808	1100	318.545	2700	693.877	9600	2125.184	5.0"	3.75'
69	32.048	405	136.223	1110	321.032	2750	705.090	9700	2144.878	6.0"	4.47"
70	32.417	410	137.635	1120	323.517	2800	716.280	9800	2164.555	7.0"	5.19"
71	32.785	415	139.045	1125	324.76	2850	727.449	9900	2184.216	8.0"	5.89"
72	33.151	420	140.453	1130	326.000	2900	738.598	10	2203.861	9.0"	6.58"
73	33.517	425	141.859	1140	328.480	2950	749.725	12.5	2688.967	$1.0(10)^{12}$	7.28"
74	33.883	430	143.262	1150	330.958	3000	760.833	15	3164.780	1.5"	$1.08(10)^{11}$
75	34.247	435	144.664	1160	333.433	3050	771.922	17.5	3633.368	2.0"	1.42"
76	34.611	440	146.064	1170	335.906	3100	782.992	20	4095.800		
77	34.974	445	147.461	1175	337.142	3150	794.042	25	5005.726		
78	35.336	450	148.856	1180	338.376	3200	805.075	30	5899.508		

Table 4.4 Dimensionless Water Influx W_{eD} for Several Values of r_e/r_R, i.e. r_a/r_e (Van Everdingen and Hurst W_{eD})

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD}=3.0$		$r_{eD} = 3.5$	
t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
$5.0(10)^{-2}$	0.276	$5.0(10)^{-2}$	0.278	$1.0(10)^{-1}$	0.408	$3.0(10)^{-1}$	0.755	1.00	1.571
6.0"	0.304	7.5"	0.345	1.5"	0.509	4.0"	0.895	1.20	1.761
7.0'	0.330	$1.0(10)^{-1}$	0.404	2.0"	0.599	5.0"	1.230	1.40	1.940
8.0'	0.354	1.25"	0.458	2.5"	0.681	6.0"	1.143	1.60	2.111
9.0"	0.375	1.50"	0.507	3.0"	0.758	7.0"	1.256	1.80	2.273
$(1.1X10)^{-1}$	0.395	1.75"	0.553	3.5"	0.829	8,0'	1.363	2.00	2.427
1.1'	0.414	2.00"	0.597	4.0"	0.897	9.0"	1.465	2.20	2.574
1.2"	0.431	2.25"	0.638	4.5"	0.962	1.00	1.563	2.40	2.715
1.3"	0.446	2.50"	0.678	5.0"	1.024	1.25	1.791	2.60	2.849
14'	0.461	2.75"	0.715	5.5"	1.083	1.50	1.997	2.80	2.976
1.5'	0.474	3.00"	0.751	6.0"	1.140	1.75	2.184	3.00	3.098
1.6"	0.486	3.25"	0.785	6.5"	1.195	2.00	2.353	3.25	3.242
1.3″	0.497	3.50"	0.817	7.0"	1.248	2.25	2.507	3.50	3.379
1.8"	0.507	3.75'	0.848	7.5"	1.299	2.50	2.646	3.75	3.507
1.9"	0.517	4.00"	0.877	8.0"	1.348	2.75	2.772	4.00	3.628
2.0"	0.525	4.25"	0.905	8.5"	1.395	3.00	2.886	4.25	3.742
2.1"	0.533	4.50"	0.932	9.0"	1.440	3.25	2.990	4.50	3.850
2.2"	0.541	4.75"	0.958	9.5"	1.484	3.50	3.084	4.75	3.951
2.3"	0.548	5 00"	0.993	1.00	1.526	3.75	3.170	5.00	4.047
2.4"	0.554	5.50"	1.028	1.10	1.605	4.00	3.247	5.50	4.222
2.5"	0.559	6.00"	1.070	1.20	1.679	4.25	3.317	6.00	4.378
2.6"	0.565	6.50"	1.108	1.30	1.747	4.50	3.381	6.50	4.516
2.8"	0.574	7.00"	1.143	1.40	1.811	4.75	3.439	7.00	4.639
3.0"	0.582	7.50"	1.174	1.50	1.870	5.00	3.491	7.50	4.749
3.2"	0.588	8.00"	1.203	1.60	1.924	5.50	3.581	8.00	4.846
3.4"	0.594	9.00"	1.253	1.70	1.975	6.00	3.656	8.50	4.932
3.6"	0.599	1.00"	1.295	1.80	2.022	6.50	3.717	9.00	5.009
3.8"	0.603	1.10	1.330	2.00	2.106	7.00	3.767	9.50	5.078
4.0"	0.606	1.20	1.358	2.20	2.178	7.50	3.809	10.00	5.138
4.5"	0.613	1.30	1.382	2.40	2.241	8.00	3.843	11.00	5.241
5.0"	0.617	1.40	1.402	2.60	2.294	9.00	3.894	12.00	5.321
6.0"	0.621	1.60	1.432	2.80	2.340	10.00	3.928	13.00	5.385
7.0"	0.623	1.70	1.444	3.00	2.380	11.00	3.951	14.00	5.435
8.0"	0.624	1.80	1.453	3.40	2.444	12.00	3.967	15.00	5.476
		2.00	1.468	3.80	2.491	14.00	3.985	16.00	5.506
		2.50	1.487	4.20	2.575	16.00	3.993	17.00	5.531
		3.00	1.495	4.60	2.551	18.00	3.997	18.00	5.551
		4.00	1.499	5.00	2.570	20.00	3.999	20.00	5.579
		5.00	1.500	6.00	2.599	22.00	3.999	25.00	5.611

(continued)

Table 4.4 (continued)

$r_{eD} = 1.5$		$r_{eD} = 2.0$		$r_{eD} = 2.5$		$r_{eD}=3.0$		$r_{eD} = 3.5$	
t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
				7.00	2.613	24.00	4.000	30.00	5.621
				8.00	2.619			35.00	5.624
				9.00	2.622			40.00	5.625
				10.00	2.624				

$r_{eD} = 4.0$		$r_{eD} = 4.5$		$r_{eD} = 5.0$		$r_{eD} = 6$	
t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
2.00	2.442	2.50	2.835	3	3.2	6	5.148
2.20	2.598	3.00	3.196	3.5	3.54	6.5	5.44
2.40	2.748	3.50	3.537	4	3.88	7	5.724
2.60	2.893	4.00	3.859	4.5	4.19	7.5	6.002
2.80	3.034	4.50	4.165	5	4.5	8	6.273
3.00	3.170	5.00	4.454	5.5	4.79	8.5	6.537
3.25	3.334	5.50	4.727	6	5.07	9	6.795
3.50	3.493	6.00	4.986	6.5	5.35	9.5	7.047
3.75	3.645	6.50	5.231	7	5.61	10	7.293
4.00	3.792	7.00	5.164	7.5	5.85	10.5	7.533
4.25	3.932	7.50	5.684	8	6.09	11	7.767
4.50	4.068	8.00	5.892	8.5	6.33	12	8.22
4.75	4.198	8.50	6.089	9	6.55	13	8.651
5.00	4.323	9.00	6.276	9.5	6.76	14	9.063
5.50	4.560	9.50	6.453	10	6.97	15	9.456
6.00	4.779	10.00	6.621	11	7.35	16	9.829
6.50	4.982	11.00	6.930	12	7.71	17	10.19
7.00	5.169	12.00	7.208	13	8.04	18	10.5
7.50	5.343	13.00	7.457	14	8.34	19	10.9
8.00	5.504	14.00	7.680	15	8.62	20	11.2
8.50	5.653	15.00	7.880	16	8.88	22	11.7
9.00	5.790	16.00	8.060	18	9.34	24	12.3
9.50	5.917	18.00	8.365	20	9.73	25	12.5
10.00	6.035	20.00	8.611	22	10.1	31	13.7
11.00	6.246	22.00	8.809	24	10.4	35	14.4
12.00	6.425	24.00	8.968	26	10.6	39	14.9
13.00	6.580	26.00	9.097	28	10.8	51	16.1
14.00	6.712	28.00	9.200	30	11	60	16.6
15.00	6.825	30.00	9.283	34	11.3	70	16.9
16.00	6.922	34.00	9.404	38	11.5	80	17.1
17.00	7.004	38.00	9.481	42	11.6	90	17.3
18.00	7.076	42.00	9.532	46	11.7	100	17.4
20.00	7.189	46.00	9.565	50	11.8	110	17.4
22.00	7.272	50.00	9.586	60	11.9	120	17.5
24.00	7.332	60.00	9.612	70	12	130	17.5

(continued)

Table 4.4 (continued)

$r_{eD} = 4.0$		$r_{eD} = 4.5$		$r_{eD} = 5.0$		$r_{eD} = 6$	
t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
26.00	7.377	70.00	9.621	80	12	140	17.5
30.00	7.434	80.00	9.623	90	12	150	17.5
34.00	7.464	90.00	9.624	100	12	160	17.5
38.00	7.481	100.00	9.625	120	12	180	17.5
42.00	7.490					200	17.5
46.00	7.494					220	17.5
50.00	7.499						

$r_{eD} = 7$		$r_{eD} = 8$		$r_{eD} = 9$		$r_{eD} = 10$	
t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
9	6.861	9	6.86	10	7.417	15	9.965
9.5	7.127	10	7.4	15	9.945	20	12.32
10	7.389	11	7.92	20	12.26	22	13.22
11	7.902	12	8.43	22	13.13	24	14.95
12	8.397	13	8.93	24	13.98	26	14.95
13	8.876	14	9.42	26	14.79	28	15.78
14	9.341	15	9.9	26	15.59	30	16.59
15	9.791	16	10.4	30	16.35	32	17.38
16	10.23	17	10.8	32	17.1	34	18.16
17	10.65	18	11.3	34	17.82	36	18.91
18	11.06	19	11.7	36	18.52	38	19.65
19	11.46	20	12.1	38	19.19	40	20.37
20	11.85	22	13	40	19.85	42	21.07
22	12.58	24	13.7	42	20.48	44	21.76
24	13.27	26	14.5	44	21.09	46	22.42
26	13.92	28	15.2	46	21.69	48	23.07
28	14.53	30	15.9	48	22.26	50	23.71
30	15.11	34	17	50	22.8	52	24.33
35	16.39	38	18	52	23.4	54	24.94
40	17.49	40	19	54	23.9	56	25.53
45	18.43	45	20	56	24.4	58	26.11
50	19.24	50	21	58	24.9	60	26.67
60	20.51	55	22	60	25.4	65	28.02
70	21.45	60	23	65	26.5	70	29.29
80	22.13	70	25	70	27.5	75	30.49
90	22.63	80	26	75	28.5	80	31.61
100	21	90	27	80	29.4	85	32.67
120	23.47	100	28	85	30.2	90	33.66
140	23.71	120	29	90	30.9	95	34.6
160	23.85	140	30	95	31.6	100	35.48
180	23.92	160	31	100	32.3	120	38.51
200	23.96	180	31	120	34.4	140	40.89

(continued)

Table 4.4 (continued)

$r_{eD} = 7$		$r_{eD} = 8$		$r_{eD} = 9$		$r_{eD} = 10$	
t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}	t_D	W_{eD}
500	24	200	31	140	35.9	160	42.75
		240	31	160	37	180	44.21
		280	31	180	37.9	200	45.36
		320	31	200	38.4	240	46.95
		360	31	240	39.2	280	47.94
		400	32	280	39.6	320	48.54
		500	32	320	39.8	360	48.91
				360	39.9	400	49.14
				400	39.9	440	49.28
				440	40	480	49.36
				480	40		

Table 4.5 Van Everdingen & Hurst aquifer model calculation

Time (yr)	Pressure (psi)	ΔP (psi)	t_D	W_{eD}	W_e
0	p_i		–	–	0
1	p_1	$\Delta p_0 = \frac{p_i - p_1}{2}$	t_{D1}	W_{eD1}	W_{e1}
2	p_2	$\Delta p_1 = \frac{p_i - p_2}{2}$	t_{D2}	W_{eD2}	W_{e2}
3	p_3	$\Delta p_2 = \frac{p_1 - p_3}{2}$	t_{D3}	W_{eD3}	W_{e3}
4	p_4	$\Delta p_3 = \frac{p_2 - p_4}{2}$	t_{D4}	W_{eD4}	W_{e4}
5	p_5	$\Delta p_4 = \frac{p_3 - p_5}{2}$	t_{D5}	W_{eD5}	W_{e5}
6	p_6	$\Delta p_5 = \frac{p_4 - p_6}{2}$	t_{D6}	W_{eD6}	W_{e6}

$$W_{eD}(t_D) = \frac{1.2838\sqrt{t_D} + 1.19328 t_D + 0.269872 {t_D}^{1.5} + 0.00855294 {t_D}^{2}}{1 + 0.616599\sqrt{t_D} + 0.0413008 t_D} \quad 0.01 < t_D < 200, \ \pm 0.02\%$$

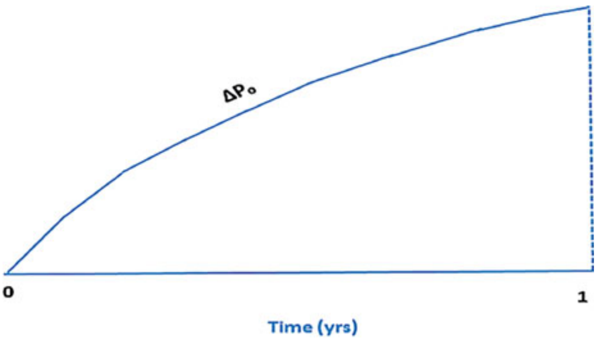
$$W_{eD}(t_D) = \frac{-4.29881 + 2.02566 t_D}{\ln t_D}, t_D \geq 200, \ \pm 0.07\%$$

Explanation of how the water influx is calculated in the above table

At time, t = 0, there is no water influx from the aquifer into the reservoir

At the end of the first year (t = 1)

The water influx is due to the expansion of the first pressure drop, Δp_0 as shown in the figure below. Based on time step given in the table, the first pressure drop is only active for just 1 year of production.



$$W_e(t_1) = C\Delta p_0 W_{eD}(T_D)$$

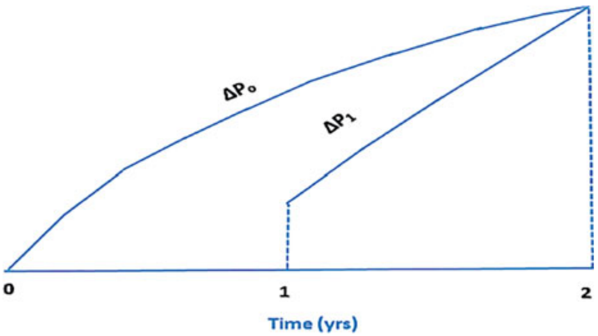
$$W_e(t_1) = C\Delta p_0 W_{e(\Delta p_0)}$$

At the end of the second year (t = 2)

The total water influx is the summation of the water influx due to the first and second pressure (Δp_0 & Δp_1) respectively. Therefore, the cumulative water influx at this period is given as:

$$W_e(t_2) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)}$$

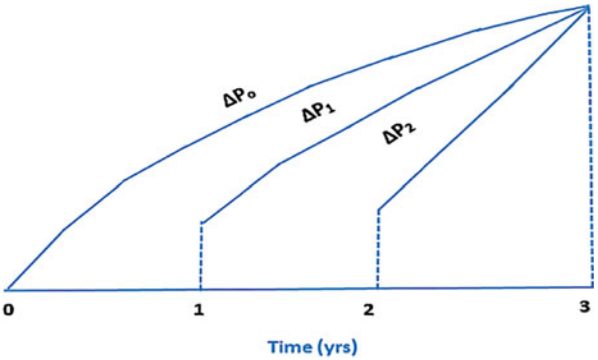
In calculating this, we have to note that the first pressure (Δp_0) is active for 2 years, which implies that t = 2 will be used to calculate the dimensionless time (t_D) which is then used to determine (W_{eD}). The second pressure drop (Δp_1) is only active for a year (t = 1). These are given as:



$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_2}$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_1}$$

At the end of the third year (t = 3)



Δp_0 is active for 3 years
Δp_1 is active for 2 years
Δp_2 is active for 1 year

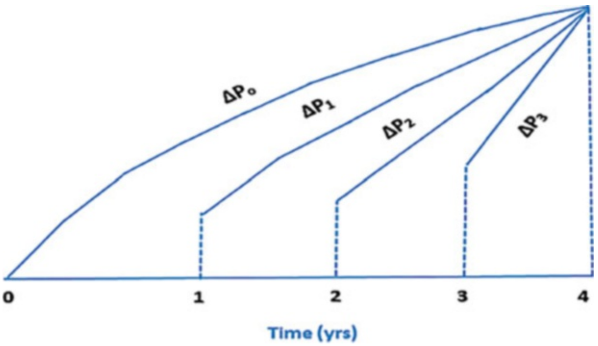
$$W_e(t_3) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_3}$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_2}$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_1}$$

At the end of the fourth year (t = 4)



Δp_0 is active for 4 years
Δp_1 is active for 3 years
Δp_2 is active for 2 years
Δp_3 is active for 1 year

$$W_e(t_4) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)} + W_{e(\Delta p_3)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_4}$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_3}$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_2}$$

$$W_{e(\Delta p_3)} = C\Delta p_3 W_{eD@t_1}$$

Note that as the time step increases, the water influx is updated accordingly with the corresponding pressure.

Example 4.3

A reservoir-aquifer system substend an angle of 140° at the centre and the aquifer radius is estimated to be 112,000 ft. using the reservoir pressure and dimensionless water influx values in Table 4.6, find the water influx at each time step, using Hurst van Everdingen and compare result with fetkovich models.

Additional Data:

Aquifer permeability, K_w	*200 mD*
Water viscosity, μ_w	*0.55 cp*
Reservoir thickness, h	*100 ft*
Drainage radius, r_e	*14,000 ft*
Total compressibility, C_t	*$2.50 * 10^{-6}$ psi^{-1}*
Dimensionless radius, r_{eD}	*8*
Porosity, ∅	*19%*

Solution

$$f = \frac{\theta}{360} = \frac{140}{360} = 0.3889$$

$$C = 1.119 f \varnothing h c_{tw} r_e^{\,2}\ (bbl/psi)$$

$$C = 1.119*0.3889*0.19*100*2.50*10^{-6}*(14000)^2 = 4051.52\ (bbl/psi)$$

$$\Delta p_0 = \frac{p_i - p_1}{2} = \frac{4014 - 3941}{2} = 36.5$$

$$\Delta p_1 = \frac{p_i - p_2}{2} = \frac{4014 - 3870}{2} = 72$$

Table 4.6 Pressure decline and dimensionless water influx data of Example 4.3

Time (yrs)	0	1	2	3	4	5
Pressure (psia)	4014	3941	3870	3800	3732	3664
$W_{D(tD)}$	0	6.8712	11.2763	14.8854	17.8373	20.2521

$$\Delta p_2 = \frac{p_1 - p_3}{2} = \frac{3941 - 3800}{2} = 70.5$$

$$\Delta p_3 = \frac{p_2 - p_4}{2} = \frac{3870 - 3732}{2} = 69$$

$$\Delta p_4 = \frac{p_3 - p_5}{2} = \frac{3800 - 3664}{2} = 68$$

There is no need to calculate the dimensionless time and water influx since they are already provided in the table above.

The water influx at each year is calculated as:

At the end of the first year (t = 1)

$$W_e(t_1) = C\Delta p_0 W_{eD}(T_D)$$

$$W_e(t_1) = 4051.52*36.5*6.8712 = 1016116.35\ bbl$$

At the end of the second year (t = 2)

$$W_e(t_2) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_2} = 4051.52*36.5*11.2763 = 1667544.66\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_1} = 4051.52*72*6.8712 = 2004393.90\ bbl$$

$$\therefore W_e(t_2) = 1667544.66 + 2004393.90 = 3671938.56\ bbl$$

At the end of the third year (t = 3)

$$W_e(t_3) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_3} = 4051.52*36.5*14.8854 = 2201260.097\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_2} = 4051.52*72*11.2763 = 3289403.158\ bbl$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_1} = 4051.52*70.5*6.8712 = 1962635.698\ bbl$$

$$\therefore W_e(t_3) = 2201260.097 + 3289403.158 + 1962635.698 = 7453298.95\ bbl$$

At the end of the fourth year (t = 4)

$$W_e(t_4) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)} + W_{e(\Delta p_3)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_4} = 4051.52*36.51*17.8373 = 2638511.168\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_3} = 4051.52*72*14.8854 = 4342211.698\ bbl$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_2} = 4051.52*70.5*11.2763 = 3220873.926\ bbl$$

$$W_{e(\Delta p_3)} = C\Delta p_3 W_{eD@t_1} = 4051.52*69*6.8712 = 1920877.491\ bbl$$

$$\begin{aligned}\therefore W_e(t_4) &= 2638511.168 + 4342211.698 + 3220873.926 + 1920877.491\\ &= 12122474.28\ bbl\end{aligned}$$

At the end of the fourth year (t = 5)

$$W_e(t_5) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)} + W_{e(\Delta p_3)} + W_{e(\Delta p_4)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_5} = 4051.52*36.5*20.2521 = 2994890.269\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_4} = 4051.52*72*17.8373 = 5203308.794\ bbl$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_3} = 4051.52*70.5*14.8854 = 4251748.954\ bbl$$

$$W_{e(\Delta p_3)} = C\Delta p_3 W_{eD@t_2} = 4051.52*69*11.2763 = 3152344.693\ bbl$$

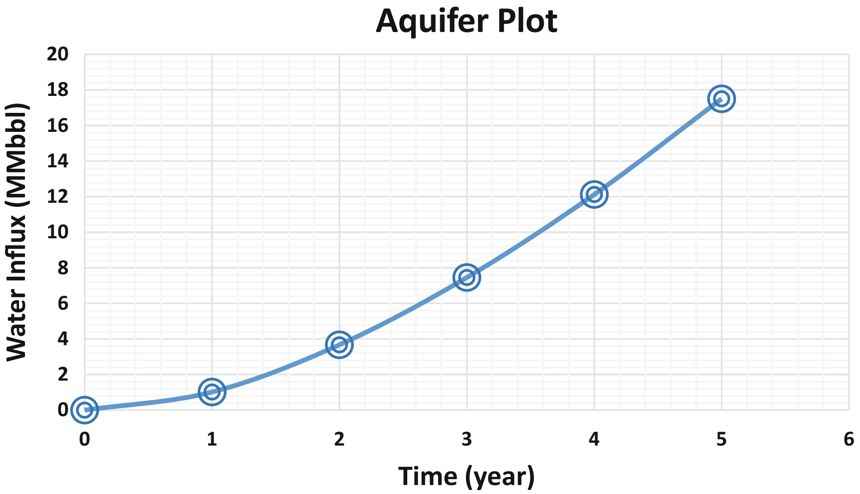
$$W_{e(\Delta p_4)} = C\Delta p_4 W_{eD@t_1} = 4051.52*68*6.8712 = 1893038.687\ bbl$$

$$\begin{aligned}W_e(t_5) &= 2994890.269 + 5203308.794 + 4251748.954 + 3152344.693\\ &\quad + 1893038.687\\ &= 17495331.4\ bbl\end{aligned}$$

The result is tabulated below

Time (yrs)	Pressure (psia)	ΔP (psia)	W_{eD}	W_e (bbl)
0	4014		0	
1	3941	36.5	6.8712	1016116.35
2	3870	72	11.2763	3671938.56
3	3800	70.5	14.8854	7453298.95
4	3732	69	17.8373	12122474.28
5	3664	68	20.2521	17495331.4

The cumulative water influx in plot below



Example 4.4
An active water drive reservoir has a cumulative water production of 28,640 bbl was estimated after the first year of production. The pressure profile (Tables 4.7), reservoir and aquifer properties are given as:

Calculate:

- Calculate the thickness of the aquifer sand
- Cumulative water influx for the next four years

Solution

$$f = \frac{\theta}{360} = \frac{120}{360} = 0.3333$$

$$C = 1.119 f \varnothing h c_{tw} {r_e}^2 \ (bbl/psi)$$

$$C = 1.119*0.3333*0.22*h*3.80*10^{-6}*(9500)^2 = 28.1397h$$

$$C = 28.1397*h \ (bbl/psi)$$

Calculation of the pressure drop

$$\Delta p_0 = \frac{p_i - p_1}{2} = \frac{5027 - 5016}{2} = 5.5$$

$$\Delta p_1 = \frac{p_i - p_2}{2} = \frac{5027 - 5001}{2} = 13$$

Table 4.7 Pressure decline data of Example 4.4

Time (yrs)	0	1	2	3	4	5
Pressure (psia)	5027	5016	5414	4966	4818	4775

Aquifer permeability, K_w	*160 mD*
Water viscosity, μ_w	*0.78 cp*
Drainage radius, r_e	*9500 ft*
Aquifer radius, r_a	*47,500 ft*
Total compressibility, C_t	*$3.80 * 10^{-6}$ psi^{-1}*
Porosity, ∅	*22%*
Aquifer angle	*120°*

$$\Delta p_2 = \frac{p_1 - p_3}{2} = \frac{5016 - 4966}{2} = 25$$

$$\Delta p_3 = \frac{p_2 - p_4}{2} = \frac{5001 - 4818}{2} = 91.5$$

$$\Delta p_4 = \frac{p_3 - p_5}{2} = \frac{4966 - 4775}{2} = 95.5$$

Calculation of dimensionless time

$$t_D = \frac{2.309kt}{\mu_w \varnothing_w c_{tw} {r_e}^2} \quad (t \ in \ years)$$

$$t_D = \frac{2.309 * 160 * t}{0.78 * 0.22 * 3.8 * 10^{-6} * 9500^2} = 6.2776 * t$$

The dimensionless time is calculated thus as:

$$@t = 1, t_D = 6.2776 * 1 = 6.2776$$

$$@t = 2, t_D = 6.2776 * 2 = 12.5552$$

$$@t = 3, t_D = 6.2776 * 3 = 18.8328$$

$$@t = 4, t_D = 6.2776 * 4 = 25.1104$$

$$@t = 5, t_D = 6.2776 * 5 = 31.388$$

Calculation of dimensionless aquifer

From Table 4.1, based on the calculated dimensionless time, we have to interpolate to get the dimensionless aquifer corresponding to the calculated dimensionless time as shown below.

tD	WeD
6	5.153
6.2776	**5.3168**
7	5.743

tD	WeD
12	8.457
12.5553	**8.7385**
13	8.964

tD	WeD
18	11.386
18.8329	**11.7766**
19	11.855

tD	WeD
25	14.573
25.1105	**14.6216**
26	15.013

tD	WeD
31	17.167
31.3882	**17.3312**
32	17.59

The water influx at each year is calculated as:
At the end of the first year (t = 1)

$$W_e(t_1) = C\Delta p_0 W_{eD}(T_D) = 28640\ bbl$$

$$28640 = 28.1397 * h * 5.5 * 5.3168$$

$$h = \frac{28640}{28.1397 * 5.5 * 5.3168} = 34.8049\ ft$$

Therefore,

$$C = 28.1397 * 34.8049 = 979.3994\ (bbl/psi)$$

At the end of the second year (t = 2)

$$W_e(t_2) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_2} = 979.3981*5.5*8.7385 = 47071.65\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_1} = 979.3981*13*5.3168 = 67694.52\ bbl$$

$$\therefore W_e(t_2) = 47071.65 + 67694.52 = 114766.17\ bbl$$

At the end of the third year (t = 3)

$$W_e(t_3) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_3} = 979.3981*5.5*11.7766 = 63436.8815\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_2} = 979.3981*13*8.7385 = 111260.1139\ bbl$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_1} = 979.3981*25*5.3168 = 130181.5955\ bbl$$

$$\therefore W_e(t_3) = 63436.8815 + 111260.1139 + 130181.5955 = 304878.59\ bbl$$

At the end of the fourth year (t = 4)

$$W_e(t_4) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)} + W_{e(\Delta p_3)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_4} = 979.3981*5.5*14.6216 = 78762.02\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_3} = 979.3981*13*11.7766 = 149941.74\ bbl$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_2} = 979.3981*25*8.7385 = 213961.76\ bbl$$

$$W_{e(\Delta p_3)} = C\Delta p_3 W_{eD@t_1} = 979.3981*91.5*5.3168 = 476464.64\ bbl$$

$$\therefore W_e(t_4) = 78762.02 + 149941.74 + 213961.76 + 476464.64 = 919130.152\ bbl$$

At the end of the fourth year (t = 5)

$$W_e(t_5) = W_{e(\Delta p_0)} + W_{e(\Delta p_1)} + W_{e(\Delta p_2)} + W_{e(\Delta p_3)} + W_{e(\Delta p_4)}$$

$$W_{e(\Delta p_0)} = C\Delta p_0 W_{eD@t_5} = 979.3981*5.5*17.3312 = 93357.794\ bbl$$

$$W_{e(\Delta p_1)} = C\Delta p_1 W_{eD@t_4} = 979.3981*13*14.6216 = 186164.77 bbl$$

$$W_{e(\Delta p_2)} = C\Delta p_2 W_{eD@t_3} = 979.3981*25*11.7766 = 288349.49\ bbl$$

$$W_{e(\Delta p_3)} = C\Delta p_3 W_{eD@t_2} = 979.3981*91.5*8.7385 = 783100.03\ bbl$$

$$W_{e(\Delta p_4)} = C\Delta p_4 W_{eD@t_1} = 979.3981*95.5*5.3168 = 497293.69\ bbl$$

$$W_e(t_5) = 93357.794 + 186164.77 + 288349.49 + 783100.03 + 497293.69 = 1848265.787\ bbl$$

The result is tabulated below

Time (yrs)	Pressure (psia)	ΔP (psia)	t_D	W_{eD}	W_e (bbl)
0	5027				0
1	5016	5.5	6.2776	5.3168	28,640
2	5001	13	12.5553	8.7385	114766.016
3	4966	25	18.8329	11.7766	304878.575
4	4818	91.5	25.1105	14.6216	919130.152
5	4775	95.5	31.3882	17.3312	1848265.787

4.2.5 Carter-Tracy Model

This method is an approximate solution to the diffusivity equation. It can be combined conveniently with a suitable material balance equation to predict the performance of water-drive reservoirs. The Carter-Tracy aquifer models can be applied to both finite and infinite-acting aquifers. It can be applied to both radial and linear aquifers and also applies to edge-water drive reservoirs only. Mathematically, it is calculated as

$$\boldsymbol{W_e(t_{Dj}) = W_e(t_{Dj-1}) + \left[\frac{C\Delta P(t_{Dj}) - W_e(t_{Dj-1})P_D'(t_{Dj})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right](t_{Dj} - t_{Dj-1})}$$

Where C = the van Everdingen-Hurst water influx constant as defined above, t_D = the dimensionless time, j = refers to the current time step, j − 1 = refers to the previous time step, ΔP_j = total pressure drop, $P_i - P_j$, psi, P_D = dimensionless pressure and P'_D = dimensionless pressure derivative.

The dimensionless pressure and first pressure derivative are given below

$$t_{Dj} = \frac{0.000264kt}{\mu_w \varnothing_w c_{tw} r_e^2}\ (t\ in\ hours) = \frac{0.006336kt}{\mu_w \varnothing_w c_{tw} r_e^2}\ (t\ in\ days)$$

$$= \frac{0.192432kt}{\mu_w \varnothing_w c_{tw} r_e^2}\ (t\ in\ months) = \frac{2.309kt}{\mu_w \varnothing_w c_{tw} r_e^2}\ (t\ in\ years)$$

$$\Delta P_n = P_i - P_n$$

$$P_D(t_D) = 2\sqrt{\frac{t_D}{\pi}}\; t_D \le 0.01$$

$$P_D(t_D) = \frac{370.529\sqrt{t_D} + 137.582t_D + 5.69549{t_D}^{1.5}}{328.834 + 265.488\sqrt{t_D} + 45.2157t_D + {t_D}^{1.5}}\; 0.01 < t_D < 500$$

$$P_D(t_D) = \left[½ \ln t_D + 040454\right]\left[1 + \frac{1}{2t_D} + \frac{1}{4t_D}\right]\; t_D \ge 500$$

$${P_D}'(t_D) = \frac{1}{\sqrt{\pi t_D}}\; t_D \le 0.01$$

$${P_D}'(t_D) = \frac{716.441 + 46.7984\sqrt{t_D} + 270.038t_D + 71.0098{t_D}^{1.5}}{1269.86\sqrt{t_D} + 1204.73t_D + 618.618{t_D}^{1.5} + 538.072{t_D}^2 + 142.41{t_D}^{2.5}}$$

$$0.01 < t_D < 500$$

$${P_D}'(t_D) = \left[½ \ln t_D + 040454\right]\left[\frac{1}{2t_D} - \frac{1}{2{t_D}^2}\right]\; t_D \ge 500$$

4.2.5.1 Steps in Calculating Carter-Tracy's Aquifer Model

Step 1: Calculate the total pressure drop at each time step
Step 2: Calculate the dimensionless time at each time step
Step 3: Calculate the dimensionless pressure and pressure derivative at each time step
Step 4: Calculate the water influx at each time step (Table 4.8)

Table 4.8 Carter-Tracy aquifer model calculation

Time (yr)	Pressure (psi)	t_{Dj}	$\Delta P(t_{Dj})$ (psi)	$P_D(t_{Dj})$	$P'_D(t_{Dj})$	$W_e(t_{Dj})$
0	P_i	-	-	-	-	0
1	P_1	t_{D1}	$\Delta P(t_{D1}) = P_i - P_1$	$P_D(t_{D1})$	$P'_D(t_{D1})$	$W_e(t_{D1})$
2	P_2	t_{D2}	$\Delta P(t_{D2}) = P_i - P_2$	$P_D(t_{D2})$	$P'_D(t_{D2})$	$W_e(t_{D2})$
3	P_3	t_{D3}	$\Delta P(t_{D3}) = P_i - P_3$	$P_D(t_{D3})$	$P'_D(t_{D3})$	$W_e(t_{D3})$
4	P_4	t_{D4}	$\Delta P(t_{D4}) = P_i - P_4$	$P_D(t_{D4})$	$P'_D(t_{D4})$	$W_e(t_{D4})$
5	P_5	t_{D5}	$\Delta P(t_{D5}) = P_i - P_5$	$P_D(t_{D5})$	$P'_D(t_{D5})$	$W_e(t_{D5})$
6	P_6	t_{D6}	$\Delta P(t_{D6}) = P_i - P_6$	$P_D(t_{D6})$	$P'_D(t_{D6})$	$W_e(t_{D6})$

Example 4.5
Repeat Example 4.3 using the Carter Tracy's aquifer model to calculate the cumulative water influx at each time step.

Solution

Step 1: Calculate the total pressure drop at each time step by applying

$$\Delta P(t_{Di+1}) = P_i - P_{i+1}$$

Time (t_i) (yrs)	P_i (psia)	ΔP(t_i) (psia)
0	4014	0
1	3941	73
2	3870	144
3	3800	214
4	3732	282
5	3664	350

Step 2: Calculate the dimensionless time at each time step using

$$t_D = \frac{2.309kt}{\mu_w \varnothing_w c_{tw} {r_e}^2} \ (t \ in \ years)$$

$$= \frac{2.309 * 200 * t}{0.55 * 0.19 * 2.5 \times 10^{-6} * (14000)^2} = 9.0187 t_i$$

Time (t_i) (yrs)	P_i (psia)	ΔP(t_i) (psia)	t_D
0	4014	0	0
1	3941	73	9.0187
2	3870	144	18.0374
3	3800	214	27.0561
4	3732	282	36.0748
5	3664	350	45.0935

Step 3: Calculate the dimensionless pressure and pressure derivative at each time step.

Since the dimensionless time in the above table is greater than 0.01 and less than 500, these equations apply for dimensionless pressure and pressure derivative

$$P_D(t_D) = \frac{370.529\sqrt{t_D} + 137.582 t_D + 5.69549 {t_D}^{1.5}}{328.834 + 265.488\sqrt{t_D} + 45.2157 t_D + {t_D}^{1.5}} \ 0.01 < t_D < 500$$

$$P_D{}'(t_D) = \frac{716.441 + 46.7984\sqrt{t_D} + 270.038t_D + 71.0098{t_D}^{1.5}}{1269.86\sqrt{t_D} + 1204.73t_D + 618.618{t_D}^{1.5} + 538.072{t_D}^{2} + 142.41{t_D}^{2.5}}$$

$$0.01 < t_D < 500$$

Time (t_i) (yrs)	P_i (psia)	ΔP(t_i) (psia)	t_D	$P_D(t_D)$	$P_D{}'(t_D)$
0	4014	0	0	0	0
1	3941	73	9.0187	1.60654	0.04742
2	3870	144	18.0374	1.91249	0.02515
3	3800	214	27.0561	2.09875	0.01719
4	3732	282	36.0748	2.23343	0.01307
5	3664	350	45.0935	2.33910	0.01055

Step 4: Calculate the water influx at each time step using

$$W_e\left(t_{Dj}\right) = W_e\left(t_{Dj-1}\right) + \left[\frac{C\Delta P\left(t_{Dj}\right) - W_e\left(t_{Dj-1}\right)P_D{}'\left(t_{Dj}\right)}{P_D\left(t_{Dj}\right) - \left(t_{Dj-1}\right)P_D{}'\left(t_{Dj}\right)}\right]\left(t_{Dj} - t_{Dj-1}\right)$$

After year 1

$$W_e(t_{D1}) = W_e(t_{D0}) + \left[\frac{C\Delta P(t_{D1}) - W_e(t_{D0})P_D{}'(t_{D1})}{P_D(t_{D1}) - (t_{D0})P_D{}'(t_{D1})}\right](t_{D1} - t_{D0})$$

$$W_e(t_{D0}) = 0$$

$$C = 1.119*0.3889*0.19*100*2.50*10^{-6}*(14000)^2 = 4051.52\ (bbl/psi)$$

$$W_e(t_{D1}) = 0 + \left[\frac{(4051.52*73) - (0*0.04742)}{1.60654 - (0*0.04742)}\right](9.0187 - 0) = 1660325.526\ bbl$$

After year 2

$$W_e(t_{D2}) = W_e(t_{D1}) + \left[\frac{C\Delta P(t_{D2}) - W_e(t_{D1})P_D{}'(t_{D2})}{P_D(t_{D2}) - (t_{D1})P_D{}'(t_{D2})}\right](t_{D2} - t_{D1})$$

$$W_e(t_{D1}) = 1660325.525\ bbl$$

$$\begin{aligned} W_e(t_{D2}) &= 1660325.526 \\ &\quad + \left[\frac{(4051.52*144) - (1660325.526*0.02515)}{1.91249 - (9.0187*0.02515)}\right](18.0374 - 9.0187) \\ &= 4558333.626\ bbl \end{aligned}$$

After year 3

$$W_e(t_{D3}) = W_e(t_{D2}) + \left[\frac{C\Delta P(t_{D3}) - W_e(t_{D2})P_D{}'(t_{D3})}{P_D(t_{D3}) - (t_{D2})P_D{}'(t_{D3})}\right](t_{D3} - t_{D2})$$

$$W_e(t_{D2}) = 4558333.626\ bbl$$

$$\begin{aligned}W_e(t_{D3}) &= 4558333.626\\&\quad + \left[\frac{(4051.52*214) - (4558333.626*0.01719)}{2.09875 - (18.0374*0.01719)}\right](27.0561 - 18.0374)\\&= 8534856.871\ bbl\end{aligned}$$

After year 4

$$W_e(t_{D4}) = W_e(t_{D3}) + \left[\frac{C\Delta P(t_{D4}) - W_e(t_{D3})P_D{}'(t_{D4})}{P_D(t_{D4}) - (t_{D3})P_D{}'(t_{D4})}\right](t_{D4} - t_{D3})$$

$$W_e(t_{D3}) = 8534856.871\ bbl$$

$$\begin{aligned}W_e(t_{D4}) &= 8534856.871\\&\quad + \left[\frac{(4051.52*282) - (8534856.871*0.01307)}{2.23343 - (27.0561*0.01307)}\right](36.0748 - 27.0561)\\&= 13481153.46\ bbl\end{aligned}$$

After year 5

$$W_e(t_{D5}) = W_e(t_{D4}) + \left[\frac{C\Delta P(t_{D5}) - W_e(t_{D4})P_D{}'(t_{D5})}{P_D(t_{D5}) - (t_{D4})P_D{}'(t_{D5})}\right](t_{D5} - t_{D4})$$

$$W_e(t_{D4}) = 13481153.46\ bbl$$

$$\begin{aligned}W_e(t_{D5}) &= 13481153.46\\&\quad + \left[\frac{(4051.52*350) - (13481153.46*0.01055)}{2.33910 - (36.0748*0.01055)}\right](45.0935 - 36.0748)\\&= 19356081.34\ bbl\end{aligned}$$

Time (t_i) (yrs)	P_i (psia)	ΔP(t_i) (psia)	t_D	$P_D(t_D)$	$P_D^{'}(t_D)$	W_e
0	4014	0	0	0	0	0
1	3941	73	9.0187	1.60654	0.04742	1660325.526
2	3870	144	18.0374	1.91249	0.02515	4558333.626
3	3800	214	27.0561	2.09875	0.01719	8534856.871
4	3732	282	36.0748	2.23343	0.01307	13481153.46
5	3664	350	45.0935	2.33910	0.01055	19356081.34

Comparing the result obtained from Carter-Tracy's method with van Everdingen method as presented in Table 4.9:

Table 4.9 Comparative result of van Everdingen and Carter-Tracy models

Time (year)	Van Everdingen & Hurst Model (bbl)	Carter-Tracy's Model (bbl)
0	0	0
1	1016116.35	1660325.526
2	3671938.56	4558333.626
3	7453298.95	8534856.871
4	12122474.3	13481153.46
5	17495331.4	19356081.34

4.2.6 Fetkovich Aquifer Model

Fetkovich (1971) proposed a model to simplify water influx calculations further. This method uses a pseudo-steady-state aquifer productivity index (PI) and an aquifer material balance to represent the system compressibility. Like the Carter-Tracy method, Fetkovich's model eliminates the use of superposition and therefore, it is much simpler than van Everdingen-Hurst method. However, because Fetkovich neglects the early transient time period in these calculations, the calculated water influx will always be less than the values predicted by the previous two models.

The Fetkovich model applies to finite-acting aquifers; the model is applicable to both radial and linear aquifers. The Fetkovich aquifer model applies to edge-water and bottom-water drive reservoirs, while the Carter-Tracy aquifer model applies to edge-water drive reservoirs. In edge-water drive, water influx occurs around the flanks of the reservoir. In bottom-water drive, the reservoir is underlain by the aquifer which encroaches vertically into the reservoir. These are represented in the Fig. 4.4.

Fetkovich used an inflow equation similar to fluid flow from a reservoir to a well, to model the water influx to the reservoir. Assuming constant pressure at the original reservoir/aquifer boundary, the rate of water influx is derived as follow:

The inflow equation is given as:

$$q = j\Delta P$$

For water

$$q_w = j\left(\overline{P_a} - P\right) = \frac{dW_e}{dt}$$

Where

The pseudo steady-state productivity index is calculated as:

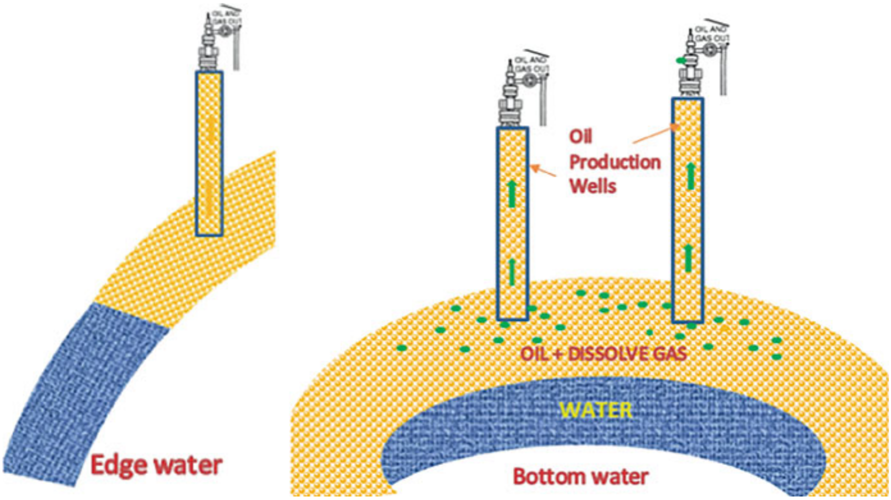


Fig. 4.4 Edge and bottom water drive

$$j = \frac{7.08 \times 10^{-3} fkh}{\mu_w \left[\ln\left(\frac{r_a}{r_e}\right) - 0.75\right]}$$

Where q_w = water influx rate, j = aquifer productivity index, P = Pressure at the reservoir fluid contact i.e. inner aquifer boundary pressure, $\overline{P_a}$ = average pressure in the aquifer & W_e = cumulative water influx.

The total aquifer influx due to the total pressure drop is:

$$W_e = c_t W_i \left(P_i - \overline{P_a}\right)$$

Expanding this equation, we have

$$W_e = c_t W_i P_i - c_t W_i \overline{P_a}$$

$$\overline{P_a} = \frac{c_t W_i P_i - W_e}{c_t W_i} = \frac{c_t W_i P_i}{c_t W_i} - \frac{W_e}{c_t W_i} = P_i - \frac{W_e}{c_t W_i} = P_i \left[1 - \frac{W_e}{c_t W_i P_i}\right]$$

Where

$$W_{ei} = c_t W_i P_i$$

Recall:

$$W_i = \frac{\pi({r_a}^2 - {r_e}^2)h\varnothing}{5.615}, f = \frac{\theta}{360}$$

Therefore, the initial volume of water in the aquifer is calculated as:

$$W_{ei} = \frac{\bar{c}f\pi({r_a}^2 - {r_e}^2)h\varnothing P_i}{5.615}$$

Hence,

$$\overline{P_a} = P_i\left[1 - \frac{W_e}{W_{ei}}\right]$$

By differentiation keeping all initial variables (P_i & W_{ei}) constant, we have

$$\frac{d\overline{P_a}}{dt} = P_i\left[0 - \frac{1}{W_{ei}}\frac{dW_e}{dt}\right] = -\frac{P_i}{W_{ei}}\frac{dW_e}{dt}$$

$$\frac{dW_e}{dt} = -\frac{W_{ei}}{P_i}\frac{d\overline{P_a}}{dt}$$

Substitute into the above equation

$$j\left(\overline{P_a} - P\right) = \frac{dW_e}{dt} = -\frac{W_{ei}}{P_i}\frac{d\overline{P_a}}{dt}$$

$$\frac{d\overline{P_a}}{\left(\overline{P_a} - P\right)} = \frac{-jP_i}{W_{ei}}dt$$

Integrating with respect to time, t

$$\ln\left(\overline{P_a} - P\right) = \frac{-jP_it}{W_{ei}} + C$$

The constant C can be evaluated at initial condition. That is

$$t = 0, W_e = 0, \overline{P_a} = P_i$$

$$\ln(P_i - P) = \frac{-jP_i(0)}{W_{ei}} + C$$

$$\therefore C = \ln(P_i - P)$$

Hence, the equation is:

$$\ln\left(\overline{P_a} - P\right) = \frac{-jP_it}{W_{ei}} + \ln\left(P_i - P\right)$$

$$\ln\left(\overline{P_a} - P\right) - \ln\left(P_i - P\right) = \frac{-jP_it}{W_{ei}}$$

$$\ln\left\{\frac{\left(\overline{P_a} - P\right)}{\left(P_i - P\right)}\right\} = \frac{-jP_it}{W_{ei}}$$

$$\frac{\left(\overline{P_a} - P\right)}{\left(P_i - P\right)} = e^{\frac{-jP_it}{W_{ei}}}$$

$$\left(\overline{P_a} - P\right) = \left(P_i - P\right)e^{\frac{-jP_it}{W_{ei}}}$$

Substituting into

$$\frac{dW_e}{dt} = j\left(\overline{P_a} - P\right) = j(P_i - P)e^{\frac{-jP_it}{W_{ei}}}$$

By integration with respect to time, it yields:

$$W_e = \frac{W_{ei}}{P_i}\left(P_i - P\right)\left(1 - e^{\frac{-jP_it}{W_{ei}}}\right)$$

Therefore, the general water influx equation for a time step is:

$$\Delta W_{e_n}{}^{k} = \frac{W_{ei}}{P_i}\left[\bar{P}_{a_{n-1}} - \bar{P}_n\right]\left(1 - e^{\frac{-jP_i\Delta t_n}{W_{ei}}}\right)$$

Where $\bar{P}_n$ is the average reservoir boundary pressure calculated as:

$$\bar{P}_n = \frac{P_{n-1} + P_n}{2}$$

Where $\bar{P}_{a_{n-1}}$ is the average aquifer pressure at the end of the previous time step calculated as:

$$\bar{P}_{a_{n-1}} = P_i\left(1 - \frac{\sum_{j=1}^{n-1} W_{e_j}}{W_{ei}}\right), \sum_{j=1}^{n-1} W_{e_j} = W_{e_1} + W_{e_2} + \ldots$$

The incremental water influx is calculated as:

Table 4.10 Fetkovich aquifer model calculation

Time (yr)	P (psi)	$\bar{P}_n$ (psi)	$\bar{P}_{a_{n-1}}$ (psi)	$\bar{P}_{a_{n-1}} - \bar{P}_n$ (psi)	ΔW_e	W_e
0	p_i	p_i	p_i	0	0	$W_{e0}=0$
1	p_1	$\bar{P}_{n1} = \frac{p_i + p_1}{2}$	$\bar{P}_{a_1} = P_i\left(1 - \frac{W_{e0}}{W_{ei}}\right)$	$\bar{P}_{a_1} - \bar{P}_{n1}$	ΔW_{e1}	W_{e1}
2	p_2	$\bar{P}_{n2} = \frac{p_1 + p_2}{2}$	$\bar{P}_{a_2} = P_i\left(1 - \frac{W_{e1}}{W_{ei}}\right)$	$\bar{P}_{a_2} - \bar{P}_{n2}$	ΔW_{e2}	W_{e2}
3	p_3	$\bar{P}_{n3} = \frac{p_2 + p_3}{2}$	$\bar{P}_{a_3} = P_i\left(1 - \frac{W_{e2}}{W_{ei}}\right)$	$\bar{P}_{a_3} - \bar{P}_{n3}$	ΔW_{e3}	W_{e3}
4	p_4	$\bar{P}_{n4} = \frac{p_3 + p_4}{2}$	$\bar{P}_{a_4} = P_i\left(1 - \frac{W_{e3}}{W_{ei}}\right)$	$\bar{P}_{a_4} - \bar{P}_{n4}$	ΔW_{e4}	W_{e4}
5	p_5	$\bar{P}_{n5} = \frac{p_4 + p_5}{2}$	$\bar{P}_{a_5} = P_i\left(1 - \frac{W_{e4}}{W_{ei}}\right)$	$\bar{P}_{a_5} - \bar{P}_{n5}$	ΔW_{e5}	W_{e5}
6	p_6	$\bar{P}_{n6} = \frac{p_5 + p_6}{2}$	$\bar{P}_{a_6} = P_i\left(1 - \frac{W_{e5}}{W_{ei}}\right)$	$\bar{P}_{a_6} - \bar{P}_{n6}$	ΔW_{e6}	W_{e6}

$$W_{e_n}{}^k = \sum_{j=1}^{n-1} \Delta W_{ej} + \Delta W_{e_n}{}^k$$

The stepwise approach is given in Table 4.10
The last column of the above table is calculated as:

$$W_{e1} = \Delta W_{e1}$$

$$W_{e2} = \Delta W_{e1} + \Delta W_{e2}$$

$$W_{e3} = \Delta W_{e1} + \Delta W_{e2} + \Delta W_{e3}$$

$$W_{e4} = \Delta W_{e1} + \Delta W_{e2} + \Delta W_{e3} + \Delta W_{e4}$$

$$W_{e5} = \Delta W_{e1} + \Delta W_{e2} + \Delta W_{e3} + \Delta W_{e4} + \Delta W_{e5}$$

$$W_{e6} = \Delta W_{e1} + \Delta W_{e2} + \Delta W_{e3} + \Delta W_{e4} + \Delta W_{e5} + \Delta W_{e6}$$

Example 4.6

Repeat Example 4.3 using the Fetkovich's aquifer model to calculate the cumulative water influx at each time step.

Solution

$$f = \frac{\theta}{360} = \frac{140}{360} = 0.3889$$

$$W_{ei} = \frac{\bar{c}f\pi({r_a}^2 - {r_e}^2)h\varnothing P_i}{5.615}$$

$$W_{ei} = \frac{2.5*10^{-6}*0.3889*3.142*\left(112000^2 - 14000^2\right)*100*0.19*4014}{5.615}$$

$$= 512344362.5\ bbl = 512.3444\ MMbbl$$

$$j = \frac{7.08 \times 10^{-3} fkh}{\mu_w\left[\ln\left(\frac{r_a}{r_e}\right) - 0.75\right]} = \frac{7.08 \times 10^{-3}*0.3889*200*100}{0.55*\left[\ln\left(\frac{112000}{14000}\right) - 0.75\right]} = 75.3129\ bbl/d/psi$$

Calculate

$$1 - e^{\frac{-jP_i\Delta tn}{W_{ei}}}$$

Since the time step is uniform, that is

$$\Delta t_n = t_1 - t_0 = t_2 - t_1 = t_3 - t_3 = 1yr = 365\ days$$

Hence,

$$1 - e^{\frac{-jP_i\Delta tn}{W_{ei}}} = 1 - e^{\frac{-75.3129*4014*365}{512344362.5}} = 0.1938$$

$$\Delta W_{e_n} = \frac{W_{ei}}{P_i}\left[\bar{P}_{a_{n-1}} - \bar{P}_n\right]\left(1 - e^{\frac{-jP_i\Delta tn}{W_{ei}}}\right)$$

$$\Delta W_{e_n} = \frac{512344362.5}{4014}\left[\bar{P}_{a_{n-1}} - \bar{P}_n\right]*0.1938 = 24736.5066\left[\bar{P}_{a_{n-1}} - \bar{P}_n\right]$$

At t = 1 year, the water influx is calculated as:

$$W_{e0} = 0$$

$$\bar{P}_{a_{n-1}} = P_i\left(1 - \frac{\sum_{j=1}^{n-1} W_{e_j}}{W_{ei}}\right)$$

$$\bar{P}_{a_1} = P_i\left(1 - \frac{W_{e0}}{W_{ei}}\right) = 4014*\left(1 - \frac{0}{512344362.5}\right) = 4014\,psia$$

$$\bar{P}_{n1} = \frac{p_i + p_1}{2} = \frac{4014 + 3941}{2} = 3977.5\,psia$$

$$\bar{P}_{a_1} - \bar{P}_{n1} = 4014 - 3977.5 = 36.5\,psia$$

$$\therefore W_{e_1} = \Delta W_{e_1} = 24736.5066\left[\bar{P}_{a_1} - \bar{P}_{n1}\right] = 24736.5066*36.5 = 902882.49\ bbl$$

At t = 2 year, the water influx is calculated as:

$$W_{e1} = 902882.49$$

$$\bar{P}_{a_2} = P_i\left(1 - \frac{W_{e1}}{W_{ei}}\right) = 4014 * \left(1 - \frac{902882.49}{512344362.5}\right) = 4006.926\,psia$$

$$\bar{P}_{n2} = \frac{p_1 + p_2}{2} = \frac{3941 + 3870}{2} = 3905.5\,psia$$

$$\bar{P}_{a_2} - \bar{P}_{n2} = 4006.926 - 3905.5 = 101.426\,psia$$

$$\Delta W_{e_2} = 24736.5066\left[\bar{P}_{a_2} - \bar{P}_{n3}\right] = 24736.5066 * 101.426 = 2508924.918\ bbl$$

$$\therefore W_{e2} = \Delta W_{e1} + \Delta W_{e2} = 2508924.918 + 902882.49 = 3411807.408\ bbl$$

At t = 3 year, the water influx is calculated as:

$$W_{e2} = 3411807.408\ bbl$$

$$\bar{P}_{a_3} = P_i\left(1 - \frac{W_{e2}}{W_{ei}}\right) = 4014 * \left(1 - \frac{3411807.408}{512344362.5}\right) = 3987.2699\,psia$$

$$\bar{P}_{n3} = \frac{p_2 + p_3}{2} = \frac{3870 + 3800}{2} = 3835\,psia$$

$$\bar{P}_{a_3} - \bar{P}_{n3} = 3987.2699 - 3835 = 152.2699\,psia$$

$$\therefore \Delta W_{e_3} = 24736.5066\left[\bar{P}_{a_3} - \bar{P}_{n3}\right] = 24736.5066 * 152.2699 = 3766625.386\ bbl$$

$$\begin{aligned}\therefore W_{e3} &= \Delta W_{e1} + \Delta W_{e2} + \Delta W_{e3} = 2508924.918 + 902882.49 + 3766625.386\\ &= 7178432.794\ bbl\end{aligned}$$

The subsequent water influx for year 4 and 5 are presented in the table below applying the same procedure

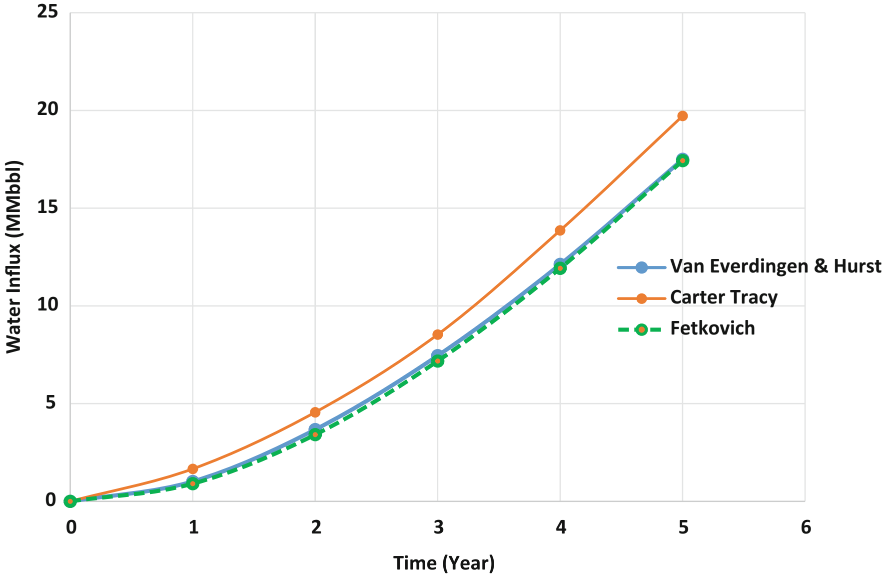
Time (yr)	P (psi)	$\bar{P}_n$ (psi)	$\bar{P}_{a_{n-1}}$ (psi)	$\bar{P}_{a_{n-1}} - \bar{P}_n$ (psi)	ΔW_e (bbl)	W_e (bbl)
0	4014	4014	4014	0	0	0
1	3941	3977.5	4014	36.5	902882.49	902882.4909
2	3870	3905.5	4006.93	101.4263	2508924.92	3411807.408
3	3800	3835	3987.27	152.2699	3766625.39	7178432.794
4	3732	3766	3957.76	191.76	4743472.00	11921904.79
5	3664	3698	3920.59	222.5969	5506269.58	17428174.37

Time (year)	Van Everdingen & Hurst Model (bbl)	Carter-Tracy's Model (bbl)	Fetkovich Model (bbl)
0	0	0	0
1	1016116.35	1660325.526	902882.4909
2	3671938.56	4558333.626	3411807.408

(continued)

Time (year)	Van Everdingen & Hurst Model (bbl)	Carter-Tracy's Model (bbl)	Fetkovich Model (bbl)
3	7453298.95	8534856.871	7178432.794
4	12122474.3	13481153.46	11921904.79
5	17495331.4	19356081.34	17428174.37

The results are plotted in the figure below. This shows that there is a closeness in value between the Van Everdingen and Fetkovich model with little deviation from the Carter-Tracy model but that does not mean that Carter-Tracy model cannot estimate water influx well. In some reservoir, Carter-Tracy model fits the aquifer model used in matching historical data. Thus, these aquifer models are tested on the reservoir to see which matches the past field performance with a minimum tolerance of error.



Exercises

1. What does it imply for a reservoir pressure profile to show a gradual decline?
2. Which of the aquifer model provides an exact solution to the radial diffusivity equation?
3. Which of the aquifer model provides an approximate solution to the radial diffusivity equation?
4. What is the primary difference between Carter-Tracy and Van Everdingen & Hurst Techniques?

5. State the aquifer models that can be applied to both finite and infinite-acting aquifers
6. Which of the aquifer models can be combined with material balance equation?
7. Why is Van Everdingen & Hurst aquifer model difficult to program in computer model?
8. State the aquifer model that can be applied to both radial and linear aquifers and also applies to edge-water drive reservoirs only.
9. Explain the concept of superposition as it applies to water influx model
10. Distinguish between van Everdingen-Hurst and Fetkovich model in terms of transient state flow
11. Which of the aquifer models calculates water influx less than the values predicted by other models?
12. A reservoir whose aquifer is strongly supported by edge water is best fitted with which aquifer model
13. Which of the aquifer model applies to finite-acting aquifers only?
14. State the aquifer model that assumes constant pressure at the original reservoir/aquifer boundary

Ex 4.1 Calculate the water influx at the end of 1, 2 & 5 years into a circular reservoir with an aquifer of infinite extent. Effective water permeability and compressibility are 100 mD and 1.0×10^{-6} psi^{-1} respectively, reservoir viscosity is 0.84, the radius of the reservoir is 2100 ft, reservoir thickness is 27.5 ft with porosity of 22%. The initial and current reservoir pressure are 2700psig and 2380psig respectively.

Ex 4.2 Given the following reservoir and aquifer information with an infinite-acting aquifer:

$$r_w = 0.235ft,\ \ r_e = 2200ft,\ \ k_w = 120mD,\ \ h = 40ft,\ \ P_i = 4260psi,\ \ ,\ \ \mu_w = 0.65cp,\ \ \varnothing = 0.25,\ \ c_{tw} = 1.07\times10^{-6},\ \ \theta = 270^{\circ}$$

The pressure history is given as

Time (years)	Pressure (psi)
0	4260
1	4235
1.5	4203
2	4175
2.5	4115

Ex 4.3 Using the radial aquifer data provided below, determine the cumulative water influx at each time step, using Hurst van Everdingen and compare result with Fetkovich & Carter-Tracy models.

Time (yrs)	0	1	2	3	4	5
Pressure (psia)	2987	2962	2927	2882	2837	2793
$W_{D(tD)}$	0	5.7126	9.0465	11.4326	13.1035	14.3835

Additional Data: $K_w = 275$ mD, $\mu_w = 0.94$ cp, h = 56 ft, aquifer radius, $r_a = 48{,}000$ ft, reservior radius, $r_o = 14{,}000$ ft, porosity = 22%, encraochment angle of 1200, total compressibility = $7.50 \text{ x } 10^{-6}$ psi^{-1}.

Ex 4.4 Calculate the cumulative water influx at each time given in the below of a finite reservoir sustended at encraochment angle of 120° with the following properties $K_w = 78$ mD, $\mu_w = 0.73$ cp, h = 128 ft, aquifer radius, $r_a = 30{,}000$ ft, reservior radius, $r_o = 6000$ ft, porosity = 18%, total compressibility = 7.98×10^{-6} psi^{-1}.

Time (yrs)	0	1	2	3	4	5
Pressure (psia)	2850	2610	2400	2220	2070	1950

References

Carter RD, Tracy GW (1960) An improved method for calculating water influx. Trans AIME 219:415–417

Fetkovitch MJ (1971) A simplified approach to water influx calculations-finite aquifer systems. J Pet Technol 23:814–828

Schilthuis RJ (1936) Active oil and reservoir energy. Trans AIME 118:37

Van Everdingen, A. F (1953) The skill effect and its influence on the production capacity of a well. Trans. AIME, 198: 171–176

Chapter 5
Material Balance

Learning Objectives:
Upon completion of this chapter, students/readers should be able to:

- Understand the concept of the material balance equation
- Describe the basic assumptions and limitations of the material balance equation
- Describe the data required for performing material balance equation
- Identify the sources where the input parameters for material equation are obtained
- Describe the uses of material balance equation
- Derive the general material balance equation for gas and oil reservoirs
- Identify each component of the general material balance equation
- Briefly describe the various reservoir drive mechanism
- Describe the data requirement for characterizing the drive mechanism
- Represent/reduce the general material equation to various reservoir types
- Understand how to determine gas-oil contact and oil-water contact from Material balance equation
- Incorporate the Carter-Tracy aquifer model into the material balance equation for undersaturated reservoir
- Calculate the various primary drive mechanism in any reservoir and recommend the secondary recovery techniques for a particular reservoir.
- Determine stock tank oil initially in place and free gas initially in place

S. Okotie, B. Ikporo, *Reservoir Engineering*,
https://doi.org/10.1007/978-3-030-02393-5_5

Nomenclature

Parameter	*Symbol*	*Unit*
Initial gas formation volume factor	β_{gi}	*cuft/scf*
Gas formation volume factor	β_g	*cuft/scf*
Cumulative water influx	W_e	*bbl*
Cumulative water produced	W_p	*bbll*
Cumulative gas produced	G_p	*scf*
Cumulative oil produced	N_p	*Stb*
Stock tank oil initially In place	N	*stb*
Stock tank gas initially in place	G	*scf*
Initial solution gas-oil ratio	R_{si}	*scf/stb*
Solution gas-oil ratio	R_s	*scf/stb*
Cumulative produce gas-oil ratio	R_p	*Scf/stb*
Bottom hole (wellbore) flowing pressure	P_{wf}	*psia*
Initial reservoir pressure	P_i	*psia*
Oil formation volume factor	β_o	*rb/stb*
Initial oil formation volume factor	β_{oi}	*rb/stb*
Water formation volume factor	β_w	*rb/stb*
Gas formation volume factor	β_g	*cuft/scf*
Initial gas formation volume factor	β_{gi}	*cuft/scf*
Reservoir temperature	T	$^\circ R$
Total fluid compressibility	C_t	$psia^{-1}$
Oil isothermal compressibility	C_o	$psia^{-1}$
Effective oil isothermal compressibility	C_{oe}	$psia^{-1}$
Water & rock compressibility	C_w & C_r	$psia^{-1}$
Gas deviation factor at depletion pressure	z	–
Gas/oil sand volume ratio or gas cap size	m	–
Connate & initial water saturation	S_{wi} & S_{wc}	*- or %*
Residual gas saturation to water displacement	S_{grw}	*- or %*
Residual oil-water saturation	S_{orw}	-
Pore volume of water-invaded zone	PV_{water}	ft^3
Reservoir pore volume	PV	ft^3
Flow rate	q	*stb/d*
Viscosity	μ	*cp*
Formation permeability	k	*mD*
Reservoir thickness	h	*ft*
Area of reservoir	A	*acres*
Wellbore radius	r_w	*ft*
Recovery factor	RF	*%*
Pressure drop	ΔP	*psi*
Initial & current gas expansion factor	E_i & E	*scf/cuft*

5.1 Introduction

Globally, there are different techniques applied in the oil and gas industry to estimate hydrocarbon reserves. These techniques include the analogy, volumetric, decline curve, material balance and reservoir simulation. The application of these techniques is dependent on the volume and quality of data available with some level of uncertainties. In Chap. 2, we have established that the analogy method is applied by comparing factors for the current field or wells while the volumetric or geologic method combined the extent of the reservoir (area), the pore volume of the reservoir rock, the content of fluid within the reservoir pore volume and PVT properties.

When production and pressure data from the field become available, decline curve analysis and material balance calculations become the predominant methods of calculating reserves since the hydrocarbon reserve estimation is a continuous process for a field that is producing. These methods greatly reduce the uncertainties in reserves estimation; however, during early depletion, caution should be exercised in using them.

Material balance equation (MBE) makes use of the basic concept of conservation of mass which states that the cumulative observed production, expressed as an underground withdrawal, must be equal to the expansion of the fluids in the reservoir resulting from a finite pressure drop or expressed as the mass of fluids originally in place equal to mass of fluid remaining plus the mass of fluid produced. MBE is seen by the Reservoir Engineers as the basic tool for interpreting and predicting the performance of oil and gas reservoirs. It helps engineers to get a feel of the reservoir. To better understand this subject, several textbooks and materials were consultated. these are: Craft & Hawkins (1991), Dake (1978, 1994), Mattar & Aderson (2005), Numbere (1998), Pletcher (2002), Steffensen (1992), Matter & McNeil (1998), Tracy (1955) & Tarek (2010).

5.1.1 Assumptions of Material Balance Equation

To apply the material balance equation, there are several assumptions made by the engineers to successfully carry out an evaluation on oil and gas reservoirs. These are:

- The reservoir is considered to be a tank
- Pressure, temperature, and rock and fluid properties are not space dependent
- Uniform hydrocarbon saturation and pressure distribution (homogenous reservoir)
- Thermodynamic equilibrium always attained.
- Isothermal condition apply
- Production data is reliable

5.1.2 Limitations of Material Balance Equation

The implication/limitation of the above stated assumptions in evaluating reservoir performance is that, material balance uses a model that is existing as an imagination of the reservoir to actually tell or forecast the behaviour of the reservoir. This is established as a result of the production of hydrocarbon from the reservoir with natural energy or by gas or water injection. These implications are given below:

- It is considered to be a tank model with a zero dimension which implies that it does not reflect the area drained
- the shape or geometry of the reservoir
- the manner in which the wells drilled into the reservoirs are positioned and orientation are not considered
- the dynamic effects of fluid are not considered
- the heterogeneous nature of the reservoir and no time parameters

These implications lead to the statement made by Warner et al. (1979) that the material balance method has some limitations, though it can be used as a pre-processing tool to infer fluid in place, drive mechanisms and identify aquifer for a more sophisticated tool "reservoir simulation". This sophisticated tool gives an insight into dynamic rock and fluid properties for evaluation of past reservoir performance, prediction of future reservoir performance, and reserves estimation.

5.2 Data Requirement in Performing Material Balance Equation

5.2.1 Production Data

- Cumulative oil, gas and water volume produced
- cumulative gas-oil ratio

5.2.2 PVT Properties

- Oil, gas and water formation volume factor
- Compressibility of water
- Solution Gas-Oil Ratio

5.2.3 Reservoir Properties

- Rock Compressibility
- Connate water saturation

5.2.4 *Other Terms*

- Initial volume of oil in reservoir
- Initial gas cap
- Water and gas injection if any

5.3 Sources of Data Use for the MBE

Parameter	Sources
PVT Data	from PVT reports, correlations
Production data	well and reservoir records
Oil and gas in place	from volumetric estimate and geological model
connate water saturation	from petrophysics (core and log)
water compressibility	from correlation or measured
pore compressibility	from correlation or measured
reservoir pressures	from pressure surveys
water influx	calculation or history

5.4 Uses of Material Balance Equation

However, despite the assumptions and limitations of the material balance approach, there some basic uses which could guide reservoir engineers prior to full field reservoir study. These are:

- Determination of the hydrocarbon in place, gas cap size etc.
- Reservoir pressure estimation from historical production and/or injection schedule.
- Predict the future performance of the reservoir and the average production of the wells sunk into the reservoir for a given pressure schedule
- Determine the presence, type and size of an aquifer.
- Estimation of fluid contacts (Gas/Oil, Water/Oil, Gas/Water).
- Material balance equation can be used to calculate fluid saturation as production increases

5.5 PVT Input Calculation

The PVT properties can either be obtained from the laboratory analysis or generated from existing correlations. Some of these developed correlations are given below.

The initial oil formation volume factor is given as:

$$= \frac{\textit{Relative total volume at initial pressure}}{\textit{Shrinkage factor of reservoir fluid fron bubble point condition to tank oil at } 60^{\circ}F}$$

Mathematically:

$$B_{oi} = \frac{V}{C_b}$$

$$B_o = 1 + (B_{oi} - 1)\frac{P}{P_b}$$

$$R_s = \left(\frac{P}{P_b}\right)R_{si}$$

5.5.1 Standing Correlations

$$A = \left(\frac{R_{sb}}{\gamma_g}\right)^{0.83} \times 10^{(0.00091T - 0.0125API)}$$

$$P_b = 18.2(A - 1.4)$$

$$R_s = \gamma_g\left[10^x\left(\frac{P}{18.2} + 1.4\right)\right]^{1.2048}$$

If $P \leq P_b$ then

$$B_o = 0.972 + 0.000147\left[R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25T\right]^{1.175}$$

Else if $P > P_b$ then

$$\rho_{ob} = \frac{62.4\gamma_o + 0.0136\gamma_g R_{sb}}{B_{ob}}$$

$$C_o = 10^{-6} exp\left[\frac{\rho_{ob} + 0.004347(P - P_b) - 79.1}{7.141 \times 10^{-4}(P - P_b) - 12.983}\right]$$

$$B_o = B_{ob}[1 - C_o(P - P_b)]$$

5.5.2 Glaso Correlations

If $P \le P_b$ then

$$b = R_s \left(\frac{\gamma_g}{\gamma_o}\right)^{0.526} + 0.968T$$

$$a = -6.58511 + 2.91329 \log b - 0.27683 (\log b)^2$$

$$B_o = 1 + 10^a$$

Else if $P > P_b$ then

$$A = 10^{-5}\left(5R_{sb} + 17.2T - 1180\gamma_{gc} + 12.61API - 1433\right)$$

$$\gamma_{gc} = \gamma_g \left[1 + 5.912 \times 10^{-5} API \times T_{sp} \log\left(\frac{P_{sp}}{114.7}\right)\right]$$

Oil compressibility is $C_o = {}^{A}/_{P}$

$$B_o = B_{ob}[1 - C_o(P - P_b)]$$

$$X = 2.8869 - (14.1811 - 3.3093 \log P)^{0.5}$$

$$R_s = \gamma_g \left[10^X \left(\frac{API^{0.989}}{T^{0.172}}\right)\right]^{1.2255}$$

$$A = \left(\frac{R_{sb}}{\gamma_g}\right)^{0.816} \times \frac{T^{0.172}}{API^{0.989}}$$

5.5.3 Al-Marhouns

$$R_s = \left\{185.483208 {\gamma_g}^{1.87784} {\gamma_o}^{-3.1437} (T + 460)^{-1.32657} P\right\}^{1.398441}$$

If $P \le P_b$ then

$$A = {R_s}^{0.74239} {\gamma_g}^{0.323294} {\gamma_o}^{-1.20204}$$

$$B_o = 0.497069 + 8.62963 \times 10^{-4}(T + 460) + 1.82594 \times 10^{-3}A + 3.18099 \times 10^{-6}A^2$$

Else if $P > P_b$ then

$$A = 10^{-5}\left(5R_{sb} + 17.2T - 1180\gamma_{gc} + 12.61API - 1433\right)(9.36)$$

$$\gamma_{gc} = \gamma_g\left[1 + 5.912 \times 10^{-5}API \times T_{sp}\log\left(\frac{P_{sp}}{114.7}\right)\right]$$

Oil compressibility is $c_o = A/P$

$$B_o = B_{ob}[1 - C_o(P - P_b)]$$

5.5.4 Petrosky and Farshad Correlations

$$R_s = \left\{\left(\frac{P}{112.727} + 12.34\right) \times {\gamma_g}^{0.8439}10^x\right\}^{1.73184}$$

If $P \le P_b$ then

$$A = \left\{{R_s}^{0.3738}\left(\frac{{\gamma_g}^{0.2914}}{{\gamma_o}^{0.6265}}\right) + 0.24626T^{0.5371}\right\}^{3.0936}$$

$$B_o = 1.0113 + 7.2046 \times 10^{-5}A$$

Else if $P > P_b$ then

$$A = 4.1646 \times 10^{-7}{R_{sb}}^{0.069357}{\gamma_g}^{0.1885}API^{0.3272}T^{0.6727}$$

$$B_o = B_{ob}exp\left\{-A\left(P^{0.4094} - {P_b}^{0.4094}\right)\right\}$$

The gas formation volume factor is calculated as:

$$B_{gi} = \frac{1}{5.615E_i} = \frac{0.0283z_iT}{5.615P_i}\ in\ (bbl/scf)$$

$$B_g = \frac{1}{5.615E} = \frac{0.0283zT}{5.615P} \ in\ (bbl/scf)$$

The compressibility factor, z is calculated as follow

$$T_c = 168 + \left(325 * \gamma_g\right) - \left(12.5 * {\gamma_g}^2\right)$$

$$P_c = 677 + \left(15 * \gamma_g\right) - \left(37.5 * {\gamma_g}^2\right)$$

$$T_r = \frac{(T + 460)}{T_c}$$

$$P_{ri} = \frac{P_i}{P_c}$$

Therefore, z is calculated as a function of the pseudo reduced properties

$$z(P_r, T_r)$$

5.6 Derivation of Material Balance Equations

5.6.1 Gas Reservoir Material Balance Equation

5.6.1.1 Dry Gas Reservoir Without Water Influx

Applying the law of conservation of mass on Fig. 5.1, it states that the mass of the gas initially in place in the reservoir is equal to the amount of gas produced plus the amount of gas remaining in the reservoir. Recall that gas expands to fill the shape of its container. Hence, in terms of volume balance, it simply states that the volume of gas originally in place at the reservoir conditions is equal to the volume of gas remaining in the reservoir at the new pressure-temperature conditions after some amount of gas has been produced. Since the pressure of the reservoir has dropped with a corresponding decrease in the volume of gas due to the amount that have been produced, therefore the remaining amount of gas in the reservoir would have expanded to occupy the same volume as that initially in place. Mathematically, we have that;

$$GB_{gi} = \left(G - G_p\right)B_g$$

$$GB_{gi} = GB_g - G_pB_g$$

$$G\left(B_g - B_{gi}\right) = G_pB_g$$

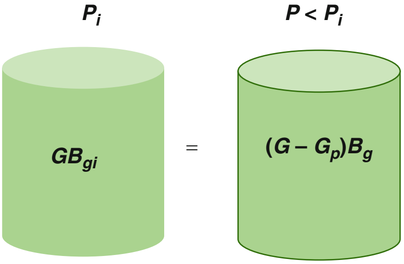


Fig. 5.1 Gas reservoir material balance

Recall

$$B_{gi} = \frac{0.0283 z_i T_i}{P_i}$$

$$B_{gi} = \frac{0.0283 zT}{P}$$

$$G\left[\frac{\cancel{0.0283} z_i \cancel{T}}{P_i}\right] = \left(G - G_p\right)\left[\frac{\cancel{0.0283} z \cancel{T}}{P}\right]$$

$$G\frac{z_i}{P_i} = \left(G - G_p\right)\frac{z}{P}$$

$$\therefore \frac{P}{z} = \frac{P_i}{z_i}\left[\frac{G - G_p}{G}\right] = \frac{P_i}{z_i}\left[1 - \frac{G_p}{G}\right] = \frac{P_i}{z_i} - \frac{1}{G}\frac{P_i}{z_i}G_p$$

A plot of $^{P}/_{z}$ *versus* G_p gives the x-intercept as the initial gas in place and the y-intercept as $^{P_i}/_{z_i}$ (Fig. 5.2)

Example 5.1
A volumetric reservoir at a temperature of 170 °F, specific gas gravity of 0.68 with an initial pressure of 3800 psi located in NDU has produced 520 MMscf of gas at a decline pressure of 2750 psi. Calculate:

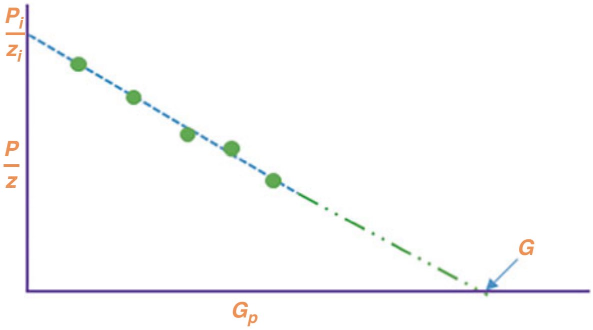
- The gas initially in place
- Remaining reserves at 2750 psi and an abandonment pressure of 600 psi
- The recovery factor at 2750 psi and abandonment pressure

Solution
If yg < = 0.7 Then

$$T_c = 168 + \left(325 * \gamma_g\right) - \left(12.5 * {\gamma_g}^2\right)$$

$$T_c = 168 + (325 * 0.68) - \left(12.5 * 0.68^2\right) = 383.22^{\circ}\mathrm{R}$$

Fig. 5.2 Plot of $^{P}/_{z}$ *versus* G_p



$$P_c = 677 + (15*\gamma_g) - (37.5*{\gamma_g}^2)$$

$$P_c = 677 + (15*0.68) - (37.5*0.68^2) = 669.86 \text{ psia}$$

$$T_r = \frac{(T+460)}{T_c} = \frac{(170+460)}{383.22} = \frac{630}{383.22} = 1.64$$

$$P_{ri} = \frac{P_i}{P_c} = \frac{3800}{669.86} = 5.67$$

Calculate z from Fig. 3.4

$$z_i(P_{ri}, T_r) = z_i(5.67, 1.64) = 0.89$$

$$P_r = \frac{P}{P_c} = \frac{2750}{669.86} = 4.11$$

$$z(P_r, T_r) = z(4.11, 1.64) = 0.84$$

At abandonment

$$P_r = \frac{P_a}{P_c} = \frac{600}{669.86} = 0.89$$

$$z(P_r, T_r) = z(0.89, 1.64) = 0.94$$

Therefore,

$$B_{gi} = \frac{0.0283 z_i T}{P_i} = \frac{0.0283*0.89*(170+460)}{3800} = 0.004176\ cuft/scf$$

$$B_g = \frac{0.0283zT}{P} = \frac{0.0283*0.83*(170+460)}{2750} = 0.005381\ cuft/scf$$

$$B_{ga} = \frac{0.0283z_aT}{P_a} = \frac{0.0283*0.94*(170+460)}{600} = 0.027932\ cuft/scf$$

The gas initially in place

$$G = \frac{G_pB_g}{\left(B_g - B_{gi}\right)} = \frac{520*10^6*0.005381}{0.005381 - 0.004176} = 2322091286\ scf = 2.322\ MMMscf$$

The gas produced at abandonment is:

$$G_{pa} = \frac{G\left(B_{ga} - B_{gi}\right)}{B_{ga}} = \frac{2322091286(0.027932 - 0.004176)}{0.027932} = 1974924839\ scf$$

The remaining gas at 2750 psi

$$= 2322091286 - 520000000 = 1802091286\ scf = 1802.09\ MMscf$$

The remaining gas at abandonment pressure of 600 psi
$=2322091286 - 1974924839 = 347166447\ scf = 347.17\ MMscf$

$$RF@2750\ psi = \frac{G_P}{G} = \frac{520000000}{2322091286} = 0.2239\%$$

$$RF@abandonment = \frac{G_{Pa}}{G} = \frac{1974924839}{2322091286} = 0.8505 = 85.05\%$$

Therefore, at the abandonment pressure, the NDU reservoir can only recovery 85.05% of the gas initially in place.

Example 5.2
A 1100 acres volumetric gas reservoir is characterized with a temperature of 170 °F, reservoir thickness of 50 ft., average porosity of 0.15, initial water saturation of 0.39. The 5 years production history is represented in the table below

Time (yrs)	Reservoir pressure (psia)	Compressibility factor, z	Cum. gas production Gp (MMMscf)
0	1920	0.8542	0.00
1	1850	0.8672	1.36
2	1802	0.8802	2.41
3	1720	0.8932	3.50
4	1638	0.9072	4.95
5	1475	0.9230	6.84

Calculate the gas initially in place (GIIP) using material balance equation and compare your result with volumetric estimate.

Solution
Volumetric estimate of GIIP

$$GIIP = \frac{43560Ah\o(1 - s_{wc})}{B_{gi}}$$

But

$$B_{gi} = \frac{0.0283z_iT}{P_i} = \frac{0.0283*0.8542*(170 + 460)}{1920} = 0.00793\ cuft/scf$$

$$GIIP = \frac{43560*1100*50*0.15*(1 - 0.39)}{0.00793} = 2.764*10^{10}scf = 27.64\ MMMscf$$

Material balance estimate of GIIP

Time (yrs)	Reservoir pressure, P (psia)	Compressibility factor, z	Cum. gas production Gp (MMMscf)	P/z
0	1920	0.8542	0.00	2247.717
1	1850	0.8672	1.36	2133.303
2	1802	0.8802	2.41	2047.262
3	1720	0.8932	3.50	1925.661
4	1638	0.9072	4.95	1805.556
5	1475	0.9230	6.84	1598.05

A plot of $^P/_z$ *versus* G_p gives the intercept as GIIP = 25.3 MMMscf (Fig. 5.3)

Example 5.3
Prior to the commencement of FUPRE dry gas reservoir production with a gas gravity of 0.69 and a reservoir temperature of 120 °F. It was observed that the initial reservoir pressure was not determined and the field has 6 years of production history as shown in the table below.

Time (yrs)	Reservoir pressure, P(psia)	Cum. gas production Gp (MMMscf)
1	3465	1790
2	3385	3807
3	3270	4560
4	3201	5820
5	3105	7465
6	3018	9451

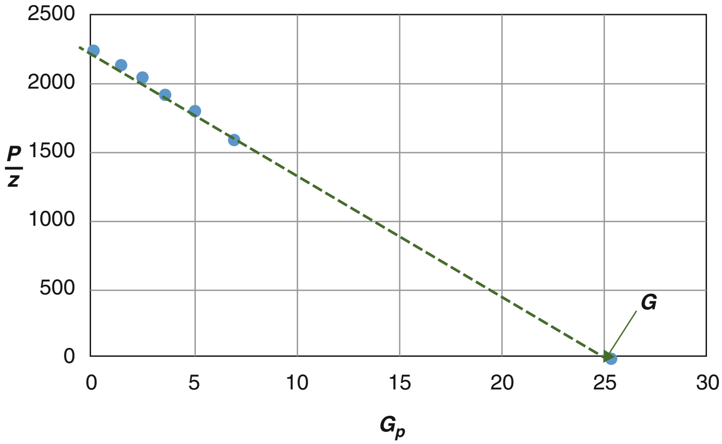


Fig. 5.3 Material balance estimate of GIIP

Determine the following:

I. Initial reservoir pressure
II. Initial gas in place
III. What will be the average reservoir pressure at the completion of a contract calling for 25MMcuft/day for 6 years in addition to the 9451MMscf produced to the sixth year?

Solution

To solve this problem, we must determine the gas deviation factor at each pressure drop to calculate P/z values. Thus, we apply correlations to calculate the pseudocritical properties of the gas as follows:

Since yg < = 0.7, Therefore

$$T_c = 168 + \left(325{*}\gamma_g\right) - \left(12.5{*}{\gamma_g}^2\right)$$

$$T_c = 168 + (325{*}0.69) - \left(12.5{*}0.69^2\right) = 386.2988^{\circ}\mathrm{R}$$

$$P_c = 677 + \left(15{*}\gamma_g\right) - \left(37.5{*}{\gamma_g}^2\right)$$

$$P_c = 677 + (15{*}0.69) - \left(37.5{*}0.69^2\right) = 669.4963\ \mathrm{psia}$$

For P = 3465 psia

$$T_r = \frac{(T + 460)}{T_c} = \frac{(120 + 460)}{386.2988} = \frac{580}{386.2988} = 1.50$$

$$P_r = \frac{P}{P_c} = \frac{3465}{669.4963} = 5.18$$

Calculate z from Fig. 3.4 (Chap. 3)

$$z(P_r, T_r) = z(5.18, 1.50) = 0.82$$

Time (yrs)	Reservoir pressure, P(psia)	Cum. gas production Gp (MMscf)	Compressibility factor, $z\left(P_r = \frac{P}{669.4963}, T_r = 1.50\right)$	P/z
1	3465	2290	0.82	4225.61
2	3385	3007	0.81	4179.01
3	3270	4560	0.80	4087.50
4	3201	5820	0.795	4026.42
5	3105	7465	0.784	3960.46
6	3018	9351	0.78	3869.23

(i) Initial pressure

From the Fig. 5.4

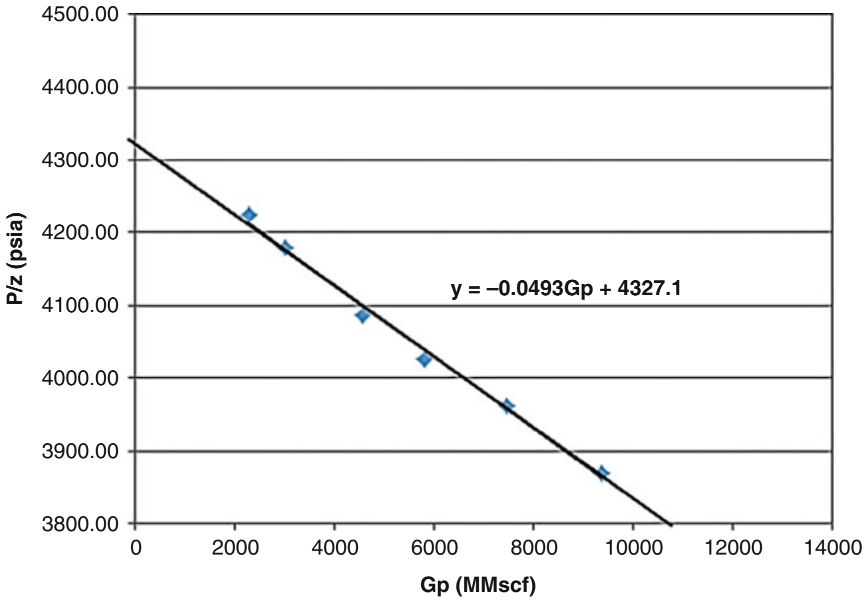


Fig. 5.4 Plot of P/z versus Gp

$$\frac{P_i}{z_i} = 4327 psia \;\; at \; G_p = 0 MMscf$$

The pseudo reduce pressure in terms of gas deviation factor is given as

$$\frac{P_{ri}}{z_i} = \frac{{}^{P_i}/_{z_i}}{P_c} = \frac{4327}{669.4963} = 6.463$$

Thus, from Fig. 3.4

$$z_i(P_r, T_r) = z_i(6.463, 1.50) = 0.89$$

$$P_i = \frac{P_i}{z_i} * z_i = 4327 * 0.89 = 3851.03 \, psia$$

The line of best fit of (P/z) versus G_p is a straight line and the corresponding equation is:

$$\frac{P}{z} = -0.0493 G_p + 4327$$

Where the slope is calculated from the Fig. 5.4 as

$$slope = \frac{\Delta\left(\frac{P}{z}\right)}{\Delta\left(G_p\right)} = \frac{4327 - 3800}{10700} = 0.0493 \, psia/MMScf \; (negative \; slope)$$

The intercept from Fig. 5.4 is

$$\frac{P_i}{z_i} = 4327 psia \;\; at \; G_p = 0 MMscf$$

(ii) Initial gas in place

$$\frac{P}{z} = -0.0493 G_p + 4327$$

When

$$\frac{P}{z} = 0.0 psia, \quad G = G_p$$

$$G = \frac{4327}{0.0493} = 87768.76 MMscf$$

(iii) The average reservoir pressure at the completion of a contract is calculated as:

Given

$$G_p = \frac{25MMscf}{day} for\ 6\ years$$

Assume that we have complete 365 production days per year

$$\therefore\ G_p = \frac{25MMscf}{day} = \frac{25MMscf}{day} * \frac{365days}{1year} * 6years = 54750MMscf$$

Therefore, the cumulative production at the end of the contract = historical production of 6 years + constant production of 25MMscf to the end of contract

$$9351 + 54750 = 64101MMscf$$

When Gp = 64101MMscf

$$\frac{P}{z} = -0.0493G_p + 4327$$

$$\frac{P}{z} = -0.0493(64101) + 4327 = 1166.8207psia$$

$$\frac{P_r}{z} = \frac{P/z}{P_c} = \frac{1166.8207}{669.4963} = 1.74$$

Thus, from Fig. 3.4

$$z(P_r, T_r) = z(1.74, 1.50) = 0.84$$

$$P = \frac{P}{z} * z = 1166.8207 * 0.84 = 980.1294psia$$

5.6.1.2 Dry Gas Reservoir with Water Influx

The volume balance over Fig. 5.5 is

$$GB_{gi} = \left(G - G_p\right)B_g + \left(W_e - W_p\right)B_w$$

$$\left(G - G_p\right)B_g = GB_{gi} - \left(W_e - W_p\right)B_w$$

Divide through by G

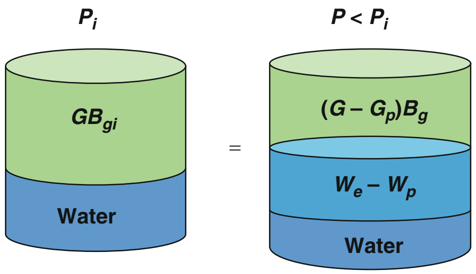


Fig. 5.5 Material balance estimate of GIIP with aquifer

$$\left[\frac{G - G_p}{G}\right] B_g = B_{gi} - \left[\frac{W_e - W_p}{G}\right] B_w$$

Divide through by B_{gi}

$$\left[\frac{G - G_p}{G}\right] \frac{B_g}{B_{gi}} = 1 - \left[\frac{W_e - W_p}{GB_{gi}}\right] B_w$$

$$\left[\frac{G - G_p}{G}\right] \frac{\frac{\cancel{0.0283}z\cancel{T}}{P}}{\frac{\cancel{0.0283}z_i\cancel{T}}{P_i}} = 1 - \left[\frac{W_e - W_p}{GB_{gi}}\right] B_w$$

$$\left[\mathbf{1} - \frac{\boldsymbol{G_p}}{\boldsymbol{G}}\right] \frac{\boldsymbol{P_i}}{\boldsymbol{z_i}} = \frac{\boldsymbol{P}}{\boldsymbol{z}} \left\{\mathbf{1} - \left[\frac{\boldsymbol{W_e - W_p}}{\boldsymbol{GB_{gi}}}\right] \boldsymbol{B_w}\right\}$$

Example 5.4

A gas reservoir with an active water drive is characterized with the following data:

Initial reservoir pressure, P_i	*3700 psia*
Current reservoir pressure, P	*3010 psia*
Initial reservoir bulk volume,	*151.76 MMcuft*
Bulk volume invaded by water at 3010 psia	*64.82 MMcuft*
Cumulative water produce, W_p	*17.3 MMstb*
Cumulative gas produce, G_p	*940.98 MMscf*
Initial gas formation volume factor, B_{gi}	*0.004938 cuft/scf*
Gas formation volume factor, B_g	*0.005103 cuft/scf*
Water formation volume factor, B_w	*1.023 rb/stb*
Porosity, ø	*20%*
Connate water saturation, S_{wc}	*22%*

Calculate:

- The initial gas in place from volumetric
- Water influx

- Water saturation at 3010 psia
- Residual gas saturation in water drive reservoir

Solution

- ***The initial gas in place from volumetric***

$$G = \frac{V_b \emptyset (1 - s_{wc})}{B_{gi}}$$

$$= \frac{151.76 * 10^6 * 0.20 * (1 - 0.23)}{0.004938} = 4732.89 * 10^6 \; scf = 4.733 \; MMMscf$$

- ***Water influx***

$$GB_{gi} = \left(G - G_p\right)B_g + \left(W_e - W_p\right)B_w$$

$$GB_{gi} = GB_g - G_pB_g + W_eB_w - W_pB_w$$

$$W_e(stb)B_w\left({}^{rb}/_{stb}\right) = G_p(scf)B_g\left({}^{cuft}/_{scf}\right) + W_e(stb)B_w\left({}^{rb}/_{stb}\right) + G(scf)\left(B_{gi} - B_g\right)\left({}^{cuft}/_{scf}\right)$$

To solve this question, we have two options of conversion. We can convert the gas formation volume factor from cuft/scf to bbl/scf or convert all production terms and water formation volume factor from bbl to scf.

$$W_e(stb)B_w\left({}^{rb}/_{stb}\right) = G_p(scf)B_g\left({}^{rb}/_{scf}\right) + W_e(stb)B_w\left({}^{rb}/_{stb}\right) + G(scf)\left(B_{gi} - B_g\right)\left({}^{rb}/_{scf}\right)$$

Recall 1 $bbl = 5.615 \; cuft$

$$B_{gi} = 0.004938 \frac{cuft}{scf} = 0.004938 \frac{cuft}{scf} * \frac{rb}{5.615 \; cuft} = 0.000879\left({}^{rb}/_{scf}\right)$$

$$B_g = 0.005103 \frac{cuft}{scf} = 0.005103 \frac{cuft}{scf} * \frac{rb}{5.615 \; cuft} = 0.000909\left({}^{rb}/_{scf}\right)$$

$$\begin{aligned} W_e(stb)B_w\left({}^{rb}/_{stb}\right) &= \left(940.98 * 10^6 * 0.000909\right) + \left(17.3 * 10^6 * 1.023\right) \\ &\quad + \left(4732.89 * 10^6 [0.000879 - 0.000909]\right) = 18411264.12 \; rb \end{aligned}$$

Therefore, water at reservoir condition

$$W_e@reservoir = W_e(stb)B_w({}^{rb}/_{stb}) = 18411264.12\ rb$$

- ***Water saturation at 3010 psia***

Since 64.82 MMscf of water has invaded the bulk rock containing 22% of connate water saturation

The original volume of connate water in the pore $= V_b s_{wc}\ \varnothing\ = 64.82 * 10^6 * 0.2 * 0.22 = 2852080\ scf$

The pore volume $= V_b\ \varnothing\ = 64.82 * 10^6 * 0.2 = 12964000\ scf$

Therefore, the water saturation of the flooded portion is:

$$s_w = \frac{Water\ volume\ remaining}{Pore\ volume}$$

The water volume remaining = connate water volume + water influx - cumulative water produced

Note, these volumes units must be consistent to reflect either surface of reservoir condition.

$$W_e@surface = {}^{18411264.12}\ {}^{rb}/_{1.023rb/stb} = 17997325.63\ stb$$

$$W_e@surface = 17997325.63\ stb * {}^{5.615}\ {}^{scf}/_{stb} = 101054983.4\ scf$$

$$W_p = 17.3*10^6 stb * {}^{5.615}\ {}^{scf}/_{stb} = 97139500\ scf$$

$$\therefore s_w = \frac{2852080 + (101054983.4\ ) - (97139500)}{12964000} = 0.5220 = 52.20\%$$

Then the residual gas saturation

$$s_{gr} = 1 - s_w = 1 - 0.5220 = 0.4780 = 47.80\%$$

5.6.1.3 Adjustment to Gas Saturation in Water Invaded Zone

The initial gas in place in reservoir volume expressed in terms of pore volume (PV) is:

$$GB_{gi} = PV(1 - s_{wi})$$

Hence

$$PV = \frac{GB_{gi}}{(1 - s_{wi})}$$

Considering the water invaded zone, the pore volume is given as:

$$\left(W_e - W_p\right)B_w = PV_{water}\left(1 - s_{wi} - s_{grw}\right)$$

Then

$$PV_{water} = \frac{\left(W_e - W_p\right)B_w}{\left(1 - s_{wi} - s_{grw}\right)}$$

The volume of trapped gas in the water invaded zone is:

$$Trapped\ gas\ volume = PV_{water}s_{grw} = \frac{\left(W_e - W_p\right)B_w}{\left(1 - s_{wi} - s_{grw}\right)}s_{grw}$$

Applying the equation of state and assuming a real gas, the number of moles, n of the volume of trapped gas in the water invaded region is calculated as:

$$n = \frac{P\frac{\left(W_e - W_p\right)B_w}{\left(1 - s_{wi} - s_{grw}\right)}s_{grw}}{zRT}$$

The adjustment to gas saturation to account for the trapped gas is:

$$s_g = \frac{remaining\ gas\ volume - trapped\ gas\ volume}{reservoir\ pore\ volume - pore\ volume\ of\ water\ invaded\ zone}$$

$$s_g = \frac{\left(G - G_p\right)B_g - \left\{\frac{\left(W_e - W_p\right)B_w}{\left(1 - s_{wi} - s_{grw}\right)}\right\}s_{grw}}{\frac{GB_{gi}}{(1 - s_{wi})} - \left\{\frac{\left(W_e - W_p\right)B_w}{\left(1 - s_{wi} - s_{grw}\right)}\right\}}$$

5.6.2 Oil Material Balance Equation

Figure 5.6 shows an initial condition of a reservoir with original gas cap and the setting when the reservoir pressure as dropped due to fluid expansion. The material balance equation uses the principle of conservation of mass. It states that the total

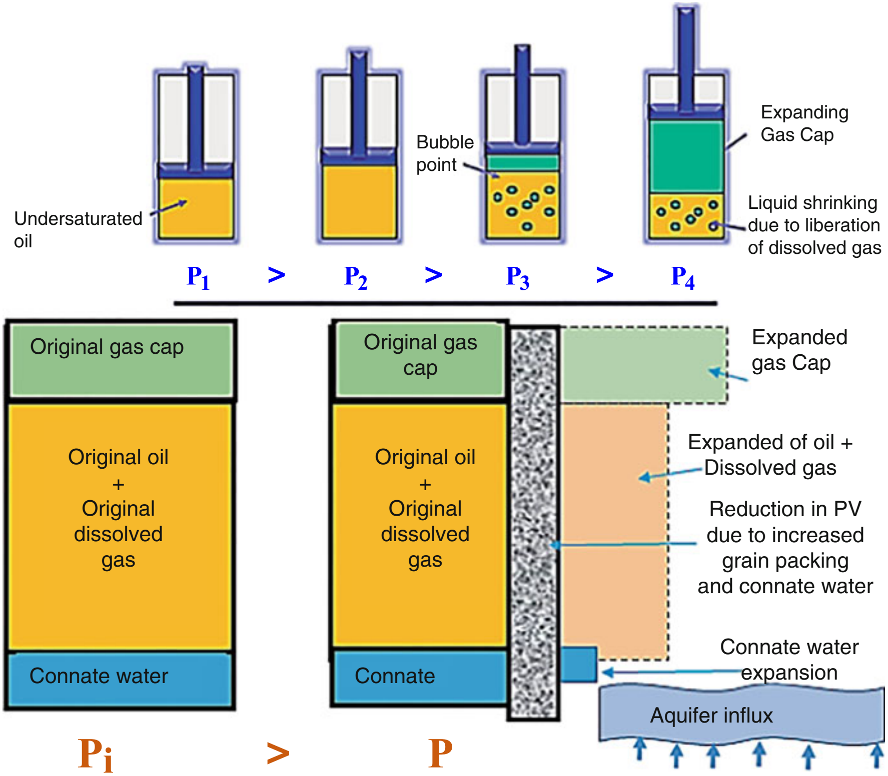


Fig. 5.6 Oil material balance system setup

amount of hydrocarbon withdrawn is equal to the sum of the expansion of the oil plus the original dissolved gas plus the primary gas plus the expansion of the connate water & decrease in pore volume plus the amount of water the encroached into the reservoir.

From the diagram, we have that

$$\begin{aligned}\textit{Total undergroud withdrawal}\ (rb) &= \textit{Expansion of the primary gas cap}\ (rb)\\ &+ \textit{Expansion of the original oil} + \textit{original dissolved gas}\ (rb)\\ &+ \textit{Expansion of connate water} + \textit{decrease in pore volume}\ (rb)\\ &+ \textit{water enroachment}(rb)\end{aligned}$$

The derivation of the general material balance is presented below

5.6.2.1 Quantity of Oil Initially in the Reservoir

$$NB_{oi}$$

5.6.2.2 Quantity of Oil Remaining in the Reservoir

$$= \left(N - N_p\right)B_{oi}$$

5.6.2.3 Expansion of the Primary Gas Cap

The gas cap size is expressed as the ratio of the initial volume of the gas (G) condition to the initial volume of the oil (N) both at stock tank. Mathematically, it is given as:

$$m = \frac{G}{N}\ (surface\ condition)$$

$$m = \frac{GB_{gi}}{NB_{oi}}(Reservior\ condition)$$

The total volume of the primary gas cap at initial pressure P_i is expressed in reservoir condition as:

$$GB_{gi} = mNB_{oi} \quad (rb)$$

At surface condition,

$$G_{@P_i} = \frac{mNB_{oi}}{B_{gi}} \quad (scf)$$

When the pressure of the reservoir decreases from P_i to P the gas volume is expressed in reservoir condition as

$$G_{@P} = G_{@P_i}B_g = \frac{mNB_{oi}}{B_{gi}}B_g \quad (rb)$$

Therefore, the expansion of the primary gas cap to the current reservoir pressure is

$$= G_{@P} - G_{@P_i}\ (rb) = \frac{mNB_{oi}}{B_{gi}} B_g - mNB_{oi}$$

$$= mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] \quad (rb)$$

5.6.2.4 The Free/Liberated Gas in the Reservoir

$$\begin{aligned}&\textit{Original gas in the reservoir}\\ &\qquad = \textit{Free gas in the reservoir} + \textit{Gas remaining in the reservoir}\\ &\qquad + \textit{Gas produced}\end{aligned}$$

$$G = G_{free} + G_{remaining} + G_{produced}$$

Recall

$$R_{si} = \frac{G}{N} \quad \rightarrow G = NR_{si}$$

$$R_p = \frac{G_p}{N_p} \quad \rightarrow G_p = N_pR_p$$

$$NR_{si} = G_{free} + \left(N - N_p\right)R_s + N_pR_p \quad (scf) \quad \textit{Surface condition}$$

Therefore, the free volume of gas in the reservoir is given as

$$G_{free} = \left[NR_{si} - \left(N - N_p\right)R_s - N_pR_p\right]B_g \quad (rb) \quad \textit{Reservoir condition}$$

5.6.2.5 The Net Water Influx into the Reservoir Is

$$\left[W_e - W_p\right]B_w$$

5.6.2.6 Expansion of Oil Zone

In the oil zone, will have the original volume of oil plus the original dissolved gas in the oil

$$@P_i \qquad N \equiv NB_{oi} \qquad\qquad @P \quad N \equiv NB_o$$

The oil expansion

$$N[B_o - B_{oi}] \qquad (rb)$$

$$@P_i \qquad G \equiv NR_{si} \qquad\qquad @P \quad G \equiv NR_s$$

The original gas expansion

$$N[R_{si} - R_s]B_g \qquad (rb)$$

Therefore, the total expansion in the oil zone is

$$N[B_o - B_{oi}] + N[R_{si} - R_s]B_g = N\big\{[B_o - B_{oi}] + [R_{si} - R_s]B_g\big\}$$

5.6.2.7 Expansion of Connate Water and Decrease in Pore Volume

The rock compressibility is expressed as

$$C_r = -\frac{1}{V_p}\frac{\partial V_{pr}}{\partial P}$$

$$C_r = \frac{1}{V_p}\frac{\Delta V_{pr}}{\Delta P}$$

$$\Delta V_{pr} = C_r V_p \Delta P$$

The connate water compressibility is expressed as

$$C_{wc} = \frac{1}{V_p}\frac{\Delta V_{pwc}}{\Delta P}$$

$$\Delta V_{pwc} = C_{wc} V_p \Delta P$$

Recall that water saturation is defined mathematically as

$$S_{wc} = \frac{V_{pwc}}{V_p}$$

The volume of the water in the pore is

$$V_{pwc} = V_p S_{wc}$$

$$\therefore\ \Delta V_{pwc} = C_{wc} S_{wc} V_p \Delta P$$

The total pore volume change is

$$\Delta V_p = \Delta V_{pr} + \Delta V_{pwc}$$

$$\Delta V_p = C_r V_p \Delta P + C_{wc} S_{wc} V_p \Delta P = V_p \Delta P [C_r + C_{wc} S_{wc}]$$

Also, the oil pore volume (original volume of oil in the reservoir) is given as

$$V_p S_{oi} = N B_{oi}$$

$$V_p = \frac{N B_{oi}}{S_{oi}} \qquad S_{oi} = 1 - S_{wc}$$

$$V_p = \frac{N B_{oi}}{1 - S_{wc}}$$

Substitute this expression into the total change in pore volume, we have

$$\Delta V_p = \frac{N B_{oi}}{1 - S_{wc}} [C_r + C_{wc} S_{wc}] \Delta P$$

Consequently, if there is gas cap, the total pore volume is adjusted to accommodate the gas volume such as

$$V_p (1 - S_{wc}) = N B_{oi} + G B_{gi} = N B_{oi} + m N B_{oi} = N B_{oi} [1 + m]$$

$$V_p = \frac{N B_{oi} [1 + m]}{1 - S_{wc}}$$

Therefore, the total change in pore volume is

$$\Delta V_p = [1 + m] \frac{N B_{oi}}{1 - S_{wc}} [C_r + C_{wc} S_{wc}] \Delta P$$

5.6.2.8 Total Underground Withdrawal

The total underground withdrawal (TUW) due to the pressure drop is the sum of the oil + gas + water production. Mathematically, it is

$$TUW = N_p\ (stb)\ oil + G_p\ (scf)\ gas + W_p\ \ (stb)\ \ water$$

At the surface condition, TUW becomes

$$TUW = N_p\ (stb) + N_pR_p\ (scf) + W_p\ \ (stb)$$

Volume of gas produced

$$N_pR_p\ (scf)$$

As the reservoir pressure (P) reduces, the volume of gas dissolved in N_p vol. of oil at P $=N_pR_s$ (*scf*)

Remainder gas is the subsurface gas withdrawal in the form of expanding liberated gas and expanding free gas

Subsurface withdrawal of gas

$= N_p[R_p - R_s]$
(scf)

Subsurface withdrawal of gas in reservoir bbls
$= N_p[R_p - R_s]B_g$ (rb)

At the reservoir condition, TUW becomes

The equivalent of this N_pR_p (*scf*) in the reservoir is $N_p[R_p - R_s]B_g$ (*rb*). Thus

$$TUW = N_pB_o\ (rb) + N_p\left[R_p - R_s\right]B_g\ (rb) + W_pB_w\ \ (rb)$$

Therefore, the total underground withdrawal is

$$= N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\} + W_pB_w \qquad (rb)$$

5.6.2.9 Quantity of Injection Gas and Water

$$= G_{inj}B_{ginj} + W_{inj}B_w$$

Substituting all into the expression of the material balance equation given as:

Total undergroud withdrawal (rb)
= Expansion of the primary gas cap (rb)
+ Expansion of the original oil + original dissolved gas (rb)
+ Expansion of connate water + decrease in pore volume (rb)
+ water enroachment(rb)

$$\begin{aligned}N_p\left\{B_o+\left[R_p-R_s\right]B_g\right\}+W_pB_w=\ & mNB_{oi}\left[\frac{B_g}{B_{gi}}-1\right]\\ & +N\left\{[B_o-B_{oi}]-[R_{si}-R_s]B_g\right\}\\ & +[1+m]\frac{NB_{oi}}{1-S_{wc}}[C_r+C_{wc}S_{wc}]\Delta P+W_eB_w\\ & +G_{inj}B_{ginj}+W_{inj}B_w\end{aligned}$$

$$\begin{aligned}&\boldsymbol{N_p\{B_o+[R_p-R_s]B_g\}+W_pB_w}\\ &\quad=N\left\{\begin{matrix}mB_{oi}\left[\dfrac{B_g}{B_{gi}}-1\right]\\ +\\ [B_o-B_{oi}]+[R_{si}-R_s]B_g\\ +\\ [1+m]\dfrac{B_{oi}}{1-S_{wc}}[C_r+C_{wc}S_{wc}]\Delta P\end{matrix}\right\}+W_eB_w+G_{inj}B_{ginj}+W_{inj}B_w\end{aligned}$$

The general material balance equation is

$$N=\frac{N_p\{B_o+[R_p-R_s]B_g\}-(W_e-W_p)B_w-G_{inj}B_{inj}-W_{inj}B_{inj}}{\left[(B_o-B_{oi})+(R_{si}-R_s)B_g+mB_{oi}\left(\dfrac{B_g}{B_{gi}}-1\right)+(1+m)B_{oi}\left(\dfrac{S_{wi}C_w+C_f}{1-S_{wi}}\right)\Delta P\right]}$$

Where $G_p=N_pR_p$

$$\begin{aligned}N_p=\ &\frac{N\left\{(B_o-B_{oi})+(R_{si}-R_s)B_g+mB_{oi}\left(\frac{B_g}{B_{gi}}-1\right)+(1+m)B_{oi}\left(\frac{S_{wi}C_w+C_f}{1-S_{wi}}\right)\Delta P\right\}}{\left(B_o-B_gR_s\right)}\\ &-\frac{(B_o-G_i)B_g+\left(W_p-W_i\right)B_w}{\left(B_o-B_gR_s\right)}\end{aligned}$$

5.7 Reservoir Drive Mechanisms

The production of hydrocarbon from a reservoir into the wellbore involves several stages of recovery. The available drive mechanisms determine the performance of the hydrocarbon reservoir. When the hydrocarbon fluids are produced by the natural energy of the reservoir, it is termed primary recovery; which is further classified based on the dominant energy responsible for primary production. There are six primary drive mechanisms, they are:

- Solution Gas (Depletion) Drive
- Water Drive
- Gas Cap Expansion (segregation) Drive
- Rock Compressibility and Connate Water Expansion Drive
- Gravity Drainage
- Combination Drive

5.7.1 *Basic Data Required to Determine Reservoir Drive Mechanism*

- Reservoir pressure and rate of decline of reservoir pressure over a period of time.
- The character of the reservoir fluids.
- The production rate.
- Gas-Oil ratio.
- Water-oil ratio.
- The cumulative production of oil, gas and water.

5.7.2 *Solution Gas (Depletion) Drive*

A solution gas or depletion drive reservoir is a recovery mechanism where the gas liberating out of the solution (oil) provides the major source of energy. We simply define it as the oil recovery mechanism that occurs when the original quantity of oil plus all its original dissolved gas expansion as a result of fluid production from its reservoir rock (Fig. 5.7).

This drive mechanism is represented mathematically as:

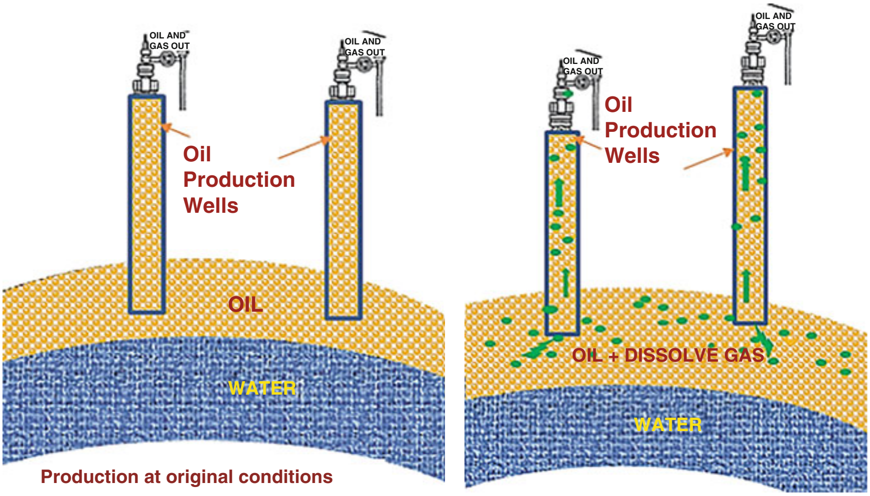


Fig. 5.7 Dissolved gas/depletion drive reservoir

$$Depletion\ Drive\ Index = \frac{Oil\ Zone\ Expansion}{Hydrocarbon\ Voidage}$$

$$\boldsymbol{DDI} = \frac{\boldsymbol{N\{(B_o - B_{oi}) + (R_{si} - R_s)B_g\}}}{\boldsymbol{N_p\{B_o + [R_p - R_s]B_g\}}}$$

5.7.2.1 Production Characteristics (Prof Onyekonwu MO, Lecture Note on Reservoir Engineering)

- *Pressure*
  - declines rapidly and steadily
  - decline rate is dependent on production rate
- *Oil Rate*
  - declines rapidly at first as oil mobility decreases
  - steady decline thereafter
- *Producing GOR*
  - Increases rapidly as free gas saturation increases.
  - Thereafter, decreases rapidly as the remaining oil contains less solution gas.
- *Water Production*

  - Mostly negligible as depletion type reservoirs are volumetric (closed) systems.

- *Ultimate Oil Recovery*
  - It may vary from less than 5% to about 30%. Thus, according to Cole (1969) these characteristics can be use to identify a depletion drive reservoir.

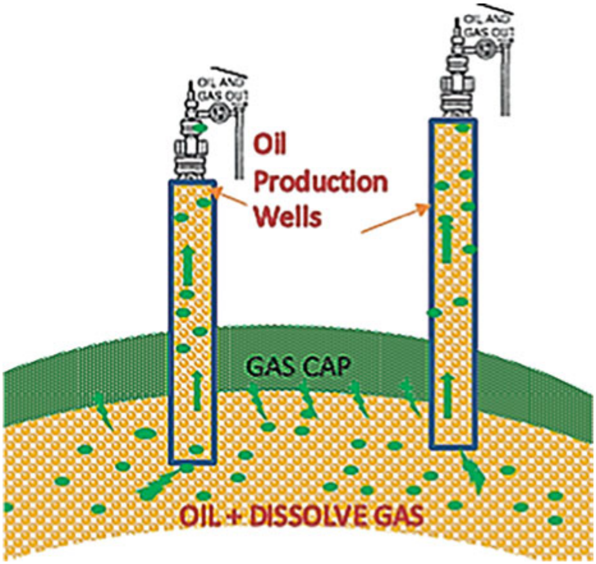
5.7.3 Gas Cap Expansion (Segregation) Drive

Segregation drive (gas-cap drive) is the mechanism wherein the displacement of oil from the formation is accomplished by the expansion of the original free gas cap as shown in Fig. 5.8.

The following are some of the points to note in a gas cap expansion drive mechanism:

- A gas cap, existing above an oil zone in the structurally higher parts of a reservoir, provides a major source of energy. The pressure at the original GOC (Fig. 5.8) is the bubble point pressure since the underlain oil is saturated.
- As pressure declines in the oil column, two things happen:
  - Some dissolved gas comes out of oil
  - Gas cap expands to replace the voidage

Fig. 5.8 Gas cap drive reservoir



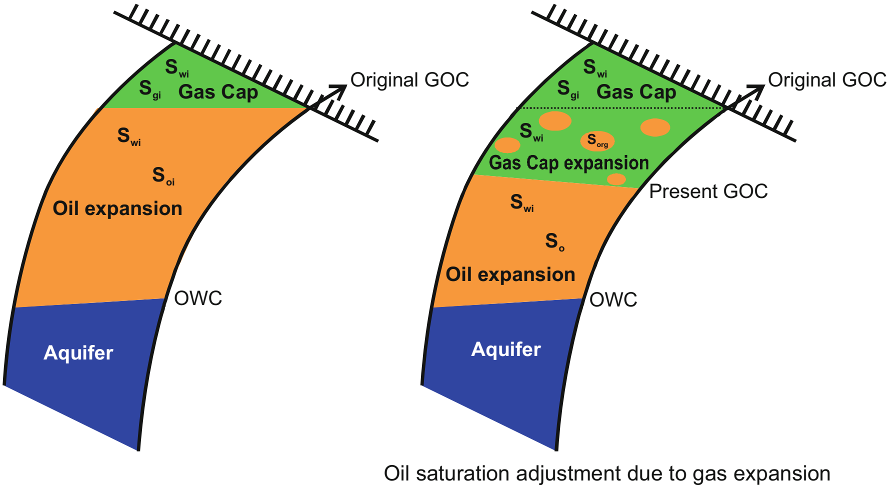


Fig. 5.9 Gas cap expansion drive reservoir

- Formation of free gas in the oil column should be minimized as much as possible. This is achieved if:
  - Gas is re-injected in the gas cap, and
  - Gas is allowed to migrate upstructure (Gravitational Segregation) (Fig. 5.9).

$$Gas\ Cap\ Drive\ Index = \frac{Gas\ Zone\ Expansion}{Hydrocarbon\ Voidage}$$

$$GDI = SDI = \frac{NmB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)}{N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\}}$$

5.7.3.1 Production Characteristics (Prof Onyekonwu MO, Lecture Note on Reservoir Engineering). The characteristics trend for gas cap reservoir listed below were comprehensively summarized by Clark (1969)

- *Pressure*
  - The reservoir pressure falls slowly and continuously
- *Oil Rate*

  - Increase in gas saturation leading to increase in the flow of gas and a drop in the effective permeability of oil.
- *Producing GOR*
  - The gas-oil ratio rises continuously in up-structure wells. As the expanding gas cap reaches the producing intervals of upstructure wells, the gas-oil ratio from the affected wells will increase to high values.
- *Water Production*
  - Absent or negligible water production
- *Ultimate Oil Recovery*
  - The expected oil recovery ranges from 20% to 40%.

5.7.4 Water Drive Mechanism

Water drive is the mechanism wherein the displacement of the oil is accomplished by the net encroachment of water into the oil zone from an underlined water body called aquifer (Fig. 5.10a).

Production of oil or gas will often change the water saturation which in turn affects the oil and gas saturation, but the amount of change varies with the reservoir drive mechanism. In an aquifer driven reservoir on an efficient water flood, as the oil is produced to the surface facilities via the production tubing, the water saturation increases accordingly to fill the space previously occupied by the withdrawn oil (Fig. 5.10b).

This mechanism is represented mathematically as

$$Water\ Drive\ Index = \frac{Net\ water\ influx}{Hydrocarbon\ Voidage}$$

$$\boldsymbol{WDI = \frac{(W_e - W_p)B_w}{N_p\{B_o + [R_p - R_s]B_g\}}}$$

5.7.4.1 Production Characteristics (Prof Onyekonwu MO, Lecture Note on Reservoir Engineering)

- ***Pressure***
  - Pressure is maintained (remains high) when water influx is active.

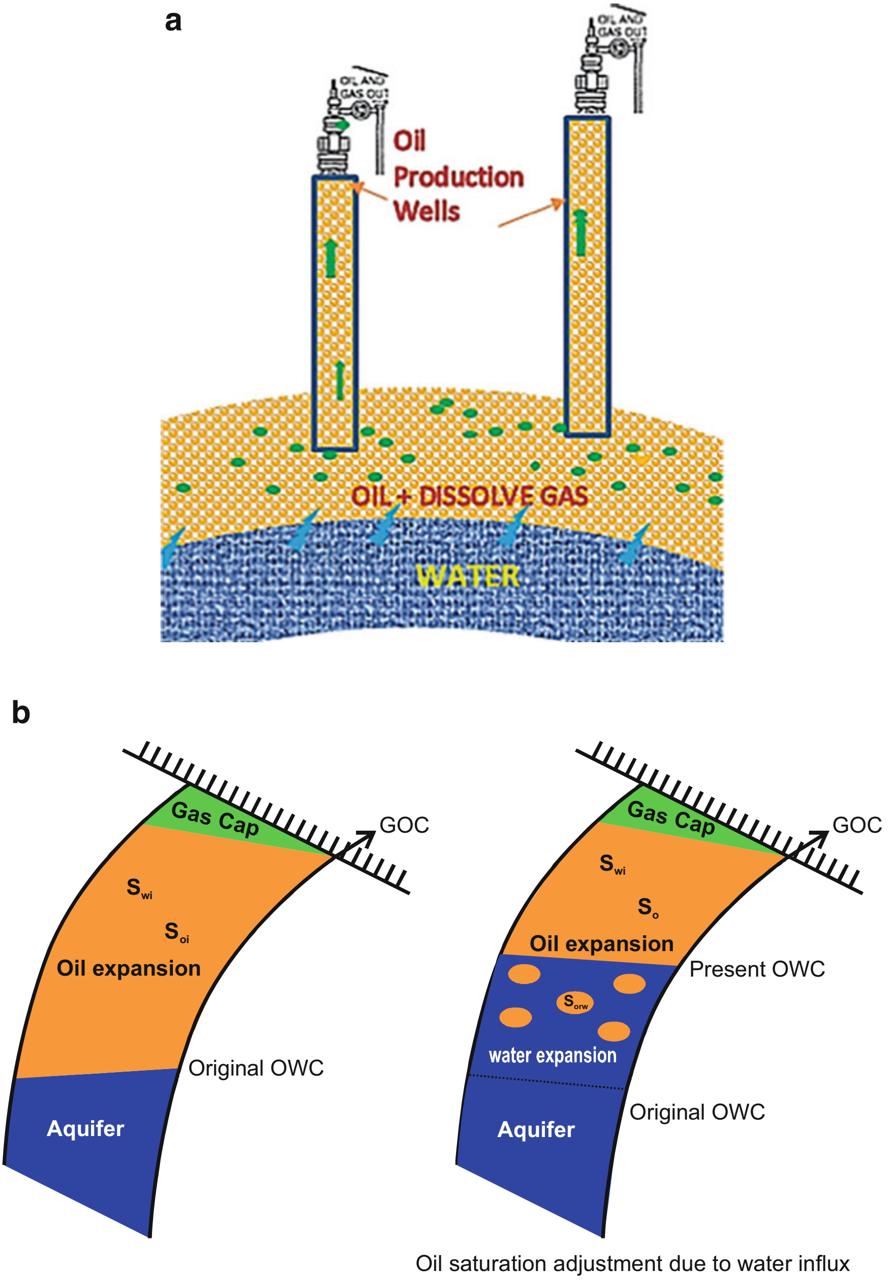


Fig. 5.10 (**a**) Water drive reservoir. (**b**) Water drive reservoir

  - Pressure declines slowly at first but then stabilizes due to increasing influx with increasing pressure differential, but not when water influx is moderate.

- ***Oil Rate***
  - Rate remains constant or gradually declines prior to water breakthrough
  - Rate decreases as water rate increases

- ***Producing GOR***
  - GOR remains constant as long as $P > P_{BP}$
  - Gradually increases if P is below the saturation pressure

- ***Water Production***
  - Dry oil until water breakthrough
  - Increasing water production to an appreciable amount from the flank wells; a sharp increase due to water coning in individual wells.

- ***Ultimate Recovery***
  - The expected oil range is 35–75%

5.7.5 Rock Compressibility and Connate Water Expansion Drive

As the reservoir pressure declines, the rock and fluid expand due to the expansion of the individual rock grains and formation compaction (individual compressibility). The compressibility of oil, rock and water is generally relatively small which makes the pressures in the undersaturated oil reservoirs to drop rapidly to the bubble point if there is no aquifer support. Sometimes, this drive mechanism is not considered or it is neglected when performing material balance calculation, especially for saturated reservoirs.

This mechanism is represented mathematically as:

$$formation\ Drive\ Index = \frac{rock\ and\ connate\ water\ expansion}{Hydrocarbon\ Voidage}$$

$$\boldsymbol{FDI} = \frac{[1+m]\frac{B_{oi}}{1-S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P}{N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\}}$$

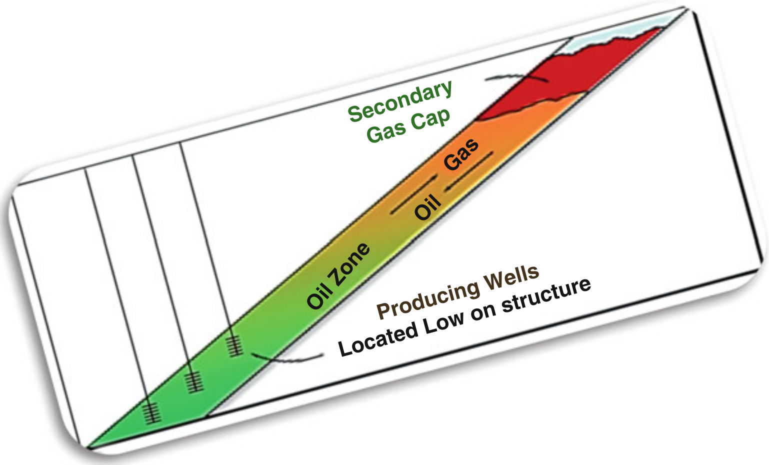


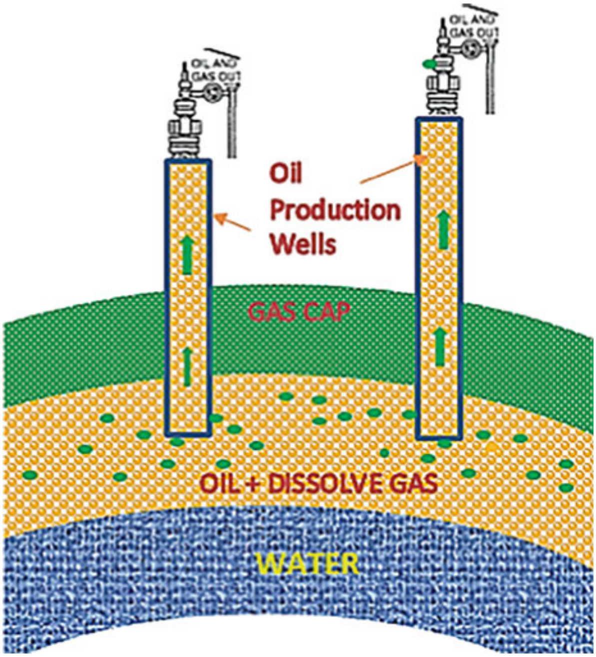
Fig. 5.11 Gravity drainage drive reservoir

5.7.6 Gravity Drainage Reservoirs (Prof Onyekonwu MO, Lecture Note on Reservoir Engineering)

- The mechanism of gravity drainage is operative in an oil reservoir as a result of difference in densities of the reservoir fluids.
- Gas coming out of solution moves updip to the crestal areas while oil moves downdip to the wells located low on the structure (Fig. 5.11).

- *Reservoir must have:*
  - High Dip
  - High Permeability
  - High Kv/Kh ratio
  - Homogeneity
  - Low Oil Viscosity
- *Production Characteristics*:
  - Formation of a secondary gas cap
  - Low GOR from structurally low wells
  - Increasing GOR from high structure wells
  - Rapid pressure decline to near dead conditions (stripper wells)
  - Little or no water production
- While rates are low, RE will be high (70–80% of the initial oil in place) eventually.
- Gravity drainage is most significant in fractured tight reservoirs.

Fig. 5.12 Combination drive reservoir



5.7.7 *Combination Drive Reservoirs*

Most oil reservoirs produce under the influence of two or more reservoir drive mechanisms, referred to collectively as a combination drive. A common example is an oil reservoir with an initial gas cap and an active water drive as shown in the Fig. 5.12.

5.7.7.1 Production Trends

The production trends of a combination drive reservoir reflect the characteristics of the dominant drive mechanism. A reservoir with a small initial gas cap and a weak water drive will behave in a way similar to a solution gas drive reservoir, with rapidly decreasing reservoir pressure and rising GORs. Likewise, a reservoir with a large gas cap and a strong water drive may show very little decline in reservoir pressure while exhibiting steadily increasing GORs and WORs. Evaluation of these production trends is the primary method a reservoir engineer has for determining the drive mechanisms that are active in a reservoir.

5.7.7.2 Recovery

The ultimate recovery obtained from a combination drive reservoir is a function of the drive mechanisms active in the reservoir. The recovery may be high or low depending on whether displacement or depletion drive mechanisms dominate. Water drive and gas cap expansion are both displacement type drive mechanisms and have relatively high recoveries. Solution gas drive is a depletion type drive and is relatively inefficient.

Recovery from a combination drive reservoir can often be improved by minimizing the effect of depletion drive mechanisms by substituting or augmenting more efficient ones through production rate management or fluid injection. To do this, the drive mechanisms active in a reservoir must be identified early in its life

5.7.7.3 Characteristics of Combination Drive Reservoirs (Prof Onyekonwu MO, Lecture Note on Reservoir Engineering)

- Gradually increasing water-cut in structurally low wells
- Pressure decline may be rapid if no strong water influx and no gas cap expansion.
- Continuously increasing GOR in structurally high wells if the gas cap is expanding
- Recovery > depletion Drive but may be less than in water drive or gas-cap drive.
- When an oil reservoir is associated with a gas cap above and an aquifer below, all drive mechanisms may be operative.
- Development strategy and well rate control are very important in the economic recovery process.

  A. If oil production rate is faster than the encroachment rates of gas cap and water advance, pressure depletion occurs in the oil zone.
  B. If oil production rate is controlled to equal voidage, it is better to have water displace oil than gas displacing oil.

     – *Danger: Oil migration into gas cap due to shrinkage of gas cap volume; some oil will be left trapped as residual.*

- RE is usually greater than recovery from depletion drive but less than water drive or gas-cap drive. The expected recovery is between 25 and 40% OOIP

5.8 Representation of Material Balance Equation under Different Reservoir Type

5.8.1 Depletion Drive Reservoir

5.8.1.1 For Undersaturated Reservoir ($P > P_b$) with No Water Influx

That is, above the bubble point; the assumptions made are:

$$m = 0, W_e = 0, R_{si} = R_s = R_p,\ \ G_p = NR_p, W_{inj} = G_{inj} = 0, K_{rg} = 0, W_p = W_e = 0$$

(*because there is no free gas in the formation*); From the general material balance equation, cancelling out all the assumed parameters gives

$$\begin{aligned} N_p\{B_o + \cancel{[R_p - R_s]B_g}\} + \cancel{W_pB_w} &= \cancel{mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]} + N\{[B_o - B_{oi}] - \cancel{[R_{si} - R_s]B_g}\} \\ &+ [1 + \cancel{m}]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P + \cancel{W_eB_w + G_{inj}B_{ginj} + W_{inj}B_w} \end{aligned}$$

It implies that

$$N = \frac{N_pB_o}{(B_o - B_{oi}) + B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P}$$

From Hawkins's equation, the isothermal compressibility of oil C_o, can be expressed as:

$$C_o = -\frac{1}{B_o}\left(\frac{\partial B_o}{\partial P}\right)_T = -\frac{1}{B_{oi}}\left(\frac{B_o - B_{oi}}{P - P_i}\right)$$

$$B_o - B_{oi} = -C_oB_{oi}(P - P_i) = C_oB_{oi}(P_i - P)$$

Put these two equations into the N equation gives:

$$N = \frac{N_pB_o}{C_oB_{oi}(P_i - P) + B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P}$$

$$N = \frac{N_p B_o}{\Delta P B_{oi}\left(C_o + \frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)}$$

$$N = \frac{N_p B_o}{B_{oi}\Delta P\left(\frac{C_o(1 - S_{wi}) + S_{wi}C_w + C_f}{1 - S_{wi}}\right)}$$

$$N = \frac{N_p B_o}{B_{oi}\Delta P\left(\frac{C_o S_o + S_{wi}C_w + C_f}{1 - S_{wi}}\right)}$$

Expressing the isothermal compressibility in terms of effective compressibility, C_{oe}. thus;

$$C_{oe} = \frac{C_o S_o + S_{wi}C_w + C_f}{1 - S_{wi}}$$

$$N = \frac{N_p B_o}{B_{oi}\Delta P C_{oe}} = \frac{N_p B_o}{B_{oi}C_{oe}(P_i - P)}$$

Therefore, the pressure at any time, t above the bubble point is given as:

$$P = P_i - \frac{N_p B_o}{B_{oi}C_{oe}N}$$

The undersaturated recovery factor is given by

$$RF = \frac{N_p}{N} = \frac{B_{oi}C_{oe}(P_i - P)}{B_o}$$

5.8.1.2 Material Balance Time Concept for Pseudo Steady State for Undersaturated Reservoir

From the expression of the isothermal compressibility in terms of effective compressibility, we can express it in terms of total compressibility, C_t

$$C_t = C_o S_o + S_{wi}C_w + C_f$$

Also recall that

$$V_{pi} = \frac{NB_{oi}}{1 - S_{wi}}$$

$$N_pB_o = \frac{NB_{oi}\Delta PC_t}{1 - S_{wi}}$$

$$N_pB_o = V_{pi}\Delta PC_t$$

$$N_p = \frac{V_{pi}\Delta PC_t}{B_o}$$

Where $V_{pi} = \text{ø}\, hA$, so we can rewrite the above equation as:

$$N_p = \frac{\text{ø} hAC_t\left(P_i - \bar{P}\right)}{B_o}$$

Rearranging gives

$$\frac{h\left(P_i - \bar{P}\right)}{B_o} = \frac{N_p}{\text{ø} hA}$$

Multiply the above equation by $\frac{2\pi k}{q\mu}$

$$\frac{2\pi k}{q\mu} \cdot \frac{h\left(P_i - \bar{P}\right)}{B_o} = \frac{2\pi k}{q\mu} \cdot \frac{N_p}{\text{ø} hA} = \frac{2\pi k}{\text{ø}\mu hA}$$

Now, write pressure drop in dimensionless pressure

$$P_i - \bar{P} = \frac{q\mu B_o}{2\pi kh} P_D$$

Also, dimensionless time base on drainage area is given as:

$$t_{AD} = \frac{kt}{\text{ø} hAC_t}$$

Recall $t = \frac{N_p}{q}$, thus; let $t = t_{mb}$ (i.e. material balance time)

$$t_{AD,mb} = \frac{kt_{mb}}{\text{ø} hC_tA} \qquad but\ t_{mb} = \frac{N_p}{q}$$

Therefore,

$$t_{AD,mb} = \frac{k}{\text{ø} hC_tA} \frac{N_p}{q}$$

Since the reservoir at this time is in the late time phase (pseudo steady state). Hence, the dimensionless pressure for this state is given as:

$$P_D = 2\pi t_{AD,mb} + \frac{1}{2}\ln\frac{A}{{r_w}^2} + \frac{1}{2}\ln\frac{2.2458}{C_A}$$

$$P_D = 2\pi t_{AD,mb} + \frac{1}{2}\ln\frac{2.2458A}{C_A {r_w}^2}$$

But

$$t_{AD,mb} = \frac{0.000264kt_{mb}}{\o hC_tA} \text{ for t in hrs and in oil field unit}$$

$$P_D = 2\pi\frac{0.000264kt_{mb}}{\o hC_tA} + \frac{1}{2}\ln\frac{A}{{r_w}^2} + \frac{1}{2}\ln\frac{2.2458}{C_A}$$

$$P_D = \frac{1.2104kt_{mb}}{\o hC_tA} + \frac{1}{2}\ln\frac{2.2458A}{C_A {r_w}^2}$$

Time is in month. Therefore, for a pseudo steady state flow; the equation becomes

$$P_i - P_{wf} = \frac{141.2q\mu B_o}{kh}\left[\frac{1.2104kt_{mb}}{\o hC_tA} + \frac{1}{2}\ln\frac{2.2458A}{C_A {r_w}^2}\right]$$

Thus, for any time t_{mb}=1, 2, 3... n months, we can get the corresponding bottom hole flowing pressure P_{wf} and these pressure obtained from the series of time generated to abandonment time will be used in the prediction stage.

This implies that;

$$P_{wf} = P_i - \frac{141.2q\mu B_o}{kh}\left[\frac{1.2104kt_{mb}}{\o hC_tA} + \frac{1}{2}\ln\frac{2.2458}{C_A {r_w}^2}\right]$$

And the material balance time is:

$$t_{mb} = \frac{\o hC_tA}{1.2104k}\left[\frac{kh\left(P_i - P_{wf}\right)}{141.2q\mu B_o} - \frac{1}{2}\ln\frac{2.2458}{C_A {r_w}^2}\right]$$

5.8.1.3 Saturated Reservoir ($P < P_b$) Without Water Influx

Assumptions are: $m = 0, W_e = 0, R_{si} \neq R_s \neq R_p,\ W_{inj} = G_{inj} = 0, C_f = C_w = 0$

$$N_p\{B_o + [R_p - R_s]B_g\} + \cancel{W_pB_w}$$
$$= \cancel{mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]} + N\{[B_o - B_{oi}] - [R_{si} - R_s]B_g\}$$
$$+ \cancel{[1+m]\frac{NB_{oi}}{1-S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P} + \cancel{W_eB_w + G_{inj}B_{ginj} + W_{inj}B_w}$$

$$N = \frac{N_p\{B_o + [R_p - R_s]B_g\}}{[(B_o - B_{oi}) + (R_{si} - R_s)B_g]} = \frac{N_pB_o + (N_pR_p - N_pR_s)B_g}{[(B_o - B_{oi}) + (R_{si} - R_s)B_g]}$$

Recall that
$G_p = N_pR_p$, hence

$$N = \frac{N_pB_o + (G_p - N_pR_s)B_g}{[(B_o - B_{oi}) + (R_{si} - R_s)B_g]}$$

$$N[(B_o - B_{oi}) + (R_{si} - R_s)B_g] = N_pB_o + G_pB_g - N_pR_sB_g$$

$$\therefore G_p = \frac{N[(B_o - B_{oi}) + (R_{si} - R_s)B_g] - N_pB_o + N_pR_sB_g}{[B_g]}$$

$$G_p = N\left[\frac{B_o - B_{oi}}{B_g} + (R_{si} - R_s)\right] - N_p\left[\frac{B_o}{B_g} + R_s\right]$$

$$N_p = \frac{N[(B_o - B_{oi}) + (R_{si} - R_s)B_g] - G_pB_g}{B_o - R_sB_g}$$

5.8.1.4 Calculation of Oil Saturation

As hydrocarbon is produced from the porous rock, water moves to replace the corresponding space or void left by the produced hydrocarbon because nature avoids vacuum. In some cases, the effects of the reservoir drive mechanisms need to be accounted for; which are presented subsequently in this chapter. Mathematically, oil saturation is given as:

$$S_o = \frac{oil\ volume\ remaining}{pore\ volume} = \frac{(N - N_p)B_o}{{}^{N}B_{oi}/_{(1-S_w)}} = \frac{(1 - S_w)(N - N_p)B_o}{NB_{oi}}$$

$$S_o = (1 - S_w)\left[1 - \frac{N_p}{N}\right]\left(\frac{B_o}{B_{oi}}\right) = (1 - S_w)[1 - RF]\left(\frac{B_o}{B_{oi}}\right)$$

5.8.2 *Gas Drive Reservoir*

Assumptions: $W_e = 0, W_{inj} = G_{inj} = 0, C_f = C_w = 0$

$$\begin{aligned} N_p\{B_o + [R_p - R_s]B_g\} &+ \cancel{W_p B_w} \\ &= mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] + N\{[B_o - B_{oi}] - [R_{si} - R_s]B_g\} \\ &+ \cancel{[1+m]\frac{NB_{oi}}{1-S_{we}}[C_f + C_{we}S_{we}]\Delta P} + \cancel{W_e B_w + G_{inj}B_{ginj} + W_{inj}B_w} \end{aligned}$$

The material balance reduces to:

$$N = \frac{N_pB_o + \left(N_pR_p - N_pR_s\right)B_g}{\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g\right] + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)}$$

$$= \frac{N_pB_o + \left(G_p - N_pR_s\right)B_g}{\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g\right] + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)}$$

$$G_p = \frac{N\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)\right] + N_p\left(R_sB_g - B_o\right)}{B_g}$$

$$N_p = \frac{N\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g\right] + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)}{B_o + \left(R_p - R_s\right)B_g}$$

5.8.2.1 Oil Saturation Adjustment Due to Gas Cap Expansion

The volume of oil in the gas-invaded zone is represented as:

$$P.V_{gas} = \frac{mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)S_{org}}{1 - S_{wi} - S_{org}}$$

To account for the effect of the gas drive or expansion or invasion, the oil saturation is updated thus as

$$\therefore S_o = \frac{\left(N - N_p\right)B_o - \frac{mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)S_{org}}{1 - S_{wi} - S_{org}}}{\frac{NB_{oi}}{(1 - S_{wi})} - \frac{mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)S_{org}}{1 - S_{wi} - S_{org}}}$$

5.8.3 Water Drive Reservoir

5.8.3.1 Undersaturated Reservoir with Water Drive

Assumptions: $W_{inj} = G_{inj} = 0, m = 0$

$$N = \frac{N_pB_o + W_pB_w - W_eB_w}{(B_o - B_{oi}) + B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P}$$

In terms of effective oil compressibility

$$N = \frac{N_pB_o + W_pB_w - W_eB_w}{B_{oi}C_{oe}(P_i - P)}$$

Where $G_p = N_pR_p$

$$N_p = \frac{N\left[(B_o - B_{oi}) + B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P\right] + \left(W_eB_w - W_pB_w\right)}{B_o}$$

5.8.3.2 Saturated Water Drive Reservoir

$$m = 0, W_{inj} = G_{inj} = 0, C_f = C_w = 0$$

$$N = \frac{N_pB_o + \left(N_pR_p - N_pR_s\right)B_g - \left(W_eB_w - W_pB_w\right)}{\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g\right]}$$

$$G_p = N_pR_p$$

$$= \frac{N\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g\right] - N_p\left(B_o + R_sB_g\right) + \left(W_eB_w - W_pB_w\right)}{\left[B_g\right]}$$

$$N_p = \frac{N\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g\right] - G_pB_g + \left(W_eB_w - W_pB_w\right)}{B_o - R_sB_g}$$

5.8.3.3 Oil Saturation Adjustment Due to Water Influx

The volume of oil in the water-invaded zone is represented as:

$$P.V_{water} = \left[\frac{W_eB_w - W_pB_w}{1 - S_{wi} - S_{orw}}\right]S_{orw}$$

To account for the effect of the water drive or expansion or invasion, the oil saturation is updated thus as

$$S_o = \frac{\left(N - N_p\right)B_o - \left[\frac{W_eB_w - W_pB_w}{1 - S_{wi} - S_{orw}}\right]S_{orw}}{\frac{NB_{oi}}{(1 - S_{wi})} - \left[\frac{W_eB_w - W_pB_w}{1 - S_{wi} - S_{orw}}\right]S_{orw}}$$

5.8.4 Combination Drive Reservoir

$$W_{inj} = G_{inj} = 0$$

$$N = \frac{N_pB_o + \left(G_p - N_pR_s\right)B_g - \left(W_e - W_p\right)B_w}{\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) + (1 + m)B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P\right]}$$

$$G_p = \frac{N\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) + (1 + m)B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P\right]}{B_g} + \frac{N_p\left(B_o - R_sB_g\right) + \left(W_e - W_p\right)B_w}{B_g}$$

$$N_p = \frac{N\left[(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right) + (1 + m)B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P\right]}{B_o + \left(R_p - R_s\right)B_g} + \frac{\left(W_e - W_p\right)B_w}{B_o + \left(R_p - R_s\right)B_g}$$

5.8.4.1 Oil Saturation Adjustment Due to Combination Drive

For the case of combination drive, both water and gas invasion zone is incorporated in the saturation equation given as:

$$S_o = \frac{\left(N - N_p\right)B_o - \frac{mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)S_{org}}{1 - S_{wi} - S_{org}} - \left[\frac{W_eB_w - W_pB_w}{1 - S_{wi} - S_{orw}}\right]S_{orw}}{\frac{NB_{oi}}{(1 - S_{wi})} - \frac{mNB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)S_{org}}{1 - S_{wi} - S_{org}} - \left[\frac{W_eB_w - W_pB_w}{1 - S_{wi} - S_{orw}}\right]S_{orw}}$$

5.9 Determination of Present GOC and OWC from Material Balance Equation

Step 1: Determine the bulk volume of the reservoir rock at each depth interval

Step 2: Make a plot of depth versus the bulk volume

Step 3: Calculate the cumulative water influx from the general material balance equation ($\boldsymbol{W_e}$)

Step 4: Calculate the volume of oil displaced by water (Net water influx into the reservoir) ($\boldsymbol{O_W = W_e - W_p}$)

Step 5: Calculate the reservoir volume liberated gas ($\boldsymbol{G_L}$)

$$G_L = \left[NR_{si} - \left(N - N_p\right)R_s\right]B_g$$

Step 6: Calculate the expansion of the primary gas cap ($\boldsymbol{G_e}$)

$$G_e = mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

Step 7: Calculate the gas drive ($\boldsymbol{G_D}$)

$$G_D = G_L + G_e$$

Step 8: Calculate the produced excess gas ($\boldsymbol{G_{pe}}$)

$$G_{pe} = \left[N_p\left(R_p - R_s\right)\right]B_g$$

Step 9: Calculate the volume of oil displaced by the gas ($\boldsymbol{O_g}$)

$$O_G = G_D - G_{pe}$$

Note the following reservoir conditions:
If O_G is negative (−ve), then oil has moved into the primary gas cap
If $G_{pe} > G_L$ then the gas cap is produced.

Step 10: Calculate the dispersed gas in the oil zone (***G_{disp}***)

$$G_{disp} = S_{gc}\left[\frac{(N - N_p)B_o}{(1 - S_{wc})}\right]$$

Note: if $G_{disp} > G_L$, *reduce* S_{gc}

Step 11: Calculate the volume of oil displaced by the primary and secondary gas cap (O_{GPS})

$$O_{GPS} = O_G - G_{disp}$$

Step 12: Calculate the gross oil sand volume flooded by water is given as:

$$GOV_w = \frac{W_e - W_p}{7758.4 * \o * F(1 - s_{wc} - s_{orw} - S_{gc})} \quad (ac - ft)$$

Step 12: Calculate the gross oil sand volume displaced by the primary and secondary gas cap given as:

$$GOV_g = \frac{O_{GPS}}{7758.4 * \o * F(1 - s_{wc} - s_{org})}$$

Step 13: Determine the present fluid contacts as follows:

Trace the value of GOV_w & GOV_g from the horizontal axis of the cumulative bulk volume plot in stage 2 to touch the curve and then read off the depth at the corresponding values. (i.e the depth corresponding to $GOV_g = GOC$ and the depth corresponding to $GOV_w = OWC$).

Example 5.5
A hydrocarbon reservoir with a large gascap has the following production and fluid data:

Initial reservoir pressure = 4630 psi	Current reservoir pressure = 4531 psi
Initial oil FVF = 1.6186 rb/stb	Initial gas exp. Factor= 287 scf/cuft
Initial solution GOR = 1164 scf/stb	gas exp. Factor= 269 scf/cuft
oil FVF =1.6015 rb/stb	Cum. Produced GOR = 2189 scf/stb

(continued)

Current solution GOR = 1135 scf/stb	Gas/oil sand volume ratio = 0.56
Cum. Oil Produced= 4.003 MMstb	Cum. Water produced = 0.045 MMstb
Connate water saturation = 15%	Formation compressibility = 3.5 × 10^{-6} psi^{-1}
STOIIP = 125 MMstb	Water compressibility = 3.5 × 10^{-6} psi^{-1}

- Determine the expansion of the various zones in MMrb
- Determine the total underground withdrawal
- Determine the volume of free gas in the reservoir
- Determine the aquifer influx.
- Calculate the percent contributions of the various fluids to the underground hydrocarbon production.
- Indicate the least and most active drive mechanism

Solution

Determine the expansion of the various zones in MMrb

$$B_{gi} = \frac{1}{5.615E_i} = \frac{1}{287*5.615} = 0.00062054\ rb/Scf$$

$$B_g = \frac{1}{5.615E} = \frac{1}{269*5.615} = 0.00066206\ rb/Scf$$

Expansion of original oil plus dissolved gas (oil zone expansion)

$$= N\{[B_o - B_{oi}] + [R_{si} - R_s]B_g\}$$

$$= 125*10^6\{[1.6015 - 1.6186] + [1164 - 1135]0.00066206\} = 262467.5\ rb$$

Expansion of primary gas cap

$$= mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

$$= 0.56*125*10^6*1.6186\left[\left(\frac{0.00066206}{0.00062054}\right) - 1\right] = 7580976.311\ rb$$

Expansion of connate water and decrease in pore volume

$$= [1+m]\frac{NB_{oi}}{1-S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P$$

$$= [1+0.56]\frac{125*10^6*1.6186}{1-0.15}\left[3.5*10^{-6} + \left(3.5*10^{-6}*0.15\right)\right](4630 - 4531)$$

$$= 147964.081\ rb$$

Total underground withdrawal

$$N_p\{B_o + [R_p - R_s]B_g\} + W_pB_w$$

$$= 4.003*10^6\{1.6015 + [2189 - 1135]0.00066206\} + 0.045*10^6*1$$
$$= 9249142.894\ rb$$

Hydrocarbon Voidage

$$N_p\{B_o + [R_p - R_s]B_g\}$$

$$= 4.003*10^6\{1.6015 + [2189 - 1135]0.00066206\} = 9204142.894\ rb$$

Volume of free gas in the reservoir

$$G_{free} = [NR_{si} - (N - N_p)R_s - N_pR_p]B_g$$

$$G_{free} = [(125*10^6*1164) - (125*10^6 - 4.003*10^6)1135$$
$$-(4.003*10^6*1164)]0.00066206 = 2323110.94\ rb$$

Water influx

$$W_e = N_p\{B_o + [R_p - R_s]B_g\} + W_pB_w - mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$
$$- N\{[B_o - B_{oi}] - [R_{si} - R_s]B_g\}$$
$$-[1 + m]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P$$

$$W_e = 9249142.894 - 7586820.68 - 262467.5 - 147964.081 = 1251890.63\ rb$$

Net water influx

$$= W_e - W_p = 1251890.63 - 0.045*10^6 = 1206890.63\ rb$$

Calculate the percent contributions of the various fluids to the underground hydrocarbon production.

$$DDI = \frac{N\{(B_o - B_{oi}) + (R_{si} - R_s)B_g\}}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{262467.5}{9204142.894} = 0.0285 = 2.85\%$$

$$FDI = \frac{[1 + m]\frac{B_{oi}}{1-S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{147964.081}{9204142.894} = 0.01607 = 1.61\%$$

$$GDI = SDI = \frac{NmB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{7586820.68}{9204142.894} = 0.8243 = 82.43\%$$

$$WDI = \frac{(W_e - W_p)B_w}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{1206890.63}{9204142.894} = 0.1311 = 13.11\%$$

Summation of drive indices 13.11 + 82.43 + 1.61 + 2.85 = 100%

- Indicate the least and most active drive mechanism

The least drive index is the expansion of rock and connate water

The most active drive index is the gas cap expansion or solution gas drive. Therefore, based on the active drive mechanism, gas injection is recommended for the pressure maintenance or secondary recovery

Example 5.6

An undersaturated reservoir producing above the bubble point had an initial pressure of 5000 psia, at which pressure the oil formation volume factor was 1.510 bbl/stb. When the pressure dropped to 4600 psia owing to the production of 100,000 stb of oil, the oil formation volume factor is 1.520 bbl/stb, the connate water saturation is 23%, water compressibility is 3.7×10^{-6} psi^{-1}, rock compressibility is 3.5×10^{-6} psi^{-1} and average porosity of 21%.

I. Determine the oil compressibility.
II. Determine the effective oil compressibility
III. Assuming a volumetric reservoir, calculate the oil in place
IV. Determine the recovery factor at 4600 psia
V. After a thorough analysis, the calculated initial oil place was 9.6 MMstb. Determine the water influx at 4600 psi after cumulative water production of 825.92 stb

Solution

Oil compressibility

$$C_o = \frac{1}{B_{oi}}\left(\frac{B_o - B_{oi}}{P_i - P}\right) = \frac{1}{1.510}\left(\frac{1.520 - 1.510}{5000 - 4600}\right) = 1.6556*10^{-5}\, psia^{-1}$$

Effective oil compressibility

$$S_o = 1 - S_{wi} = 1 - 0.23 = 0.77$$

$$C_{oe} = \frac{C_oS_o + S_{wi}C_w + C_f}{1 - S_{wi}}$$

$$= \frac{(1.6556*10^{-5}*0.77) + (3.7*10^{-6}*0.23) + (3.5*10^{-6})}{1 - 0.23}$$

$$= 2.2207*10^{-5}\, psia^{-1}$$

Oil in place

$$N = \frac{N_p B_o}{B_{oi} C_{oe}(P_i - P)}$$

$$= \frac{100000 * 1.520}{1.510 * 2.2207 * 10^{-5}(5000 - 4600)} = 11332265.91\ rb = 11.33\ MMstb$$

Recovery factor

$$RF = \frac{N_p}{N} = \frac{B_{oi} C_{oe}(P_i - P)}{B_o} = \frac{1.510 * 2.2207 * 10^{-5}(5000 - 4600)}{1.520} = 8.824 * 10^{-3}$$
$$= 0.88\%$$

Water influx

$$N = \frac{N_p B_o + W_p B_w - W_e B_w}{B_{oi} C_{oe}(P_i - P)}$$

$$W_e = N_p B_o - N B_{oi} C_{oe}(P_i - P) + W_p B_w$$

Assume $B_w = 1\ rb/stb$

$$= [100000 * 1.520] - \left[9.6 * 10^6 * 1.510 * 2.2207 * 10^{-5} * (5000 - 4600)\right] + 825.92$$

$$= 24060.8512\ rb = 24.061\ Mstb$$

Example 5.7

A hydrocarbon reservoir with a large gascap has the following production and fluid data:

Initial reservoir pressure = 4630 psi	Current reservoir pressure = 4531 psi
Initial oil FVF = 1.6186 rb/stb	Initial gas exp. Factor= 269 scf/cuft
Initial solution GOR = 1164 scf/stb	gas exp. Factor= 267 scf/cuft
oil FVF= 1.6015 rb/stb	Cum. Produced GOR = 2189 scf/stb
Current solution GOR = 1135 scf/stb	Gas/oil sand volume ratio = 4.06
Cum. Oil Produced= 4.003 MMstb	Cum. Water produced = 0.045 MMstb
Connate water saturation = 15%	Formation compressibility = 3.5×10^{-6} psi^{-1}
Cum aquifer influx = 6.56 MMstb	Water compressibility = 3.5×10^{-6} psi^{-1}

(a) Determine the correct/matching value of STOIIP (N).
(b) Calculate also the percent contributions of the various fluids to the underground hydrocarbon production

Solution

(a) Determine the correct/matching value of STOIIP (N).

Determine the expansion of the various zones in MMrb

$$B_{gi} = \frac{1}{5.615E_i} = \frac{1}{269*5.615} = 0.0006621\ rb/Scf$$

$$B_g = \frac{1}{5.615E} = \frac{1}{267*5.615} = 0.0006670\ rb/Scf$$

Expansion of original oil plus dissolved gas (**oil zone expansion**)

$$= N\{[B_o - B_{oi}] + [R_{si} - R_s]B_g\}$$

$$= N\{[1.6015 - 1.6186] + [1164 - 1135]0.0006670\} = 0.002243N\ rb$$

Expansion of primary gas cap

$$= mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

$$= 4.06*N*1.6186\left[\left(\frac{0.0006670}{0.0006621}\right) - 1\right] = 0.0486N\ rb$$

Expansion of connate water and decrease in pore volume

$$= [1 + m]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P$$

$$= [1 + 4.06]\frac{N*1.6186}{1 - 0.15}\left[3.5*10^{-6} + \left(3.5*10^{-6}*0.15\right)\right](4630 - 4531)$$

$$= 0.003839N\ rb$$

Total underground withdrawal

$$N_p\{B_o + [R_p - R_s]B_g\} + W_pB_w$$

$$= 4.003*10^6\{1.6015 + [2189 - 1135]0.0006670\} + 0.045*10^6*1$$
$$= 9269985.554\ rb$$

Hydrocarbon Voidage

$$N_p\{B_o + [R_p - R_s]B_g\}$$

$$= 4.003*10^6\{1.6015 + [2189 - 1135]0.0006670\} = 9224985.554\ rb$$

To Calculate N

$$
\begin{aligned}
N_p\{B_o + [R_p - R_s]B_g\} + W_pB_w = mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] \\
+ N\{[B_o - B_{oi}] - [R_{si} - R_s]B_g\} \\
+ [1+m]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P + W_eB_w
\end{aligned}
$$

$$9269985.554 = 0.0486N + 0.002243N + 0.003839N + 6.56*10^6$$

$$9269985.554 - 6.56*10^6 = 0.054682N$$

$$2709985 = 0.054682N$$

Therefore

$$N = \frac{2709985}{0.054682} = 49559005.78\ stb = 49.559\ MMstb$$

(b) **Calculate the percent contributions of the various fluids to the underground hydrocarbon production**.

$$DDI = \frac{N\{(B_o - B_{oi}) + (R_{si} - R_s)B_g\}}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{0.002243N}{9224985.554} = \frac{0.002243*49559005.78}{9224985.554}$$
$$= 0.0120 = 1.20\%$$

$$FDI = \frac{[1+m]\frac{B_{oi}}{1-S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{0.003839*49559005.78}{9224985.554} = 0.0206$$
$$= 2.06\%$$

$$GDI = SDI = \frac{NmB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{0.0486*49559005.78}{9224985.554} = 0.261$$
$$= 26.11\%$$

$$WDI = \frac{(W_e - W_p)B_w}{N_p\{B_o + [R_p - R_s]B_g\}} = \frac{6.56*10^6 - 0.045*10^6}{9224985.554} = 0.7062 = 70.62\%$$

Summation of drive indices 1.20 + 2.06 + 26.11 + 70.62 = 100%

- Indicate the least and most active drive mechanism

The least drive index is the expansion of the oil zone
The most active drive index is the water drive

Example 5.8
A saturated oil reservoir which has produced a cumulative gas-oil ratio of about 4100 scf/stb is presented with two cases. In the first case, the reservoir has been producing without an effort to shut-in the gas wells or to re-inject the gas and case represents a scenario where two-thirds of the original solution gas remains in the reservoir either by re-injecting the produced gas or by shutting in the high gas producers at the same pressure at which the oil formation volume factor was determined. Given the following data below, calculate the recovery factor in both cases.

$$B_{oi} = 1.383\frac{scf}{stb}, \quad B_o = 1.462\frac{scf}{stb}, B_g = 0.00274\frac{bbl}{scf}, R_{si} = 1080\frac{scf}{stb}, R_s = 820\ scf/stb$$

Solution

$$RF = \frac{N_p}{N} = \frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_o + \left[R_p - R_s\right]B_g}$$

Case 1

$$RF = \frac{N_p}{N} = \frac{(1.462 - 1.383) + (1080 - 820)0.00274}{1.462 + [4100 - 820]0.00274} = 0.0757 = 7.57\%$$

This case is a solution gas drive reservoir

Case 2
The remaining volume of solution gas in the reservoir at two-thirds of the original solution gas is

$$= \frac{2}{3}R_p = \frac{2}{3} \times 4100 = 2733.33\ scf/stb$$

The means that the produced GOR is =4100 − 2733.33 = 1366.67 *scf/stb*

$$\therefore \quad R_p = 1366.67\ scf/stb$$

$$RF = \frac{N_p}{N} = \frac{(1.462 - 1.383) + (1080 - 820)0.00274}{1.462 + [1366.67 - 820]0.00274} = 0.2674 = 26.74\%$$

This case is a gas drive reservoir.

Example 5.9

As a reservoir engineer working ABC Company, you have been given the following production and fluid data below to perform classical material balance analysis. Recommend to the management of your company, the secondary recovery method for this reservoir based on the predominant energy of a reservoir.

Initial reservoir pressure = 2740 psi	Current reservoir pressure = 2460 psi
Initial oil FVF = 1.3985 rb/stb	Initial gas exp. Factor= 198.3 scf/cuft
Initial solution GOR = 643 scf/stb	gas exp. Factor= 178.14 scf/cuft
oil FVF= 1.3578 rb/stb	Cum. Gas Produced= 15,498 MMscf
Current solution GOR = 577.3 scf/stb	Gas/oil sand volume ratio = 0.7
Cum. Oil Produced = 18.9 MMstb	Cum. Water produced = 3.3 MMstb
Connate water saturation = 17%	Formation compressibility = 3.5 × 10^{-6} psi^{-1}
STOIIP = 120 MMstb	Water compressibility = 3.5 × 10^{-6} psi^{-1}

Solution

Determine the expansion of the various zones in MMrb

$$B_{gi} = \frac{1}{5.615E_i} = \frac{1}{198.3*5.615} = 0.00089811\ rb/Scf$$

$$B_g = \frac{1}{5.615E} = \frac{1}{178.14*5.615} = 0.00099974\ rb/Scf$$

Expansion of original oil plus dissolved gas (oil zone expansion)

$$= N\left\{[B_o - B_{oi}] + [R_{si} - R_s]B_g\right\}$$

$$= 120*10^6\{[1.3578 - 1.3985] + [\,643 - 577.3]0.00099974\} = 2997950.16\ rb$$

Expansion of primary gas cap

$$= mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

$$= 0.7*120*10^6*1.3985\left[\left(\frac{0.00099974}{0.00089811}\right) - 1\right] = 13293341.15\ rb$$

Expansion of connate water and decrease in pore volume

$$= [1 + m]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P$$

$$= [1 + 0.7]\frac{120*10^6*1.3985}{1 - 0.17}\left[3.5*10^{-6} + \left(3.5*10^{-6}*0.17\right)\right](2740 - 2460)$$

$$= 394118.1933\ rb$$

Total underground withdrawal

$$N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\} + W_pB_w$$

$$\begin{aligned} &= 18.9*10^6\{1.3578 + [\,820 - 577.3]0.00099974\} + 3.3*10^6*1 \\ &= 33548257.37\ rb \end{aligned}$$

Hydrocarbon Voidage

$$N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\}$$

$$= 18.9*10^6\{1.3578 + [\,820 - 577.3]0.00099974\} = 30248257.37\ rb$$

Water influx

$$\begin{aligned} W_e = {} & N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\} + W_pB_w - mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] \\ & - N\left\{[B_o - B_{oi}] - [R_{si} - R_s]B_g\right\} \\ & -[1 + m]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P \end{aligned}$$

$$\begin{aligned} W_e &= 33548257.37 - 13293341.15 - 2997950.16 - 394118.1933 \\ &= 16862847.87\ rb \end{aligned}$$

Net water influx

$$= W_e - W_p = 16862847.87 - 3.3*10^6 = 13562847.87\ rb$$

Calculate the percent contributions of the various fluids to the underground hydrocarbon production.

$$DDI = \frac{N\left\{(B_o - B_{oi}) + (R_{si} - R_s)B_g\right\}}{N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\}} = \frac{2997950.16}{30248257.37} = 0.0991 = 9.91\%$$

$$FDI = \frac{[1 + m]\frac{B_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P}{N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\}} = \frac{394118.1933}{30248257.37} = 0.01303 = 1.30\%$$

$$GDI = SDI = \frac{NmB_{oi}\left(\frac{B_g}{B_{gi}} - 1\right)}{N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\}} = \frac{13293341.15}{30248257.37} = 0.4395 = 43.95\%$$

$$WDI = \frac{\left(W_e - W_p\right)B_w}{N_p\left\{B_o + \left[R_p - R_s\right]B_g\right\}} = \frac{13562847.87}{30248257.37} = 0.4484 = 44.84\%$$

The predominant drive mechanism of the reservoir is water and gas whose values are close. Hence, any of gas or water can be injected. Also, water alternating gas injection or simultaneous water and gas injection can be used for this field.

5.10 Combining Aquifer Models with Material Balance Equation (MBE)

One of the advantages of Carter-Tracy's model over Van Everdingen-Hurst model is that; it does not require superposition and can be easily combined with MBE. Thus, Carter-Tracy's model is combined with undersaturated MBE as follows:

$$W_e\left(t_{Dj}\right) = W_e\left(t_{Dj-1}\right) + \left[\frac{C\Delta P\left(t_{Dj}\right) - W_e\left(t_{Dj-1}\right)P_D{}'\left(t_{Dj}\right)}{P_D\left(t_{Dj}\right) - \left(t_{Dj-1}\right)P_D{}'\left(t_{Dj}\right)}\right]\left(t_{Dj} - t_{Dj-1}\right)$$

$$N = \frac{N_pB_o + W_pB_w - W_eB_w}{B_{oi}C_{oe}\Delta P}$$

$$NB_{oi}C_{oe}\Delta P\left(t_{Dj}\right) = N_p\left(t_{Dj}\right)B_o + W_p\left(t_{Dj}\right)B_w - W_e\left(t_{Dj}\right)B_w$$

$$\textit{assume that } B_w = 1$$

$$NB_{oi}C_{oe}\Delta P\left(t_{Dj}\right) = N_p\left(t_{Dj}\right)B_o + W_p\left(t_{Dj}\right) - W_e\left(t_{Dj}\right)$$

$$B_o = B_{oi}(1 + C_o\Delta P)$$

$$NB_{oi}C_{oe}\Delta P\left(t_{Dj}\right) = N_p\left(t_{Dj}\right)\left[B_{oi}(1 + C_o\Delta P)\right] + W_p\left(t_{Dj}\right) - W_e\left(t_{Dj}\right)$$

Substituting the expression of $W_e(t_{Dj})$

$$\begin{aligned} NB_{oi}C_{oe}\Delta P\left(t_{Dj}\right) = {} & N_p\left(t_{Dj}\right)\left[B_{oi}\left(1 + C_o\Delta P\left(t_{Dj}\right)\right)\right] + W_p\left(t_{Dj}\right) \\ & - \left\{W_e\left(t_{Dj-1}\right) + \left[\frac{C\Delta P\left(t_{Dj}\right) - W_e\left(t_{Dj-1}\right)P_D{}'\left(t_{Dj}\right)}{P_D\left(t_{Dj}\right) - \left(t_{Dj-1}\right)P_D{}'\left(t_{Dj}\right)}\right]\left(t_{Dj} - t_{Dj-1}\right)\right\} \end{aligned}$$

$$NB_{oi}C_{oe}\Delta P(t_{Dj}) = N_p(t_{Dj})B_{oi} + N_p(t_{Dj})B_{oi}C_o\Delta P(t_{Dj}) + W_p(t_{Dj}) - W_e(t_{Dj-1}) - \left\{\frac{C\Delta P(t_{Dj})(t_{Dj} - t_{Dj-1}) - W_e(t_{Dj-1})P_D'(t_{Dj})(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right\}$$

$$NB_{oi}C_{oe}\Delta P(t_{Dj}) = N_p(t_{Dj})B_{oi} + N_p(t_{Dj})B_{oi}C_o\Delta P(t_{Dj}) + W_p(t_{Dj}) - W_e(t_{Dj-1}) - \frac{C\Delta P(t_{Dj})(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})} + \frac{W_e(t_{Dj-1})P_D'(t_{Dj})(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}$$

$$\Delta P(t_{Dj})\left\{NB_{oi}C_{oe} + N_p(t_{Dj})B_{oi}C_o + \frac{C(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right\} = N_p(t_{Dj})B_{oi} + W_p(t_{Dj}) - W_e(t_{Dj-1}) + \frac{W_e(t_{Dj-1})P_D'(t_{Dj})(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}$$

Consider the right hand side of the above equation

$$= N_p(t_{Dj})B_{oi} + W_p(t_{Dj}) - W_e(t_{Dj-1})\left\{1 - \frac{P_D'(t_{Dj})(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right\}$$

$$= N_p(t_{Dj})B_{oi} + W_p(t_{Dj}) - W_e(t_{Dj-1})\left\{\frac{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj}) - P_D'(t_{Dj})(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right\}$$

$$= N_p(t_{Dj})B_{oi} + W_p(t_{Dj}) - W_e(t_{Dj-1})\left\{\frac{P_D(t_{Dj}) - \cancel{(t_{Dj-1})P_D'(t_{Dj})} - t_{Dj}P_D'(t_{Dj}) + \cancel{(t_{Dj-1})P_D'(t_{Dj})}}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right\}$$

$$= N_p(t_{Dj})B_{oi} + W_p(t_{Dj}) - W_e(t_{Dj-1})\left\{\frac{P_D(t_{Dj}) - t_{Dj}P_D'(t_{Dj})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right\}$$

Therefore, combining both equations, the pressure drop is given as:

$$\boldsymbol{\Delta P(t_{Dj}) = \frac{N_p(t_{Dj})B_{oi} + W_p(t_{Dj}) - W_e(t_{Dj-1})\left\{\frac{P_D(t_{Dj}) - t_{Dj}P_D'(t_{Dj})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}\right\}}{NB_{oi}C_{oe} + N_p(t_{Dj})B_{oi}C_o + \frac{C(t_{Dj} - t_{Dj-1})}{P_D(t_{Dj}) - (t_{Dj-1})P_D'(t_{Dj})}}}$$

Example 5.10
Given the following data:

Initial reservoir pressure = 4000 psi	Bubble point pressure = 1500 psi
Initial oil FVF = 1.324 rb/stb	Porosity = 0.23
Oil FVF = 1.332 rb/stb	Viscosity = 0.32
Reservoir thickness = 90 ft	Reservoir area = 1500 acres
Oil rate = 29,000 stb/day	Water FVF = 1.03 rb/stb
Oil compressibility = 1.5×10^{-5} psi^{-1}	Permeability = 150 mD
Connate water saturation = 0.25	Formation compressibility = 3.4×10^{-6} psi^{-1}
STOIIP = 148 MMstb	Water compressibility = 3.5×10^{-6} psi^{-1}

Use Carter-Tracy method to calculate the pressure drop and aquifer influx at year 1 and 2 respectively assuming there is a cumulative water of 480,570 stb and 561,802 stb at year 1 and 2 respectively.

Solution

Step 1: Calculate the reservoir radius

$$r_e = \sqrt{\frac{43560A}{\pi}} = \sqrt{\frac{43560*1500}{3.14159}} = 4560.52\,ft$$

Step 2: Calculate the aquifer influx constant, C

Assume the aquifer angle to be 360^0

$$C = 1.119 f \o h c_{tw} {r_e}^2 = \quad 1.119*0.23*90*\left(3.5*10^{-6} + 3.4*10^{-6}\right)*(4560.52)^2$$
$$= 3324.14\ rb/psi$$

Step 3: Calculate the effective oil compressibility

$$S_o = 1 - S_{wi} = 1 - 0.25 = 0.75$$

$$C_{oe} = \frac{C_o S_o + S_{wi} C_w + C_f}{1 - S_{wi}}$$

$$C_{oe} = \frac{\left(1.5*10^{-5}*0.75\right) + \left(3.5*10^{-6}*0.25\right) + \left(3.4*10^{-6}\right)}{1 - 0.25} = 2.07*10^{-5}$$

Step 4: Calculate the dimensionless time

$$t_{Dj} = \frac{2.309kt}{\mu_w \O_w c_{tw} {r_e}^2}\ (t\ in\ years)$$

At t = 1 yr

$$t_{D1} = \frac{2.309*150*1}{0.32*0.23*\left(3.5*10^{-6}+3.4*10^{-6}\right)*(4560.52)^2} = 32.79$$

At t = 2 yrs

$$t_{D2} = \frac{2.309*150*2}{0.32*0.23*\left(3.5*10^{-6}+3.4*10^{-6}\right)*(4560.52)^2} = 65.58$$

Step 5: Calculate the dimensionless pressures

Based on the criteria given above,

$$0.01 < t_D < 500$$

$$P_D(t_D) = \frac{370.529\sqrt{t_D} + 137.582t_D + 5.69549{t_D}^{1.5}}{328.834 + 265.488\sqrt{t_D} + 45.2157t_D + {t_D}^{1.5}}$$

$$P_D(t_{D1}) = \frac{\left(370.529\sqrt{32.79}\right) + (137.582*32.79) + \left(5.69549*\{32.79\}^{1.5}\right)}{328.834 + \left(265.488\sqrt{32.79}\right) + (45.2157*32.79) + (32.79)^{1.5}}$$
$$= 2.1885$$

$$P_D(t_{D2}) = \frac{\left(370.529\sqrt{65.58}\right) + (137.582*65.58) + \left(5.69549*\{65.58\}^{1.5}\right)}{328.834 + \left(265.488\sqrt{65.58}\right) + (45.2157*65.58) + (65.58)^{1.5}}$$
$$= 2.5184$$

Step 6: Calculate the dimensionless pressure derivatives

$$P_D'(t_D) = \frac{716.441 + 46.7984\sqrt{t_D} + 270.038t_D + 71.0098{t_D}^{1.5}}{1269.86\sqrt{t_D} + 1204.73t_D + 618.618{t_D}^{1.5} + 538.072{t_D}^2 + 142.41{t_D}^{2.5}}$$

$$P_D'(t_{D1}) = \frac{\begin{array}{c}716.441 + \left(46.7984\sqrt{32.79}\right) + (270.038*32.79) \\ + \left(71.0098*(32.79)^{1.5}\right)\end{array}}{\begin{array}{c}\left(1269.86\sqrt{32.79}\right) + (1204.73*32.79) + \left(618.618*(32.79)^{1.5}\right) + \\ \left(538.072*(32.79)^{2}\right) + \left(142.41*(32.79)^{2.5}\right)\end{array}}$$
$$= 0.01433$$

$$P_D{'}(t_{D2}) = \frac{716.441 + \left(46.7984\sqrt{65.58}\right) + (270.038*65.58) + \left(71.0098*(65.58)^{1.5}\right)}{\left(1269.86\sqrt{65.58}\right) + (1204.73*65.58) + \left(618.618*(65.58)^{1.5}\right) + \left(538.072*(65.58)^{2}\right) + \left(142.41*(65.58)^{2.5}\right)}$$

$$= 7.3508*10^{-3} = 0.00735$$

Step 7: Convert the average oil rate to cumulative oil production

$$N_p = q\Delta t$$

$$N_{p@t_{D1}} = 29000\,\frac{stb}{day}*1\,yr*\frac{356\,days}{1yr} = 10585000\,stb$$

$$N_{p@t_{D2}} = 29000\,\frac{stb}{day}*2\,yr*\frac{356\,days}{1yr} = 21170000\,stb$$

Step 8: Calculate the pressure drops and water influx

At t = 1 yr

$$\Delta P(t_{D1}) = \frac{N_p(t_{D1})B_{oi} + W_p(t_{D1}) - W_e(t_{D0})\left\{\frac{P_D(t_{D1}) - t_{D1}P_D{'}(t_{D1})}{P_D(t_{D1}) - (t_{D0})P_D{'}(t_{D1})}\right\}}{NB_{oi}C_{oe} + N_p(t_{D1})B_{oi}C_o + \frac{C(t_{D1} - t_{D0})}{P_D(t_{D1}) - (t_{D0})P_D{'}(t_{D1})}}$$

$$\Delta P(t_{D1}) = \frac{(10585000*1.324) + 480570 - 0\left\{\frac{2.1885 - (32.79*0.01433)}{2.1885 - (0*0.01433)}\right\}}{\left(148*10^6*1.324*2.07*10^{-5}\right) + \left(10585000*1.324*1.5*10^{-5}\right) + \frac{3324.14*(32.79 - 0)}{2.1885 - (0*0.01433)}}$$

$$= 287.48\,psi$$

$$W_e(t_{D1}) = W_e(t_{D0}) + \left[\frac{C\Delta P(t_{D1}) - W_e(t_{D0})P_D{'}(t_{D1})}{P_D(t_{D1}) - (t_{D0})P_D{'}(t_{D1})}\right](t_{D1} - t_{D0})$$

$$W_e(t_{D1}) = 0 + \left[\frac{(3324.14*287.48) - (0*0.01433)}{2.1885 - (0*0.01433)}\right](32.79 - 0)$$

$$= 14317981.87\,bbl$$

At t = 2 yrs

$$\Delta P(t_{D2}) = \frac{N_p(t_{D2})B_{oi} + W_p(t_{D2}) - W_e(t_{D1})\left\{\frac{P_D(t_{D2}) - t_{D2}P_D{}'(t_{D2})}{P_D(t_{D2}) - (t_{D1})P_D{}'(t_{D2})}\right\}}{NB_{oi}C_{oe} + N_p(t_{D2})B_{oi}C_o + \frac{C(t_{D2} - t_{D1})}{P_D(t_{D2}) - (t_{D1})P_D{}'(t_{D2})}}$$

$$\Delta P(t_{D2}) = \frac{(21170000*1.324) + 561802 - 14317981.87\left\{\frac{2.5184 - (65.58*0.00735)}{2.5184 - (32.79*0.00735)}\right\}}{\left(148*10^6*1.324*2.07*10^{-5}\right) + \left(10585000*1.324*1.5*10^{-5}\right) + \frac{3324.14*(65.58 - 32.79)}{2.5184 - (32.79*0.00735)}}$$

$$= 302.87\,psi$$

$$W_e(t_{D2}) = W_e(t_{D1}) + \left[\frac{C\Delta P(t_{D2}) - W_e(t_{D1})P_D{}'(t_{D2})}{P_D(t_{D2}) - (t_{D1})P_D{}'(t_{D2})}\right](t_{D2} - t_{D1})$$

$$W_e(t_{D2}) = 14317981.87 + \left[\frac{(3324.14*302.87) - (14317981.87*0.00735)}{2.5184 - (32.79*0.00735)}\right](65.58 - 32.79)$$

$$= 27298463.43\,bbl$$

Example 5.11

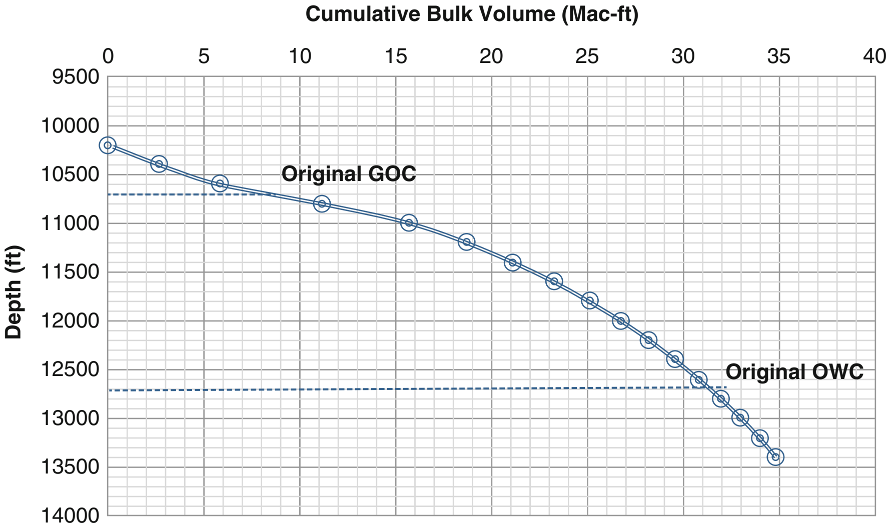
The hydrocarbon contents of a reservoir were determined from the data of cumulative bulk volume (CBV) at the indicated depths on the table below. Given the following petrophysical and PVT parameters: the gas-oil contact (GOC) = 10700ftss; the oil-water contact (OWC) = 12700ftss (1 ac-ft = 7758.4bbls). Determine the present fluid contacts.

Initial reservoir pressure = 2740 psi	Current reservoir pressure = 2460 psi
Initial oil FVF = 1.3985 rb/stb	Initial gas exp. factor = 198.3 scf/cuft
Initial solution GOR = 643 scf/stb	Gas exp. factor = 178.14 scf/cuft
Oil FVF = 1.3578 rb/stb	Cum. Produced GOR = 800 scf/stb
Current solution GOR = 577.3 scf/stb	Gas/oil sand volume ratio = 0.3
Cum. Oil produced = 19.8 MMstb	Cum. Water produced = 3.3 MMstb
Connate water saturation = 21%	Formation compressibility = 3.5 × 10^{-6} psi^{-1}
STOIIP = 125 MMstb	Water compressibility = 3.5 × 10^{-6} psi^{-1}
Porosity = 25.4%	Sand/shale factor (F) = 0.75
Critical gas saturation = 5%	Residual oil-water saturation and oil-gas saturation = 22%

Depth versus cumulative bulk volume (CBV)

Depth (ft)	CBV (M ac-ft)
10,200	0
10,400	2.697
10,600	5.794
10,800	11.134
11,000	15.724
11,200	18.698
11,400	21.141
11,600	23.243
11,800	25.096
12,000	26.752
12,200	28.248
12,400	29.607
12,600	30.848
12,800	31.986
13,000	33.031
13,200	33.992
13,400	34.876

Solution



Step 1: Determine the bulk volume of the reservoir rock at each depth interval

Step 2: Make a plot of depth versus the bulk volume

Step 3: Calculate the cumulative water influx from the general material balance equation ($\boldsymbol{W_e}$)

Determine the expansion of the various zones in MMrb

$$B_{gi} = \frac{1}{5.615E_i} = \frac{1}{198.3*5.615} = 0.00089811\ rb/Scf$$

$$B_g = \frac{1}{5.615E} = \frac{1}{178.14*5.615} = 0.00099974\ rb/Scf$$

Expansion of original oil plus dissolved gas (oil zone expansion)

$$= N\{[B_o - B_{oi}] + [R_{si} - R_s]B_g\}$$

$$= 125*10^6\{[1.3578 - 1.3985] + [\,643 - 577.3]0.00099974\} = 3122864.75\ rb$$

Expansion of primary gas cap

$$= mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

$$= 0.3*125*10^6*1.3985\left[\left(\frac{0.00099974}{0.00089811}\right) - 1\right] = 5934527.299\ rb$$

Expansion of connate water and decrease in pore volume

$$= [1+m]\frac{NB_{oi}}{1-S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P$$

$$= [1+0.3]\frac{125*10^6*1.3985}{1-0.21}\left[3.5*10^{-6} + \left(3.5*10^{-6}*0.21\right)\right](2740 - 2460)$$

$$= 341114.5079\ rb$$

Total underground withdrawal

$$N_p\{B_o + [R_p - R_s]B_g\} + W_pB_w$$

$$= 19.8*10^6\{1.3578 + [\,800 - 577.3]0.00099974\} + 3.3*10^6*1$$
$$= 34592753.54\ rb$$

Hydrocarbon Voidage

$$N_p\{B_o + [R_p - R_s]B_g\}$$

$$= 19.8*10^6\{1.3578 + [\,800 - 577.3]0.00099974\} = 31292753.54\ rb$$

Water influx

$$\begin{aligned} W_e = N_p\{B_o + [R_p - R_s]B_g\} + W_pB_w - mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] \\ - N\{[B_o - B_{oi}] - [R_{si} - R_s]B_g\} \\ -[1+m]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P \end{aligned}$$

$$\begin{aligned} W_e &= 34592753.54 \ - 5934527.299 - 3122864.75 - 341114.5079 \\ &= 25194246.98\ rb \end{aligned}$$

Step 4: Calculate the volume of oil displaced by water (Net water influx into the reservoir) ($\boldsymbol{O_W = W_e - W_p}$)

Net water influx

$$= W_e - W_p = 25194246.98 - 3.3*10^6 = 21894246.98\ \ rb$$

Step 5: Calculate the reservoir volume liberated gas ($\boldsymbol{G_L}$)

$$G_L = [NR_{si} - (N - N_p)R_s]B_g$$

$$\begin{aligned} &= [125*10^6(643) - (125*10^6 - 19.8*10^6)577.3\ ]0.00099974 \\ &= 19637932.81\ rb \end{aligned}$$

Step 6: Calculate the expansion of the primary gas cap ($\boldsymbol{G_e}$)

$$G_e = mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

$$= 0.3*125*10^6*1.3985\left[\left(\frac{0.00099974}{0.00089811}\right) - 1\right] = 5934527.299\ rb$$

Step 7: Calculate the gas drive ($\boldsymbol{G_D}$)

$$G_D = G_L + G_e = 19637932.81 + 5934527.299 = 25572460.11\ rb$$

Step 8: Calculate the produced excess gas ($\boldsymbol{G_{pe}}$)

$$G_{pe} = [N_p(R_p - R_s)]B_g$$

$$= 19.8*10^6[\ 800 - 577.3]0.00099974 = 4408313.54\ rb$$

Step 9: Calculate the volume of oil displaced by the gas ($\boldsymbol{O_g}$)

$$O_G = G_D - G_{pe}$$

$$= 25572460.11 - 4408313.54 = 21164146.57\ rb$$

Note the following reservoir conditions:
If O_G is negative (−ve), then oil has moved into the primary gas cap
If $G_{pe} > G_L$ then the gas cap is produced.
Therefore, neither of these conditions occurred.

Step 10: Calculate the dispersed gas in the oil zone (***G_{disp}***)

$$G_{disp} = S_{gc}\left[\frac{(N - N_p)B_o}{(1 - S_{wc})}\right]$$

$$G_{disp} = 0.05\left[\frac{(125*10^6 - 19.8*10^6)1.3578}{1 - 0.21}\right] = 9040541.772\ rb$$

Note: if $G_{disp} > G_L,\ reduce\ S_{gc}.$
Thus, condition did not hold.

Step 11: Calculate the volume of oil displaced by the primary and secondary gas cap (O_{GPS})

$$O_{GPS} = O_G - G_{disp}$$

$$O_{GPS} = 21164146.57 - 9040541.772 = 12123604.8\ rb$$

Step 12: Calculate the gross oil sand volume flooded by water is given as:

$$GOV_w = \frac{W_e - W_p}{7758.4*ø*F(1 - s_{wc} - s_{orw} - S_{gc})} \quad (ac - ft)$$

$$GOV_w = \frac{21894246.98}{7758.4*0.254*0.75(1 - 0.21 - 0.22 - 0.05)} = 28487.840\ rb$$

Step 12: Calculate the gross oil sand volume displaced by the primary and secondary gas cap given as:

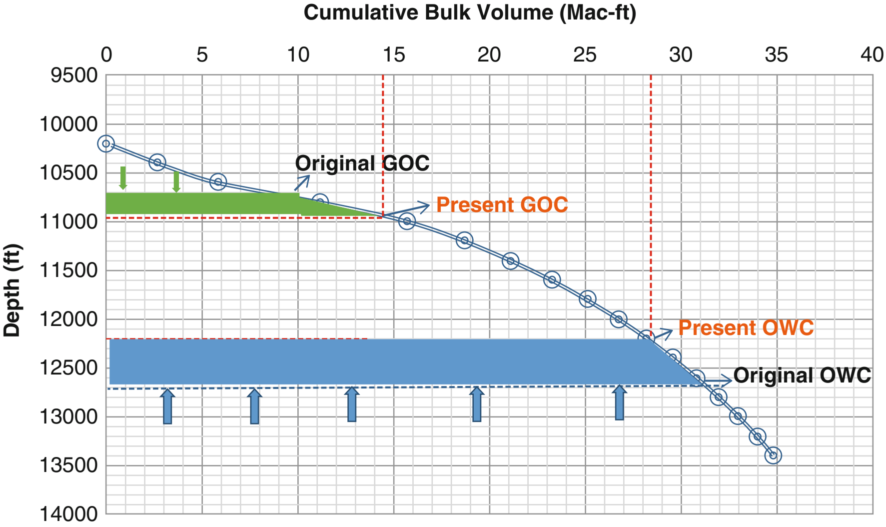
$$GOV_g = \frac{O_{GPS}}{7758.4*ø*F(1 - s_{wc} - s_{org})}$$

$$GOV_g = \frac{12123604.8}{7758.4*0.254*0.75(1-0.21-0.22)} = 14380.96\ rb$$

Step 13: Determine the present fluid contacts as follows:

Trace the value of GOV_w & GOV_g from the horizontal axis of the cumulative bulk volume plot in stage 2 to touch the curve and then read off the depth at the corresponding values. {i.e. the depth corresponding to GOV_g (14.38 *Mrb*) = *present GOC* and the depth corresponding to GOV_w (28.49 *Mrb*) = *OWC*}.

From the graph, ***present GOC* = 10942 *ft and present OWC* = 12200 *ft***



Exercises

1. How do we improve the recovery of a combination drive reservoir?
2. STOIIP is:
3. Total underground voidage is:
4. The ratio of gross gas sand volume to the gross oil sand volume is called
5. Explain gas cap size in terms of gross gas and oil sand volume
6. Free gas initially in place is:
7. In a saturated reservoir, how do you evaluate the gas initially in place?
8. Hydrocarbon voidage is:

   Write the mathematical equation for the following in reservoir condition:
9. Hydrocarbon pore volume:
10. Total underground withdrawal:

11. Hydrocarbon voidage:
12. The free/liberated gas in the reservoir:
13. The original gas expansion:
14. The volume of the water in the pore
15. The total change in pore volume:
16. The denominator in the calculation of drive indices in material balance is called:
17. Which concept is material balance based on and state the expression
18. Which of the flow geometry can a material balance method be applied
19. Mention five parameters and their sources required in performing material balance equation

S/N	Parameter	Sources

20. Explain how material balance can be used to estimation reservoir pressure from historical production and/or injection schedule:
21. Explain why the volume of gas original in place at reservoir conditions is equal to the volume of gas remaining in the reservoir at the new pressure-temperature conditions after some amount of gas have been produced.

Ex 5.1 The following information on a water-drive gas reservoir is given:

$$P_i = 3960\,psia,\ \ P = 3180\,psia,\ \ T = 150^0F,\ \ \emptyset = 18\%,\ \ s_{wc} = 24\%,\ \ \gamma_g = 0.63 \quad W_p@surface = 12.3\,MMbbl,\ \ G_p = 740.25\,MMscf,\ \ bulk\ volume = 97823.72\,acre-ft$$

Calculate the cumulative water influx

Ex 5.2 A reservoir with temperature of 230 °F, gas gravity of 0.65, the reservoir contain 85,000 acre-ft (bulk volume), porosity is 0.19, and connate water saturation is 0.27. If the reservoir pressure has declined from 3500 to 2750 psia while producing 25.8 MMMscf of gas with no water production to date. Estimate the barrels of water influx.

Ex 5.3 A 1100 acres volumetric gas reservoir is characterized with temperature of 170 °F, reservoir thickness of 50 ft., average porosity of 0.15, initial water saturation of 0.39. The 5 years production history is represented in the table below

Time (yrs)	Reservoir Pressure (psia)	Compressibility factor, z	Cum. Gas Production Gp (MMMscf)
0	2180	0.7589	0.00
1	1985	0.7651	6.96
2	1720	0.7894	14.82
3	1308	0.8232	23.50
4	985	0.8672	32.05
5	650	0.9030	36.84

- Estimate the gas initial in place
- Estimate the recoverable reserve at an abandonment pressure of 600 psi. Assume the compressibility factor at 600 psi to be equal to one
- What is the recovery factor at the abandonment pressure of 600 psia?

Ex 5.4 Calculate the total hydrocarbon withdrawal and the drive indices for a reservoir having the following production and fluid data:

Initial reservoir pressure = 2740 psi	Current reservoir pressure = 2460 psi
Initial oil FVF = 1.3985 rb/stb	Initial gas exp. factor = 198.3 scf/cuft
Initial solution GOR = 643 scf/stb	Gas exp. factor = 178.14 scf/cuft
Oil FVF = 1.3578 rb/stb	Cum. Produced GOR = 820 scf/stb
Current solution GOR = 577.3 scf/stb	Gas/oil sand volume ratio = 0.7
Cum. Oil produced = 18.9 MMstb	Cum. Water produced = 3.3 MMstb
Connate water saturation = 17%	Formation compressibility = 7.0×10^{-6} psi^{-1}
STOIIP = 120 MMstb	Water compressibility = 3.0×10^{-6} psi^{-1}

Ex 5.5 Given the following data:

Initial reservoir pressure = 3800 psi	Bubble point pressure = 1450 psi
Initial oil FVF = 1.124 rb/stb	Porosity = 0.23
Oil FVF = 1.132 rb/stb	Viscosity = 0.425
Reservoir thickness = 100 ft	Reservoir area = 1500 acres
Oil rate = 30,800 stb/day	Water FVF = 1.03 rb/stb
Oil compressibility = 1.65×10^{-5} psi^{-1}	Permeability = 162 mD
Connate water saturation = 0.24	Formation compressibility = 3.6×10^{-6} psi^{-1}
STOIIP = 137 MMstb	Water compressibility = 3.7×10^{-6} psi^{-1}

Combine the Carter-Tracy method with material balance equation to calculate the pressure drop and aquifer influx at year 1, 2, 3, 4 & 5 respectively assuming there is a cumulative water of 499,573 stb, 5,984,805 stb, 7826801.82 stb, 8907579.17 stb and 10184573.59 stb at year 1, 2, 3, 4 and 5 respectively.

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Chapter 6
Linear Form of Material Balance Equation

Learning Objectives

Upon completion of this chapter, students/readers should be able to:

- Reduce the general material balance equation to a straight line form
- Briefly describe the diagnostics plot to determine the presence of aquifer, the strength of the aquifer if present.
- Represent the material balance equation in a straight line form for an undersaturated reservoir without water influx
- Represent the material balance equation in a straight line form for an undersaturated reservoir with water influx
- Represent the material balance equation in a straight line form for a saturated reservoir without water drive
- Represent the material balance equation in a straight line form for a saturated reservoir with water drive
- Represent the material balance equation in a straight line form for gas cap drive reservoir
- Represent the material balance equation in a straight line form for combination drive reservoir
- Perform calculations in the various reservoir scenarios to determine stock tank oil initially in place, gas initially in place and gas cap size.

Nomenclature

Parameter	*Symbol*	*Unit*
Initial gas formation volume factor	β_{gi}	*cuft/scf*
Gas formation volume factor	β_g	*cuft/scf*
Cumulative water influx	W_e	*bbl*
Cumulative water produced	W_p	*bbll*

(continued)

S. Okotie, B. Ikporo, *Reservoir Engineering*,
https://doi.org/10.1007/978-3-030-02393-5_6

Parameter	*Symbol*	*Unit*
Cumulative gas produced	G_p	*scf*
Cumulative oil produced	N_p	*Stb*
Stock tank oil initially In place	N	*stb*
Stock tank gas initially in place	G	*scf*
Initial solution gas-oil ratio	R_{si}	*scf/stb*
Solution gas-oil ratio	R_s	*scf/stb*
Cumulative produce gas-oil ratio	R_p	*Scf/stb*
Bottom hole (wellbore) flowing pressure	P_{wf}	*psia*
Initial reservoir pressure	P_i	*psia*
Oil formation volume factor	β_o	*rb/stb*
Initial oil formation volume factor	β_{oi}	*rb/stb*
Water formation volume factor	β_w	*rb/stb*
Gas formation volume factor	β_g	*cuft/scf*
Initial gas formation volume factor	β_{gi}	*cuft/scf*
Reservoir temperature	T	$^\circ R$
Total fluid compressibility	C_t	$psia^{-1}$
Oil isothermal compressibility	C_o	$psia^{-1}$
Effective oil isothermal compressibility	C_{oe}	$psia^{-1}$
Water & rock compressibility	C_w & C_r	$psia^{-1}$
Gas deviation factor at depletion pressure	z	–
Gas/oil sand volume ratio or gas cap size	m	–
Connate & initial water saturation	S_{wi} & S_{wc}	*- or %*
Residual gas saturation to water displacement	S_{grw}	*- or %*
Residual oil-water saturation	S_{orw}	-
Pore volume of water-invaded zone	PV_{water}	ft^3
Reservoir pore volume	PV	ft^3
Flow rate	q	*stb/d*
Viscosity	μ	*cp*
Formation permeability	k	*mD*
Reservoir thickness	h	*ft*
Area of reservoir	A	*acres*
Wellbore radius	r_w	*ft*
Recovery factor	RF	*%*
Pressure drop	ΔP	*psi*
Initial & current gas expansion factor	E_i & E	*scf/cuft*

6.1 Introduction

The material balance equation is a complex equation for calculating the original oil in place, cumulative water influx and the original size of the gas cap as compared to the oil zone size. This complexity prompted Havlena and Odeh to express the MBE in a straight line form. This involves rearranging the MBE into a linear equation. The straight lines method requires the plotting of a variable group against another variable group selected, depending on the reservoir drive mechanism and if linear relationship does not exist, then this deviation suggests that reservoir is not performing as anticipated and other mechanisms are involved, which were not accounted for; but once linearity has been achieved, based on matching pressure and production data, then a mathematical model has been achieved. This technique of trying to match historic pressure and production rate is referred to as history matching. Thus, the application of the model to the future enables predictions of the future reservoir performance. To successfully develop this chapter, several textbooks and materials such Craft & Hawkins (1991), Dake (1994), Donnez (2010), Havlena & Odeh AS (1964), Numbere (1998), Pletcher (2002) and Steffensen (1992) were consulted.

The straight line method was first recognized by Van Everdigen et al. (2013) but with some reasons, it was never exploited. The straight line method considered the underground recoverable F, gas cap expansion function E_g, dissolved gas-oil expansion function E_o, connate water and rock contraction function $E_{f,w}$ as the variable for plotting by considering the cumulative production at each pressure.

Havlena and Odeh presented the material balance equation in a straight line form. These are presented below:

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right] + W_pB_w \Longrightarrow \text{Total underground withdrawal}$$

$$E_o = \left[(B_o - B_{oi}) + (R_{si} - R_S)B_g\right] \Longrightarrow \text{Oil and dissolved gas expansion}$$

$$E_g = B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right] \Longrightarrow \text{Gas cap gas expansion}$$

$$E_{f,w} = (1+m)B_{oi}\left[\frac{s_{wi}c_w + c_f}{1 - s_{wi}}\right]\Delta P \Longrightarrow \text{Connate \& rock expansion}$$

Using these terms, the material balance equation can be written as

$$F = N\left[E_o + mE_g + E_{f,w}\right] + W_eB_w$$

Including the injection terms

$$F = N\left[E_o + mE_g + E_{f,w}\right] + W_eB_w + W_{inj}B_{winj} + G_{inj}B_{ginj}$$

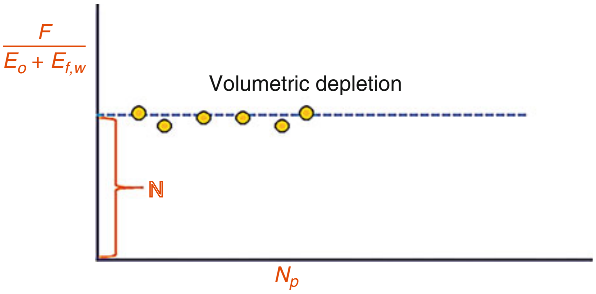
6.2 Diagnostic Plot

In evaluating the performance of a reservoir, there is need to adequately identify the type of reservoir in question based on the signature of pressure history or behaviour and the production trend. Campbell and Dake plots are the vital diagnostic tools employed to identify the reservoir type. The plots are established based on the assumption of a volumetric reservoir, and deviation from this behaviour is used to indicate the reservoir type.

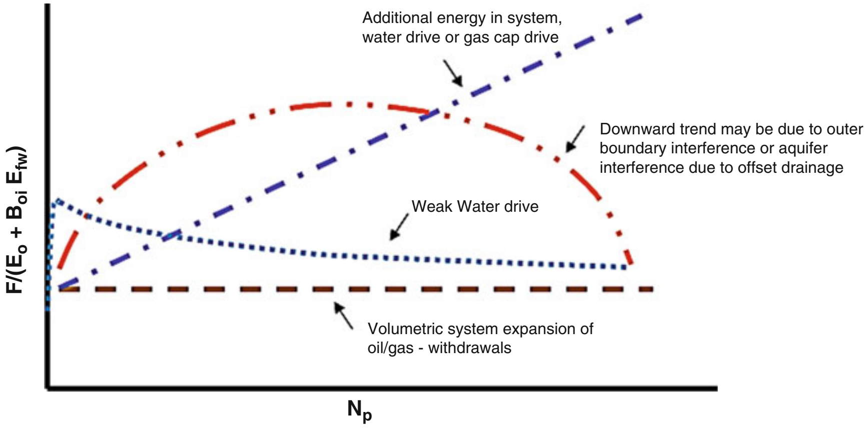
For volumetric reservoirs whose production is mainly by oil and connate water/rock expansion, the value of STOIIP, N can be calculated at every pressure where production data is given. Rearranging the material balance equation as shown below.

$$N = \frac{F}{E_o + E_{f,w}}$$

If a plot of cumulative oil production versus net withdrawal over the fluid expansions is created with a volumetric reservoir data, then the calculated values of STOIIP, N on the horizontal axis should be constant at all pressure points. In practice, this is often not the case either because there is water influx or because there may be faulty pressure or production readings.

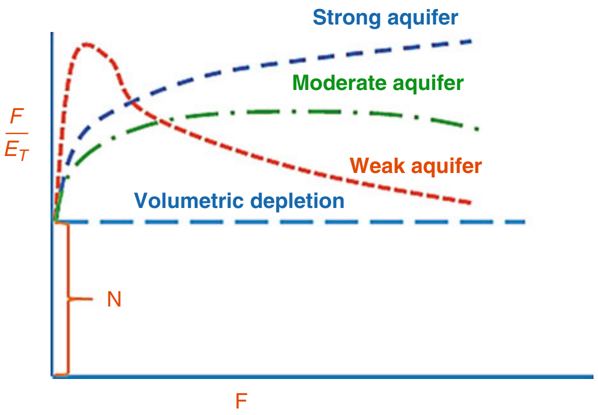


If a gas cap is present, there will be a gas expansion component in the reservoir's production. As production continues and the reservoir pressure decreased, the gas expansion term increases with an increase in the gas formation volume factor. To balance this, the withdrawal over oil/water/formation expansion term must also continue to increase. Thus, in the case of gas cap drive, the Dake plot will show a continual increasing trend.



Similarly, if water drive is present, the withdrawal over oil/water/formation expansion term must increase to balance the water influx. With a very strong aquifer, the water influx may continue to increase with time, while a limited or small aquifer may have an initial increase in water influx to the extent that it eventually decreases.

The Campbell plot is very similar to Dake's diagnostic tool, with an exception that it incorporates a gas cap if required. In the Campbell plot, the withdrawal is plotted against withdrawal over total expansion, while the water influx term is neglected. If there is no water influx, the data will plot as a horizontal line. If there is water influx into the reservoir, the withdrawal over total expansion term will increase proportionally to the water influx over total expansion. The Campbell plot can be more sensitive to the strength of the aquifer. In this version of the material balance, using only E_T neglects the water and formation compressibility (compaction) term. The Campbell plot is shown below.



6.3 The Linear Form of the Material Balance Equation

According to Tarek (2010), the linear form of MBE is presented in six scenarios to determine either m, N, G or W_e as follow:

- Undersaturated reservoir without water influx
- Undersaturated reservoir with water influx
- Saturated reservoir without water drive
- Saturated reservoir with water drive
- Gas cap drive reservoir
- Combination drive reservoir

6.3.1 Scenario 1: Undersaturated Reservoir Without Water Influx

Applying the above assumption, the equation reduces to

$$F = N\left[E_o + E_{f,w}\right]$$

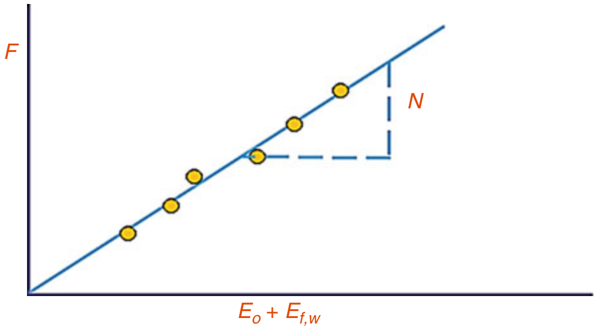
Where

$$F = N_p B_o$$

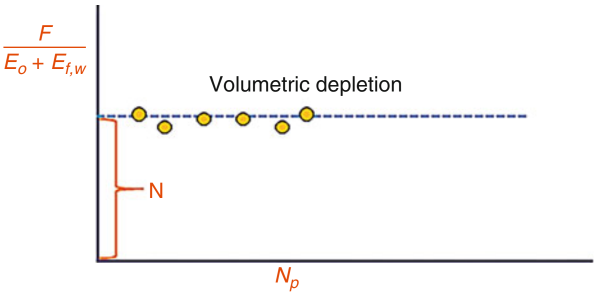
$$E_o = B_o - B_{oi}$$

$$E_{f,w} = (1 + m)B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P$$

A plot of F versus $E_o + E_{f,w}$ at every given pressure points gives a straight line that passes through the origin as shown below.



Alternatively, when $^{F}\big/_{E_o + E_{f,w}}$ $\mathit{versus}\ N_p$ is plotted at every given pressure points gives a horizontal line representing the value of N as shown below.



6.3.2 Scenario 2: Undersaturated Reservoir with Water Influx

Applying the above assumption, the equation reduces to

$$F = N\left[E_o + E_{f,w}\right] + W_e$$

$$\frac{F}{E_o + E_{f,w}} = N + \frac{W_e}{E_o + E_{f,w}}$$

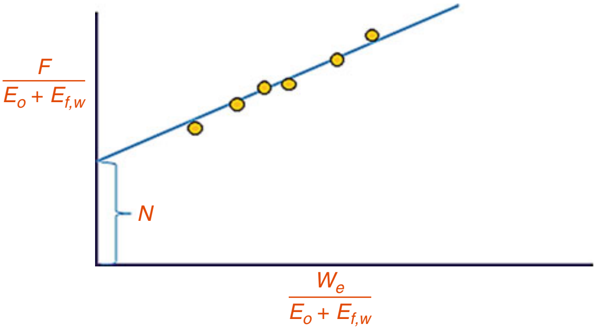
Where

$$F = N_p B_o + W_p B_w$$

$$E_o = B_o - B_{oi}$$

$$E_{f,w} = (1 + m)B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P$$

A plot of $^{F}\big/_{E_o+E_{f,w}}$ $\text{versus}\, ^{W_e}\!/_{E_o+E_{f,w}}$ gives N as the intercept and a slop of unit.



6.3.3 Scenario 3: Saturated Reservoir Without Water Influx

Applying the above assumption, the equation reduces to

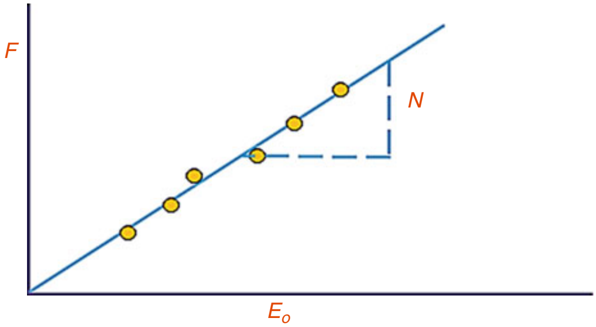
$$F = NE_o$$

Where

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right]$$

$$E_o = \left[(B_o - B_{oi}) + (R_{si} - R_S)\, B_g \right]$$

A plot of F versus E_o at every given pressure points gives a straight line that passes through the origin as shown below.



Example 6.1
Given the PVT and production data in the table below, detect if there is aquifer support and characterize the strength if there is any.

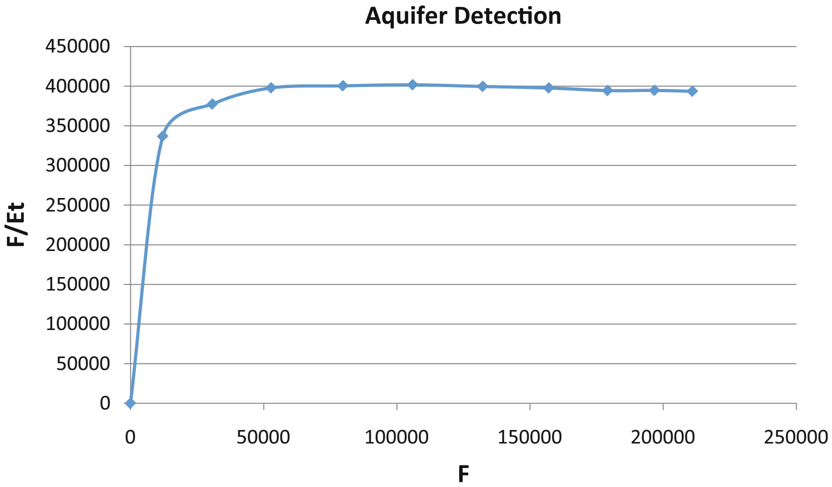
P (psia)	Rs (scf/STB)	Bo (rb/STB)	Bg (rb/STB)	Np (MMSTB)	Gp (MMSCF)	Rp
2740	650	1.404	0.0009	0	0	0
2500	592	1.374	0.001	7.88	5988.8	760
2290	545	1.349	0.0011	18.42	15565	845
2109	507	1.329	0.0012	29.15	26818	920
1949	471	1.316	0.0013	40.69	39673	975
1818	442	1.303	0.0014	50.14	51394	1025
1702	418	1.294	0.0015	58.42	62217	1065
1608	398	1.287	0.0016	65.39	71603	1095
1535	383	1.28	0.0017	70.74	79229	1120
1480	370	1.276	0.0018	74.54	85348	1145
1440	362	1.273	0.0018	77.43	89819	1160

Additional data:

Initial Pressure	2740	Cf	4.00E-06
Porosity	0.25	m	0.1
Swc	0.05	Cw	3.00E-06

Solution

P (psia)	Np (MMSTB)	E_o	E_g	E_f	E_t	F	F/E_t
2740	0	0	0	0	0	0	0
2500	7.88	0.0268	0.0075	0.0016	0.036	12124	336720
2290	18.42	0.0574	0.0211	0.003	0.0815	30761	377341
2109	29.15	0.0923	0.0362	0.0043	0.1328	52826	397787
1949	40.69	0.1411	0.0528	0.0053	0.1993	79798	400401
1818	50.14	0.1881	0.0694	0.0062	0.2638	105964	401707
1702	58.42	0.238	0.0861	0.007	0.3311	132292	399608
1608	65.39	0.2862	0.1011	0.0076	0.395	157081	397688
1535	70.74	0.3299	0.1162	0.0081	0.4543	179177	394425
1480	74.54	0.3648	0.1253	0.0085	0.4986	196786	394673
1440	77.43	0.3932	0.1344	0.0088	0.5363	211025	393488



From the above plot, it can be deduced that there is an aquifer support which corresponds to Campbell plot of moderate aquifer strength.

Example 6.2

Characterize the strength of the aquifer in the Akpet reservoir given the PVT and production data in the table below.

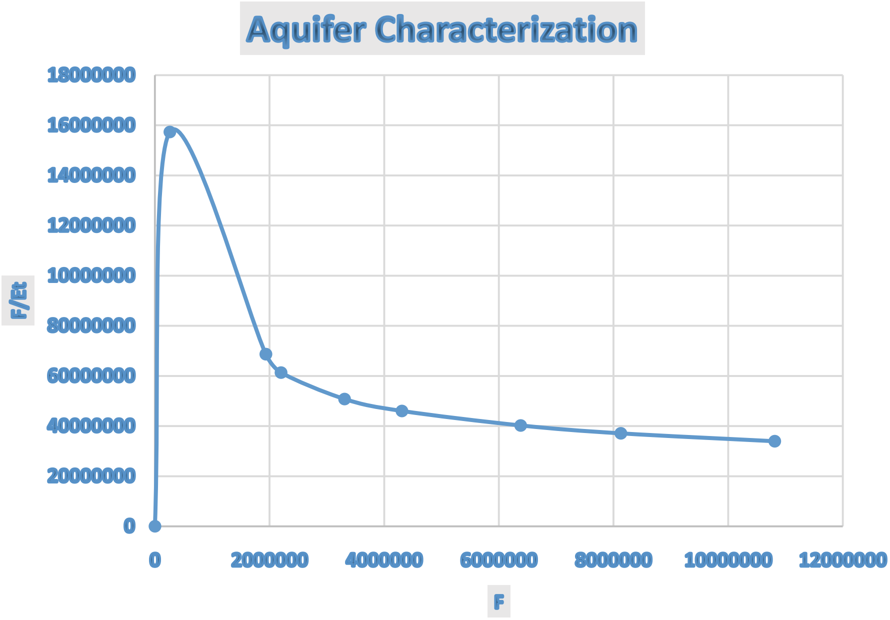
P (psia)	Np (STB)	Wp (STB)	Gp (Mscf)	Bo (RB/STB)	Rs (scf/STB)	Bg (RB/scf)	Bw (RB/STB)
3093	0	0	0	1.3101	504	0.000950	1.0334
3017	200671	0	98063	1.3113	504	0.000995	1.0336
2695	1322730	7	814420	1.2986	470.9	0.001133	1.0345
2640	1532250	10	894484	1.2942	461.2	0.001150	1.0346

(continued)

P (psia)	Np (STB)	Wp (STB)	Gp (Mscf)	Bo (RB/STB)	Rs (scf/STB)	Bg (RB/scf)	Bw (RB/STB)
2461	2170810	29	1359270	1.2809	430.7	0.001239	1.0350
2318	2579850	63	1826800	1.2700	406.2	0.001324	1.0353
2071	3208410	825	2736410	1.2489	361.7	0.001505	1.0359
1903	3592730	11138	3401290	1.2360	331.5	0.001663	1.0363
1698	4011570	97446	4222680	1.2208	294.6	0.001912	1.0367

Additional data:

Initial Pressure	3093 psi	Cf	2.28E-06 psi-1		
Porosity	0.25	m	0		
Swc	0.208	Cw	2.28E-06 psi-1		
Rp	**Eo**	**Ef,w**	**Et**	**F**	**F/et**
0	0	0	0	0	0
488.6755	0.0012	0.0004	0.0017	260083.2757	157350020.3479
615.7114	0.0260	0.0022	0.0282	1934674.4510	68677847.5603
583.7716	0.0334	0.0025	0.0359	2199143.5748	61328262.7213
626.1580	0.0617	0.0034	0.0651	3306508.8006	50788728.8326
708.1032	0.0894	0.0042	0.0936	4307427.3112	46015424.5980
852.8866	0.1530	0.0055	0.1586	6379828.9438	40232653.5191
946.7146	0.2128	0.0064	0.2192	8127367.5936	37076082.5825
1052.6253	0.3111	0.0076	0.3186	10812131.5160	33932660.4357



From the above plot, it can be deduced that there is an aquifer support which corresponds to Campbell plot of weak aquifer strength.

Example 6.3
The data in the table below represent a data from a saturated oil reservoir without an active water drive. Calculate the stock tank oil initially in place.

Time (yrs)	N_p (MMstb)	Bg (cuft/scf)	B_o (rb/stb)	R_s (cuft/stb)	R_p (cuft/stb)
0	0	0.00433	1.5533	719.9045	0
1	1.9891	0.00446	1.5440	702.4075	795.3195
2	7.0973	0.00466	1.5306	676.9572	860.8164
3	10.7186	0.00489	1.5168	650.6649	926.3133
4	18.5518	0.00525	1.4969	612.9574	954.3834
5	22.8154	0.00549	1.4854	591.0627	985.6700
6	28.0537	0.00581	1.4509	555.0749	1008.657
7	31.0359	0.00611	1.4201	525.0909	1041.227
8	34.5123	0.00641	1.3957	503.1039	1059.677

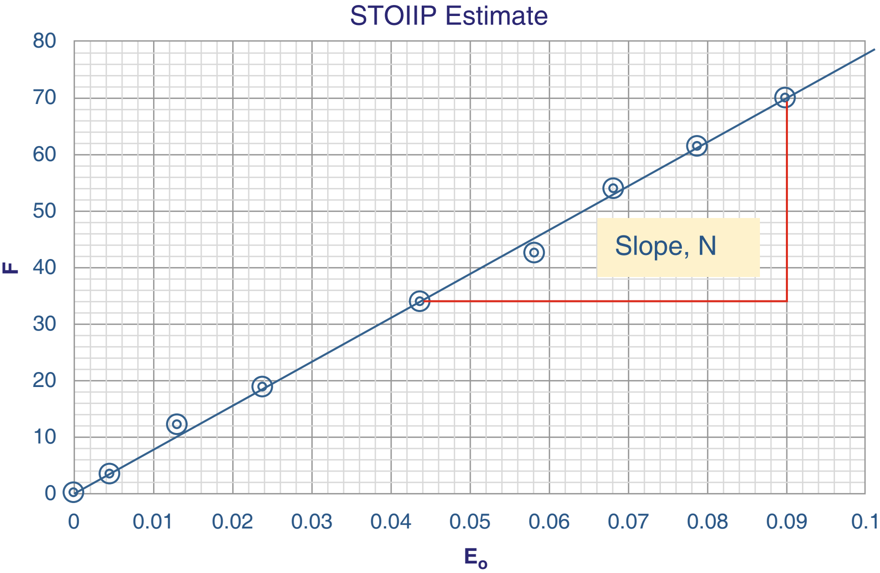
Solution
For unit consistency, B_g is converted from cuft/scf to rb/scf as:

$$1\ bbl = 5.615 cuft.$$

$$F = N_p\left[B_o + \left(R_p - R_s\right)^{B_g}/_{5.615}\right]$$

$$E_o = \left[\left(B_o - B_{oi}\right) + \left(R_{si} - R_S\right)\ ^{B_g}/_{5.615}\ \right]$$

Time (yrs)	F	E_o
0	0	0
1	3.2180	0.0046
2	11.9461	0.0129
3	18.8310	0.0238
4	33.6925	0.0436
5	42.6927	0.0581
6	53.8697	0.0682
7	61.5050	0.0788
8	70.0971	0.0899



$$Slope, N = \frac{70.0971 - 33.6925}{0.0899 - 0.0436} = 786.3954\ MMstb$$

6.3.4 Scenario 4: Saturated Reservoir with Water Influx

Applying the above assumption, the equation reduces to

$$F = NE_o + W_e$$

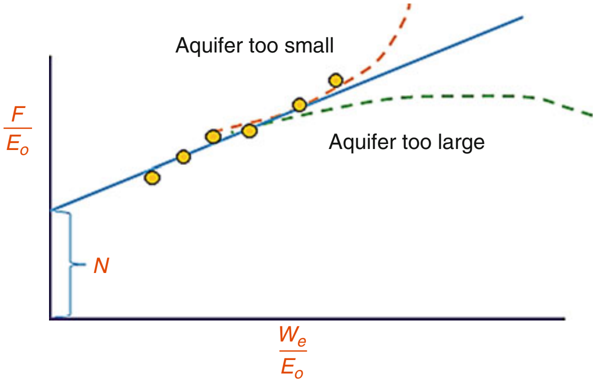
$$\frac{F}{E_o} = N + \frac{W_e}{E_o}$$

Where

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right] + W_pB_w$$

$$E_o = \left[(B_o - B_{oi}) + (R_{si} - R_S)\ B_g\ \right]$$

A plot of $^{F}/_{E_o}$ *versus* $^{W_e}/_{E_o}$ gives N as the intercept and a slope of unit



6.3.5 Scenario 5: Gas Cap Drive Reservoir

- Finding the STOIIP, N when the gas cap size, m is known
- Finding the gas cap size, m the STOIIP, N and the GIIP, G

Finding the STOIIP, N when the gas cap size, m is known

$$F = N\left[E_o + mE_g\right]$$

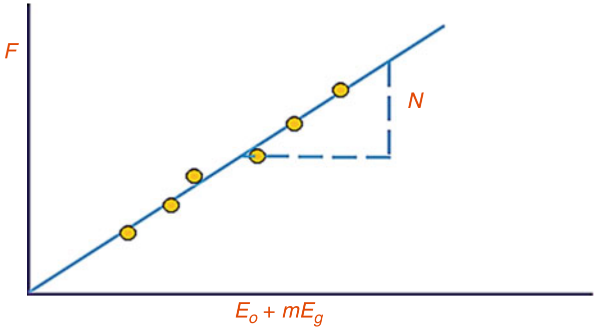
Where

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right]$$

$$E_o = \left[(B_o - B_{oi}) + (R_{si} - R_S)\, B_g\ \right]$$

$$E_g = B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

A plot of F versus $E_o + E_g$ at every given pressure points gives a straight line that passes through the origin as shown below.



Example 6.4

Find the STOIIP, N of a gas drive reservoir with a known gas cap size, m and supported by an aquifer. The tables are reservoir and production history of the field.

P_i	2560 psi	m	0.08	R_{si}	600 scf/stb	B_{gi}	0.001098 cuft/scf
B_{oi}	1.316 rb/stb	P_b	2560 psi	B_w	1.05 rb/stb		

Date	N_p (MMstb)	B_g (cuft/scf)	B_o (rb/stb)	R_s (cuft/stb)	R_p (cuft/stb)
Aug-93	21.456	0.00129	1.293	542	905
Aug-94	33.871	0.00138	1.278	498	898
Aug-95	41.871	0.00142	1.272	483	763
Aug-96	55.843	0.00148	1.267	473	659
Aug-97	71.78	0.00157	1.259	461	576
Aug-98	80.758	0.00161	1.254	452	518

Solution

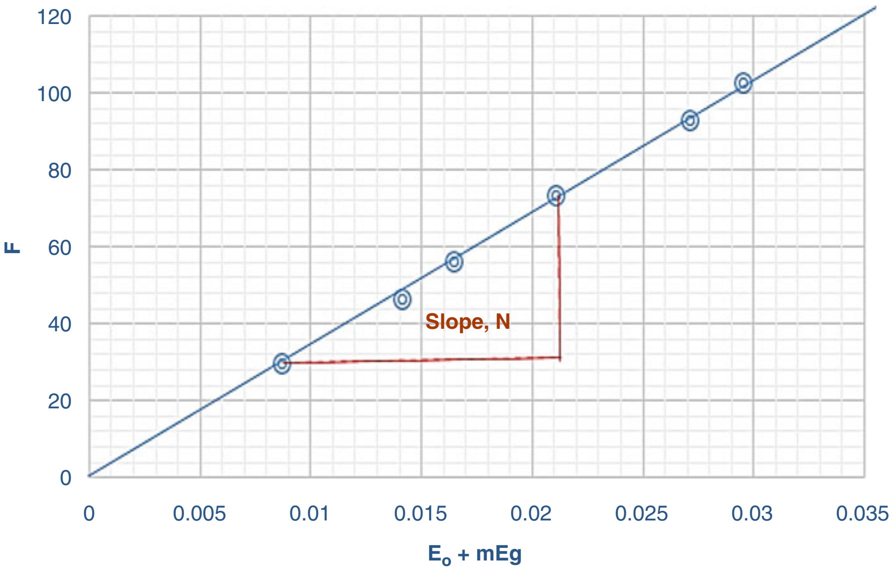
The table below is generated with the following equations:

$$F = N_p\left[B_o + \left(R_p - R_s\right)^{B_g}/_{5.615}\right]$$

$$E_o = \left[(B_o - B_{oi}) + \left(R_{si} - R_S\right)\,^{B_g}/_{5.615}\,\right]$$

$$E_g = B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

Date	F	E_o	mE_g	$E_o + mE_g$
Aug-93	29.53196	−0.00967	0.01841	0.008735
Aug-94	46.61693	−0.01293	0.027039	0.014108
Aug-95	56.22481	−0.01441	0.030874	0.016463
Aug-96	73.49083	−0.01553	0.036627	0.021102
Aug-97	92.6791	−0.01813	0.045257	0.027123
Aug-98	102.7988	−0.01956	0.049092	0.029529



From the plot of F Vs $E_o + E_g$, the STOIIP, N is given as the slope

$$Slope, N = \frac{73.49083 - 29.53196}{0.021102 - 0.008735} = 3554.401\ MMscf$$

Finding the gas cap size, m, the STOIIP, N and the GIIP, G

$$F = N\left[E_o + mE_g\right] = NE_o + mNE_g$$

$$\frac{F}{E_o} = N + mN\frac{E_g}{E_o}$$

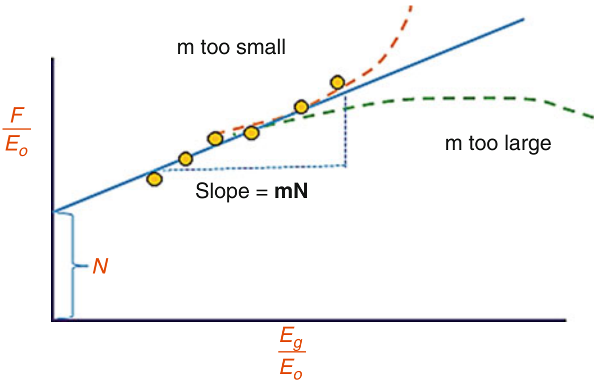
Where

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right]$$

$$E_o = \left[(B_o - B_{oi}) + (R_{si} - R_S)\ B_g\ \right]$$

$$E_g = B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

A plot of $^F/_{E_o}$ *versus* $^{E_g}/_{E_o}$ gives N as the intercept and a slop of unit

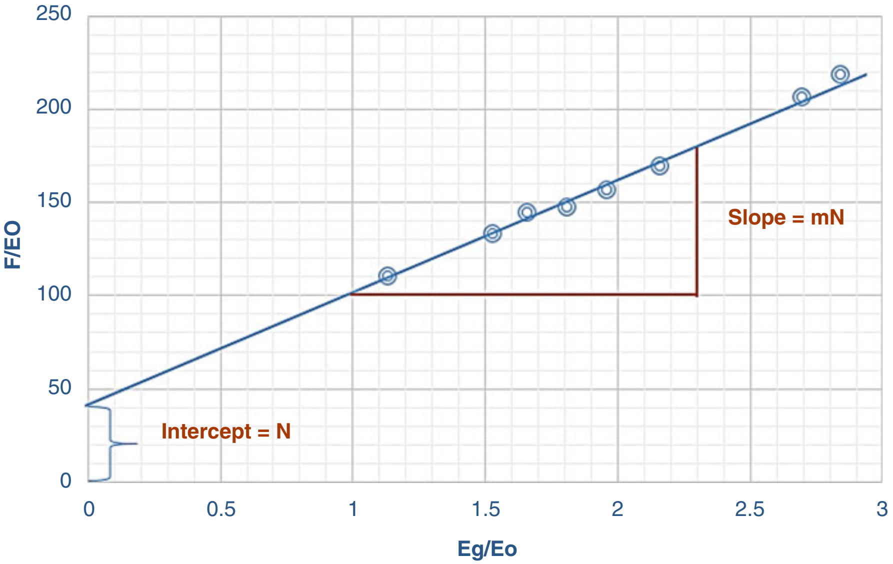


Example 6.5

Find the gas cap size, m, the STOIIP, N and the GIIP, G from the data given in the table below.

Pressure (psi)	Np (MMstb)	Bo (rb/stb)	Bg (rb/scf)	Rp (scf/stb)	Rs (scf/stb)
4200	0	1.3696	0.00095	0	640
3850	8.92	1.4698	0.00109	1249	568
3708	12.023	1.4542	0.00117	1261	535
3590	13.213	1.4423	0.00121	1370	513
3410	14.776	1.4304	0.00129	1469	477
3300	17.268	1.4185	0.00138	1505	446
2985	20.16	1.4056	0.0015	1547	419
2752	26.704	1.3827	0.00183	1645	403
2500	28.204	1.3603	0.00194	1666	362

Pressure (psi)	F	E_o	E_g	F/E_o	E_g/E_o
4200	0.0000	0.0000	0.0000	0.0000	0.0000
3850	19.7318	0.1787	0.2018	110.4312	1.1296
3708	27.6964	0.2075	0.3172	133.5089	1.5289
3590	32.7586	0.2264	0.3748	144.7126	1.6559
3410	40.0441	0.2711	0.4902	147.7262	1.8083
3300	49.7305	0.3166	0.6199	157.0667	1.9579
2985	62.4476	0.3675	0.7929	169.9255	2.1576
2752	97.6181	0.4468	1.2687	218.4778	2.8394
2500	109.7153	0.5300	1.4273	207.0021	2.6929



From the plot of F/E_o vs. E_g/E_o
The STOIIP (N) = 41 MMstb which is given as the intercept from the plot
Slope (Nm) is

$$= \frac{180 - 100}{2.3 - 1.0} = 61.5385\ MMstb$$

The gas cap size is

$$m = \frac{slope}{N} = \frac{61.5385}{41} = 1.5009 \approx 1.5$$

Recall

$$m = \frac{GB_{gi}}{NB_{oi}}$$

$$G = \frac{mNB_{oi}}{B_{gi}} = \frac{1.5009 * 41 * 1.3696}{0.00095} = 88716.7771 MMscf = 88.72 MMMscf$$

Therefore, the gas initially in place is = 88.72 MMMscf

6.3.6 Scenario 6: Combination Drive Reservoir

$$F = NE_t + W_e$$

$$\frac{F}{E_t} = N + \frac{W_e}{E_t}$$

Where

$$F = N_p\left[B_o + \left(R_p - R_s\right)B_g\right] + W_pB_w$$

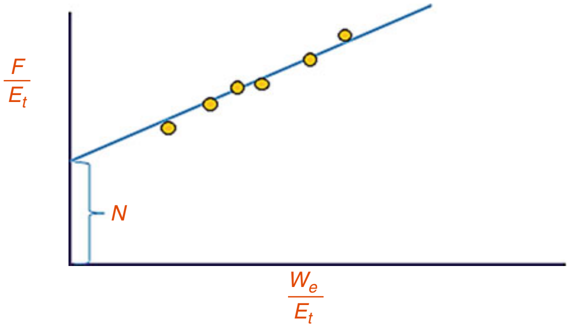
$$E_t = E_o + mE_g + E_{f,w}$$

$$E_o = \left[(B_o - B_{oi}) + (R_{si} - R_S)\ B_g\ \right]$$

$$E_g = B_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]$$

$$E_{f,w} = (1 + m)B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P$$

The plot is shown below



6.3.7 Linear Form of Gas Material Balance Equation

Havlena and Odeh also expressed the material balance equation in terms of gas production, fluid expansion and water influx as:

$$\begin{aligned} \textit{Total underground withdrawal} = {} & \textit{gas expansion} \\ & + \textit{water\&pore compaction expansion} \\ & + \textit{water influx} \end{aligned}$$

$$G_pB_g + W_pB_w = G\left(B_g - B_{gi}\right) + GB_{gi}\left[\frac{c_w s_{wi} + c_f}{1 - s_{wi}}\right]\Delta P + W_eB_w$$

$$F = G\left(E_g + E_{f,w}\right) + W_eB_w$$

Where

$$F = G_pB_g + W_pB_w$$

$$E_g = B_g - B_{gi}$$

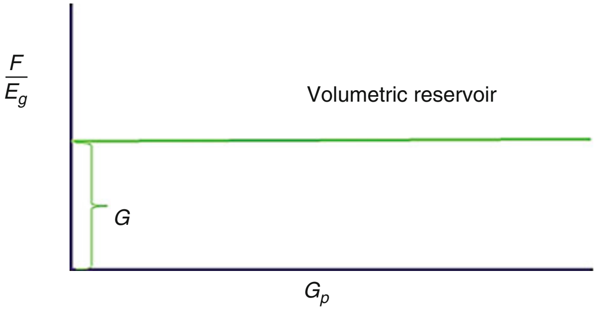
$$E_{f,w} = B_{gi}\left[\frac{c_w s_{wi} + c_f}{1 - s_{wi}}\right]\Delta P$$

Assume that the rock and water expansion term is negligible, the equation reduces to:

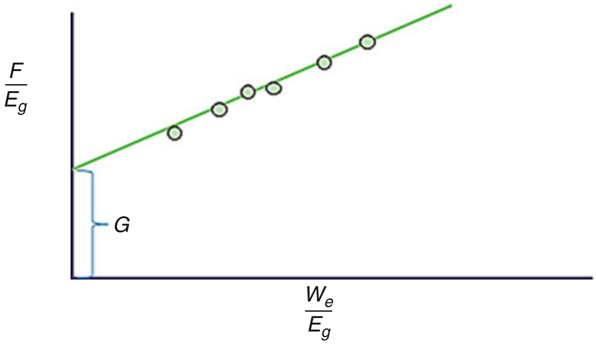
$$F = GE_g + W_eB_w$$

$$\frac{F}{E_g} = G + \frac{W_eB_w}{E_g}$$

A plot of $^F/_{E_g}$ *versus* G_p gives a horizontal line with G as the intercept



A plot of $^F/_{E_g}$ *versus* $^{W_e}/_{E_g}$ gives G as the intercept and a slop, B_w



Example 6.6
A gas field at Okuatata whose production history in the table below span for about 2 years, is currently producing without a water drive. The reservoir temperature is 160 °F. Assume that the gas deviation factor in the range of pressures given is 0.8. Calculate the original gas in the reservoir.

Time (months)	P (psia)	Gp (MMscf)
0	3500	0
6	3180	95.78
12	2805	231.89
18	2350	364.93
24	2000	498.16

Solution

$$F = GE_g$$

Where

$$F = G_p B_g$$

$$E_g = B_g - B_{gi}$$

To calculate the gas formation volume factor (See details in understanding the basis of rock and fluid properties by author), we apply the following equations:

If yg < = 0.7 Then

$$T_c = 168 + \left(325 * \gamma_g\right) - \left(12.5 * {\gamma_g}^2\right)$$

$$P_c = 677 + \left(15 * \gamma_g\right) - \left(37.5 * {\gamma_g}^2\right)$$

Else If yg > 0.7 Then

$$T_c = 187 + \left(330 * \gamma_g\right) - \left(75.5 * {\gamma_g}^2\right)$$

$$P_c = 706 - \left(51.7 * \gamma_g\right) - \left(11.1 * {\gamma_g}^2\right)$$

$$T_r = \frac{T}{T_c}$$

$$P_r = \frac{P}{P_c}$$

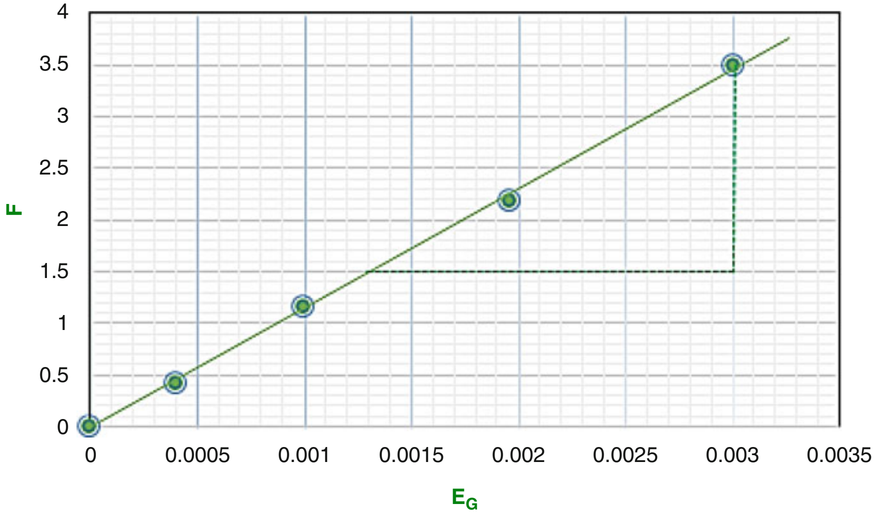
At this point, we can apply any of the correlations for gas deviation factor or read directly from the chart as a function of reduced temperature and pressure or use Papay's Correlation given as:

$$z = 1 - \left[\frac{3.52 P_r}{10^{0.9813 T_r}}\right] + \left[\frac{0.274 {P_r}^2}{10^{0.8157 T_r}}\right]$$

$$B_g = \frac{0\ 02827 zT}{P} \left({}^{ft^3}/_{scf}\right)$$

$$B_g = \frac{0\ 005053 zT}{P} \left({}^{bbl}/_{scf}\right)$$

Time (months)	P	G_p	B_g	E_g	F	F/E_g
0	3500	0	0.004011	0	0	0
6	3180	95.78	0.004414	0.000403	0.42277	1049.064
12	2805	231.89	0.005004	0.000993	1.16038	1168.557
18	2350	364.93	0.005973	0.001962	2.17973	1110.972
24	2000	498.16	0.007018	0.003007	3.49609	1162.649



The original gas in the reservoir is given as the slope of the plot of F Vs Eg

$$slope = G = \frac{3.4936 - 1.5}{0.003 - 0.0013} = 1172.71 MMscf$$

6.4 The Alternative Time Function Model

Considering the left hand side of the material balance equation

Where

$$N_p = \sum_{k=1}^{n} Q_k t_k$$

K represents time at each reservoir average pressure and n total average pressure point

$$\mathrm{F} = \sum_{k=1}^{n} Q_k t_k \left[B_o + \left(R_p - R_s\right)B_g + W_p B_w\right]$$

$$\mathrm{F} = \sum_{k=1}^{n} Q_k t_k \left[B_{ok} + \left(R_{pk} - R_{sk}\right)B_{gk} + W_{pk} B_{wk}\right]$$

$$\mathrm{F} = \frac{\sum_{k=1}^{n} Q_k \quad \sum_{k=1}^{n} t_k}{n}\left[B_{ok} + \left(R_{pk} - R_{sk}\right)B_{gk} + W_{pk} B_{wk}\right]$$

$$\mathrm{F} = \sum_{k=1}^{n} Q_k \ \left[B_{ok} + \left(R_{pk} - R_{sk}\right)B_{gk} + W_{pk}B_{wk}\right] \quad \frac{\sum_{k=1}^{n} t_k}{n}$$

Where

$$\mathrm{F}' = \sum_{k=1}^{n} Q_k \ \left[B_{ok} + \left(R_{pk} - R_{sk}\right)B_{gk} + W_{pk}B_{wk}\right]$$

Hence

$$\mathrm{F} = \mathrm{F}' \ \frac{\sum_{k=1}^{n} t_k}{n}$$

Therefore the new model

$$\mathrm{F}' \ \frac{\sum_{k=1}^{n} t_k}{n} = \mathrm{N}\left[E_o + mE_g + E_{f,w}\right] + \mathrm{W_e B_w}$$

Where $\mathrm{E_o}$, $\mathrm{E_g}$ and $\mathrm{E_{f,w}}$ are defined in equations above

Consider a case with no water drive and no original gas cap

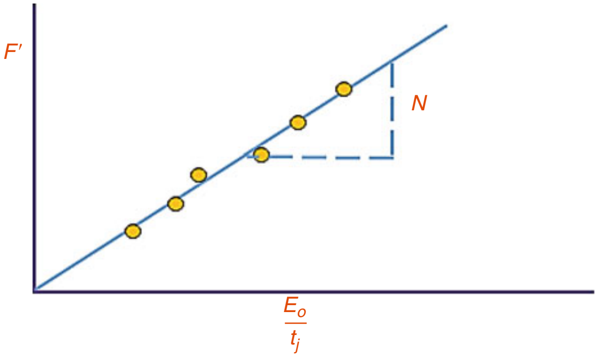
$$F' = \frac{N(E_o)}{\frac{\sum_{k=1}^{n} t_k}{n}}$$

Where

$$t_j = \frac{\sum_{k=j}^{n} t_k}{n}$$

Hence

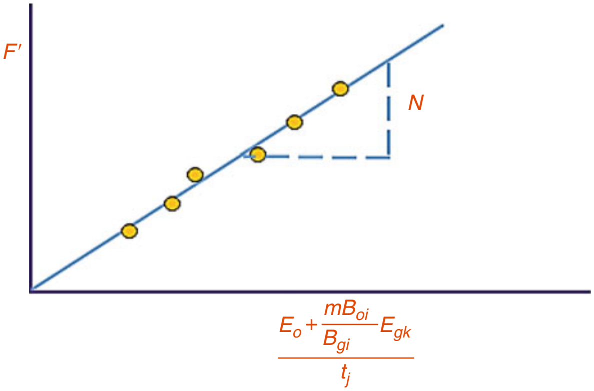
A plot of F' against ${}^{E_o}/_{t_j}$ should result in a straight line scattered data point at the initial state and stabilize into a linear form with N being the slope. It should be noted that the origin is not a must point.



6.4.1 No Water Drive, a Known Gas Cap

$$F' = N\frac{\left[E_{ok} + \frac{mB_{oi}}{B_{gi}}E_{gk}\right]}{t_j}$$

A plot of F' against $\frac{E_{ok}+\frac{mB_{oi}}{B_{gi}}E_{gk}}{t_j}$ should result in straight line with slope N



6.4.2 No Water Drive, N and M Are

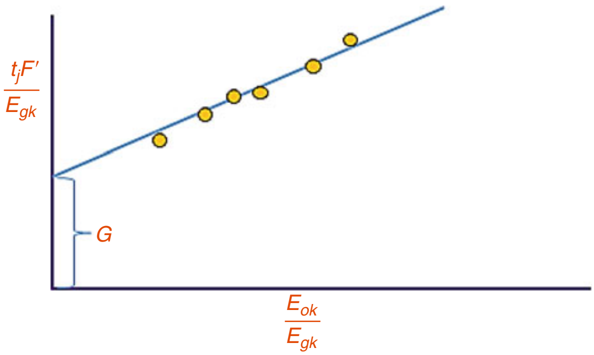
$$F' = N\frac{E_{ok}}{t_j} + G\frac{E_{gk}}{t_j}$$

Simplifying the model, the equation reduces

$$\frac{t_jF'}{E_{gk}} = N\frac{E_{ok}}{E_{ok}} + G$$

And $G = Nm\frac{B_{oi}}{B_{gi}}$= original cap gas

A plot of $\frac{t_jF'}{E_{gk}}$ against $^{E_{ok}}/_{E_{gk}}$ should result in a straight line with N being the slope and G being the intercept.



Case Study 1

Ouro Prieto reservoir which is an undersaturated reservoir with active water drive is considered with a comprehensive analysis of fluid data before building the material balance model. The reservoir has four producers with two producing water. Stanley et al. (2015) performed a comprehensive simulation studies on the Ouro Prieto reservoir and result obtained proved adequacy of the model.

Also the havlena and odeh straight model was used to validate the new model. Figures 6.1 and 6.2 show linear profiles of both models and estimated initial oil in place were compared and the difference percent show in the table below.

Result From Models

	Original MBE	Alternative MBE
Initial oil in place	2,303,332.45	2,213,838.27
Intercept	96.83	1.997
SSE	0.043%	

A model without intercept if stabilize should have intercept approximately zero or close to, though depending on the data structure. This is display in the result shown above for both the alternative and original Havlena and Odeh model. Also comparing both results in term of oil initially in place, it shows an approximate error of 0.043% which makes the model suitable for predicting reservoir performance.

6.5 Conclusion

(i) Efficient modeling of straight line MBE of a cluster reservoir is realistic using the model, hence reservoir fluid properties at each mini-reservoir should be separately considered.
(ii) Careful interest should be paid to issues relating to inconsistent average production rate and also uncertainty in the fluid properties

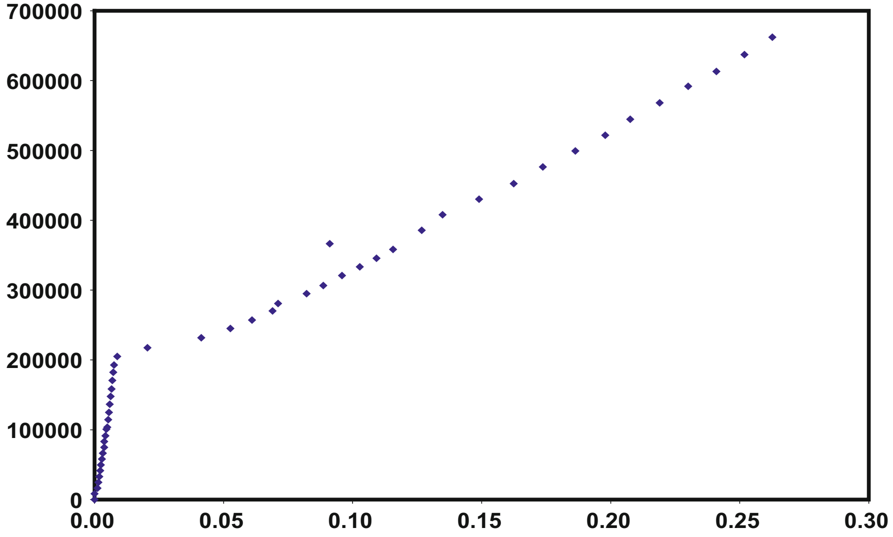


Fig. 6.1 Original material balance using F against E_0

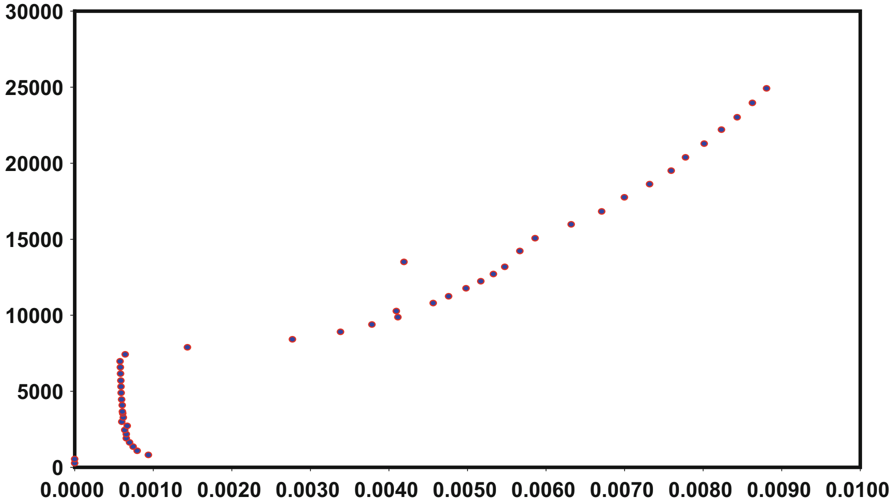


Fig. 6.2 Alternative material balance model using equation

(iii) The model is more applicable for newly discovered reservoir with less information on future production data provided fluid data from nearby reservoir with similar reservoir characteristics are available

(iv) The model duly served as a check to the original material balance Havlena and Odeh straight line MBE, therefore can be used to validate simulation result.

Case Study 2

Apply the appropriate straight line material balance equation to calculate the STOIIP in the data presented in the tables below. This data is obtained from L.P Dake (1978) Fundamentals of Reservoir Engineering, Example 9.2.

PVT data for L.P Dake Example 9.2

GOR (Rs)	650
Oil Gravity	40
(Yg)	0.7
Salinity	14000

Time (day)	Pressure (psia)	Solution GOR (scf/STB)	Oil FVF (rb/STB)	Gas FVF (rb/STB)	Oil Viscosity (cp)	Gas Viscosity (cp)
0	2740	650	1.404	0.00093	0.54	0.0148
365	2500	592	1.374	0.00098	0.589	0.01497
730	2290	545	1.349	0.00107	0.518	0.01497
1096	2109	507	1.329	0.00117	0.497	0.01497
1461	1949	471	1.316	0.00128	0.497	0.01497
1826	1818	442	1.303	0.00139	0.497	0.01497
2191	1702	418	1.294	0.00150	0.497	0.01497
2557	1608	398	1.287	0.00160	0.497	0.01497
2922	1535	383	1.280	0.00170	0.497	0.01497
3287	1480	381	1.276	0.00176	0.497	0.01497
3652	1440	364	1.273	0.00182	0.497	0.00182

Reservoir and Aquifer data

Aquifer data		**Reservoir data**	
Parameter	**Value**	**Parameter**	**Value**
Reservoir thickness	100	Temperature	115
Reservoir radius	9200	Initial pressure	2740
Aquifer radius	46000	Porosity	0.25
Emcroachment angle	140	Swc	0.05
Aquifer permeability	200	Cw	3.00E-06
		Cf	4.00E-06
Relative permeability data			
	Residual sat	**End point**	**Exponent**
Krw	0.25	0.039336	0.064557
Kro	0.15	0.8	10.5533
Krg	0.05	0.9	1

Production data of L.P Dake Example 9.2

Time (dd/mm/yyyy)	Reservoir Pressure (psia)	Cum oil Produced (MMSTB)	Cum Gas Produced (MMSCF)	Cum Water Produced (MMSTB)
1/8/1994	2740	0	0	0
1/8/1995	2500	7.88	5988.8	0
1/8/1996	2290	18.42	15564.9	0
1/8/1997	2109	29.15	26818	0
1/8/1998	1949	40.69	39672.8	0
1/8/1999	1818	50.14	51393.5	0
1/8/2000	1702	58.42	62217.3	0
1/8/2001	1608	65.39	71602.8	0
1/8/2002	1535	70.74	79228.8	0
1/8/2003	1480	74.54	85348.3	0
1/8/2004	1440	77.43	89818.8	0

Material balance analysis has been carried out on example 9.2 of L. P. Dake reservoir. The Reservoir Prediction Analysis tool, REPAT developed by Okotie and Onyekonwu (2015) was used for the analysis and compare with MBAL of Petroleum Experts Limited. The program uses a conceptual model of the reservoir to predict the reservoir behavior and reserves based on the effects of fluids produced from the reservoir. Besides, the in-place volumes calculated from this study can be subjected to static and dynamic simulation tool for validation. The reservoir pressure, PVT and production data, after careful review, served as input data into the REPAT and MBAL program.

Description of the Tool Used in the Study

The reservoir performance analysis tool (REPAT 8.5) is a package designed to help engineers to have a better understanding of reservoir behaviour, infer hydrocarbon in place, determine the best aquifer model, history match production history and perform prediction run. The tool is setup in a way that user can go from left to right on the options menu and from each option, a user can navigate from top to bottom. Thus, this tool is broken down into various components and these are:

- Setting the system/model options
- Entering PVT data and perform correlation match to select the best model
- Entering reservoir, relative permeability and aquifer data
- Entering production history data
- Performing a history match
- Performing prediction run
- Generation of report
- help

Results from REPAT and MBal

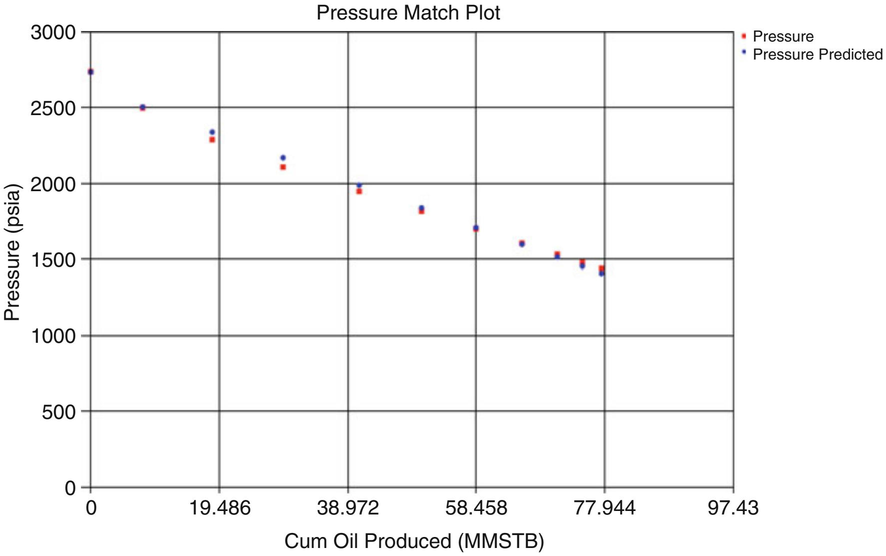
The summary of the results obtained from L.P Dake Example 9.2 analysis are as shown in the table below.

Summary of L.P Dake Example 9.2 Analysis Results

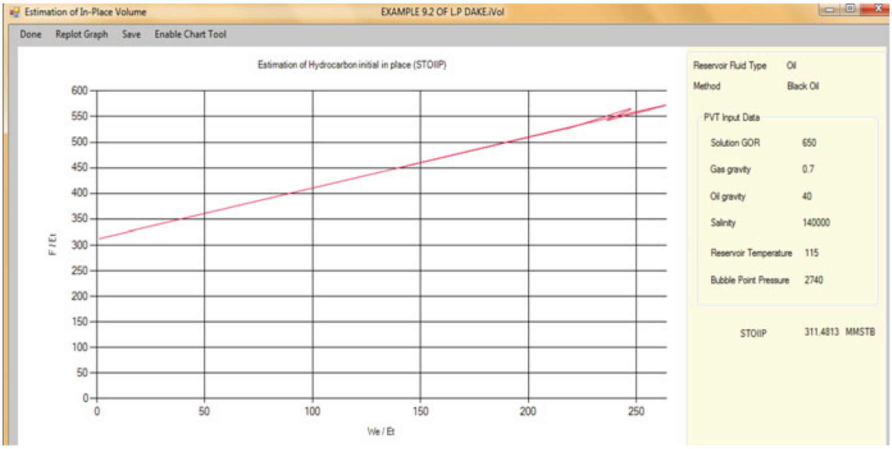
Parameter	REPAT	MBAL	L.P DAKE
Aquifer model	**Hurst-Van Everdingen**	**Hurst-Van Everdingen-Dake**	**Hurst-Van Everdingen**
Reservoir Thickness (ft)	100	100	100
Reservoir Radius (ft)	9200	9200	9200
Outer/Inner Radius	5.0761	5.1	5.00
Encroachment Angle	140	140	140
Aquifer Permeability (md)	200	327.19	200
OIIP (MMSTB)	311.48	312.79	312

Summary of Input Data for the Aquifer model of L.P Dake Example 9.2

Parameter	Value	Source
Aquifer permeability (md)	327.19	Regression in REPAT and MBAL
Encroachment Angle (deg)	140	Fault polygon
Reservoir radius (ft)	9200	Estimated from seismic map
Outer/inner radius (ratio)	5.00	Estimated from seismic map
Reservoir thickness(ft)	100	Logs



History-Prediction pressure plot



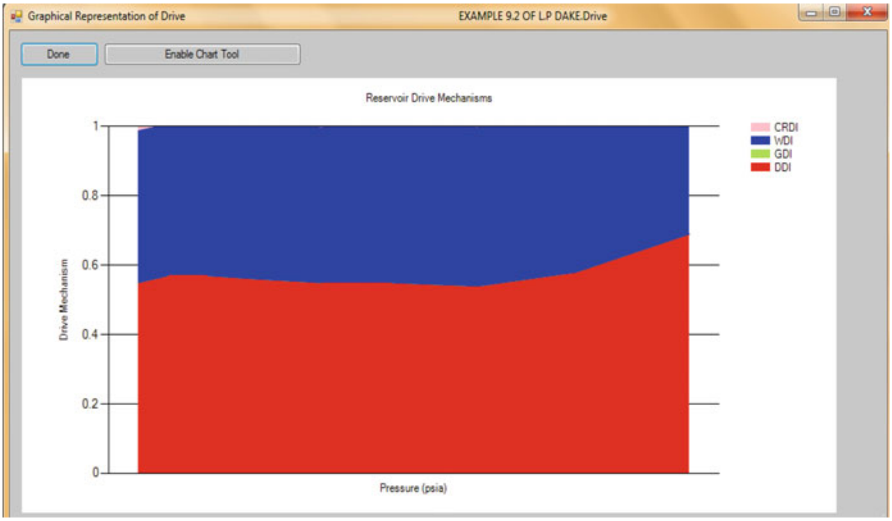
Reservoir Fluid Type Oil
Method Black Oil

PVT Input Data

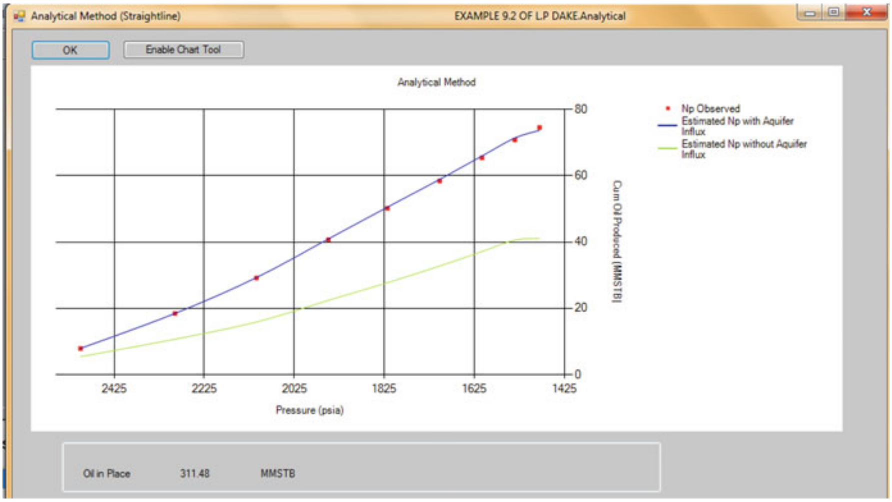
Solution GOR	650
Gas gravity	0.7
Oil gravity	40
Salinity	140000
Reservoir Temperature	115
Bubble Point Pressure	2740

STOIIP 311.4813 MMSTB

Graphical estimate of STOIIP

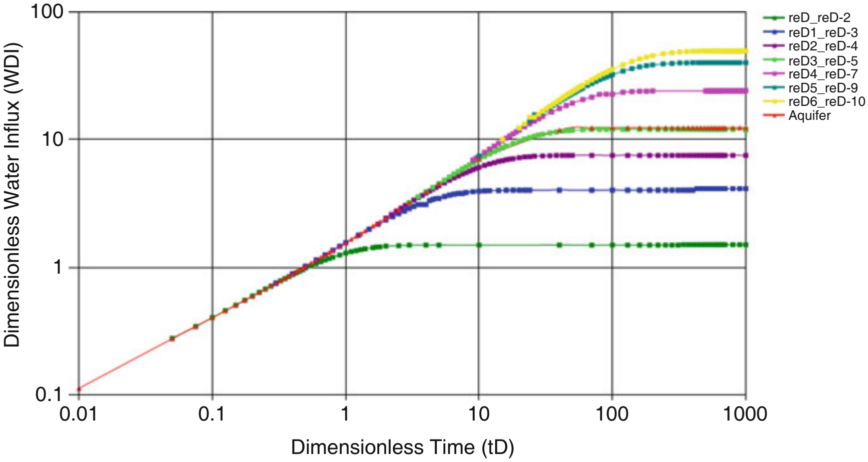


Energy plot



Oil in Place 311.48 MMSTB

History match plot



Aquifer plot

Inference from the Analysis

The Hurst-Van Everdingen model was selected as the most likely case for example 9.2 in L. P. Dake. The parameters used to obtain the history match and the OIIP from Hurst-Van Everdingen radial aquifer compare favourably with the expected values. The inferences from the Material Balance Analysis of this example using REPAT are as follows:

- The OOIP is 311.48MMSTB from the diagnostic (F/E_t Vs W_e/E_t) plot as shown in the historical match plot above

- The example 9.2 reservoir is influenced by a combination of water drive and fluid expansion drive mechanism as revealed by the energy plot
- Results from the analytical cumulative oil produced match as shown in historical match plot, indicates a Hurst –Van Everdingen radial water drive behavior, encroaching at an angle of 140^0. A good production simulation match was obtained
- The Results of the analysis indicates that the Hurst–Van Everdingen radial aquifer Influx model incorporated into the (F/E_t Vs W_e/E_t) straight-line method is the most likely aquifer model.
- Figure 6.8 the aquifer plot shows the dimensionless aquifer plot and the red line indicates example 9.2 plot

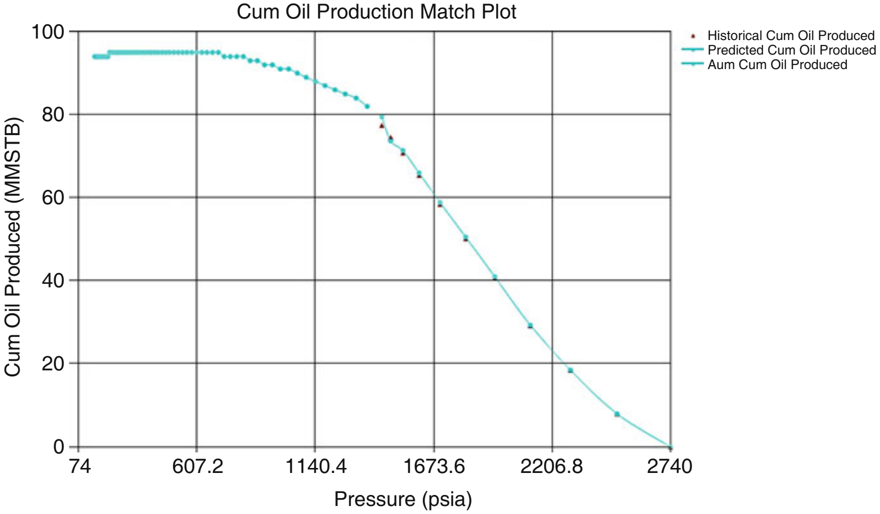
The volume obtained with REPAT using example 9.2 reservoir compares favourably with the volume reported by L.P. Dake as shown in the table above.

Constraints

Unknown aquifer characteristics and properties

Prediction Result

The figure below shows the prediction result obtained from example 9.2 after careful analysis and history match. The predicted result match perfectly well with the historical data and extrapolated to a future pressure as the reservoir declines to abandonment. REPAT has a user-defined option of prediction to control the start and end of prediction result. Hence, since the tool gave a close value of STOIIP as compared with the base case of example 9.2 and also able to match the historical data, it therefore gave a good prediction result.



Result from REPAT

6.6 Conclusions

The result obtained from the analysis of example 9.2 from fundamentals of reservoir engineering by L.P Dake using this study software "REPAT", the following conclusion can be drawn:

- The Hurst-Van Everdingen radial aquifer model was selected as the most likely case. The parameters used to obtain the history match and the OIIP compare favorably with the expected values from L.P. Dake and MBal.
- The error in STOIIP obtained from REPAT is 0.00195 and R-value of 0.99999 which is a good fit, while MBal is 0.00253 using the STOIIP in example 9.2 in L. P Dake as a base case.
- The reservoir is supported by a combination of water drive and fluid expansion drive
- The result of STOIIP obtained after regression on aquifer-reservoir radius ratio converges at 5.0761 from Hurst-Van Everdingen radial aquifer model.
- A good pressure and historical production simulation match was obtained from REPAT

Recommendations

- Results from REPAT should be compared with the result from other means of oil in place estimate such as static (geology) and simulation (eclipse).
- Prediction of cumulative water produced should be model.
- REPAT can be used as a pre-processing tool for reservoir simulation/study to infer in place volume and best aquifer model.
- It can be used as a "stand-alone" for reservoir performance
- REPAT can also be used in the academic environment.

Case Study 3
The case study presented here is a paper published by Authors of this book.

Hydraulic Communication Check
Analytical plots of the pressure and the production data as shown in Fig. 6.3 have been used to check for a possible communication across the J2 and J3 reservoir levels.

Similar SBHP and GOR trends for J2 and J3 reservoirs indicate possible communication across these levels, hence J2 and J3 was modeled as multiple tanks connected by means of a transmissibility.

Data Presentation
Ugua J2-J3 reservoirs are both saturated oil reservoir. J2 reservoir operates at a temperature of 229 deg. F and a bubble point of 4718 psi while J3 reservoir operates at a temperature of 228 deg. F and a bubble point 4711.29 psi. J2 reservoir production history spans a period of 35 years (May 1976–January 2011) while J3 spans a period of 34 years (February 1977–January 2011). The PVT and reservoir

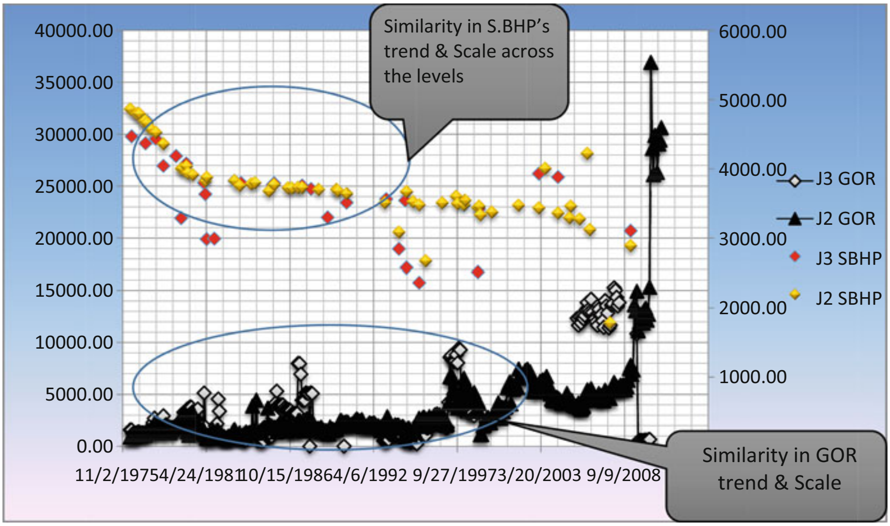


Fig. 6.3 GOR and SBHP Plots

(tank) data used in the analysis for both reservoirs are as shown in the Tables 6.1 and 6.2.

Procedure

The Havlena-Odeh and the F/Et vs. We/Et straight line plots of the graphical method incorporating various radial aquifer models were used to evaluate the aquifer properties, match the reservoir pressure and determine the gas initially in-place (GIIP). The accuracy of the results was validated with the history match of the model's pressure and production.

The analysis procedure is as follows:

- Pressure and production data is entered on a Tank basis.
- The matching facility in MBAL is used to adjust the empirical fluid property correlations to fit measured PVT laboratory data. Correlations are modified using a non-linear regression technique to best fit the measured data.
- The graphical method plot is used to visually determine the different Reservoir and Aquifer parameters. The Havlena – Odeh and the F/Et vs. We/Et straight-line plots of the graphical method were used to visually observe and determine the appropriate aquifer model and parameters.
- The non-linear regression engine of the analytical method is used in estimating the unknown reservoir and aquifer parameters and fine-tune the pressure and production match. This is done for various aquifer models and their standard deviations from the actual field data are compared
- The accuracy of the model is validated by history matching the field pressure and production data with the simulation data.

Table 6.1 Ugua J2 and J3 reservoir PVT Data

J2 Reservoir		J3 Reservoir	
Property	**Value**	**Property**	**Value**
Formation GOR (scf/STB)	1736	Formation GOR (scf/STB)	1253
Oil gravity	34.8	Oil gravity	33.7
Gas gravity	0.863	Gas gravity	0.698
Mole percent H2S (%)	0	Mole percent H2S (%)	0
Mole percent CO (%)	1.84	Mole percent CO (%)	2.75
Mole percent N2 (%)	1.09	Mole percent N2 (%)	0.07
Water salinity (ppm)	10000	Water salinity (ppm)	10000
Pb,Rs,Bo correlation	Lasater	Pb, Rs, Bo correlation	Glasso
Oil viscosity correlation	Petrosky et al	Oil viscosity correlation	Beal et al
Separator	Single stage	Separator	Single stage

Table 6.2 Ugua J2 and J3 reservoir (Tank) Properties

J2 Reservoir		J3 Reservoir	
Parameter	**Value**	**Parameter**	**Value**
Temperature (deg.F)	229	Temperature (deg.F)	228
Initial pressure (psi)	4718	Initial pressure (psi)	4711.29
Porosity	0.15	Porosity	0.15
Connate water saturation	0.15	Connate water saturation	0.15
Water compressibility (1/psi)	Use correlation	Water compressibility (1/psi)	Use correlation
Initial gas cap	0.038	Initial gas cap	0.116

6.7 Results

The summary of the result from the analysis is shown in Table 6.3.

The summary of the aquifer parameters used in the Material Balance calculations and the source of each data is depicted in Tables 6.4 and 6.5.

The Hurst-Van Everdingen Modified model was selected as the most likely case for J2 while Hurst-Van Everdingen-Dake for J3. The parameters used to obtain the history match and the OIIP from both models with the Hurst-Van Everdingen Modified and Hurst-Van Everdingen radial aquifer compare favorably with the expected values. The plots generated from the most likely case models are shown in Figs. 6.4, 6.5, 6.6, 6.7, 6.8, 6.9, 6.10, 6.11, and 6.12.

Constraints

- Unknown aquifer characteristics and properties.

Table 6.3 Summary of Ugua J2 and J3 Reservoir Analysis Results

J2 Reservoir		J3 Reservoir	
Aquifer model	**Hurst-Van Everdingen-modified**	**Aquifer model**	**Hurst-Van Everdingen-Dake**
Reservoir thickness (m)	282	Reservoir thickness (m)	96.17
Reservoir radius (m)	5000	Reservoir radius (m)	3576
Outer/inner radius	2.56	Outer/inner radius	3.93
Encroachment angle	224	Encroachment angle	139
Aquifer permeability (md)	2.48	Aquifer permeability (md)	35
OIIP (MMSTB)	125.006	OIIP (MMSTB)	80.689
GIIP (Bscf)	42.72	GIIP (Bscf)	68.7

Table 6.4 Summary of Input Data for Ugua J2 Reservoir Aquifer Model and Transmissibility

J2 Reservoir		
Paramerter	**Value**	**Source**
Aquifer permeability	2.48	Regression in MBAL
Encroachment angle (deg.)	224	Fault polygon
Reservoir radius (m)	5000	Estimation from seismic map
Outer/inner radius (ratio)	2.56	Estimation from seismic map
Reservoir thickness(m)	282	Logs
Transmissibility(Rb/day*cp/psi)	4.76925	Regression in MBAL

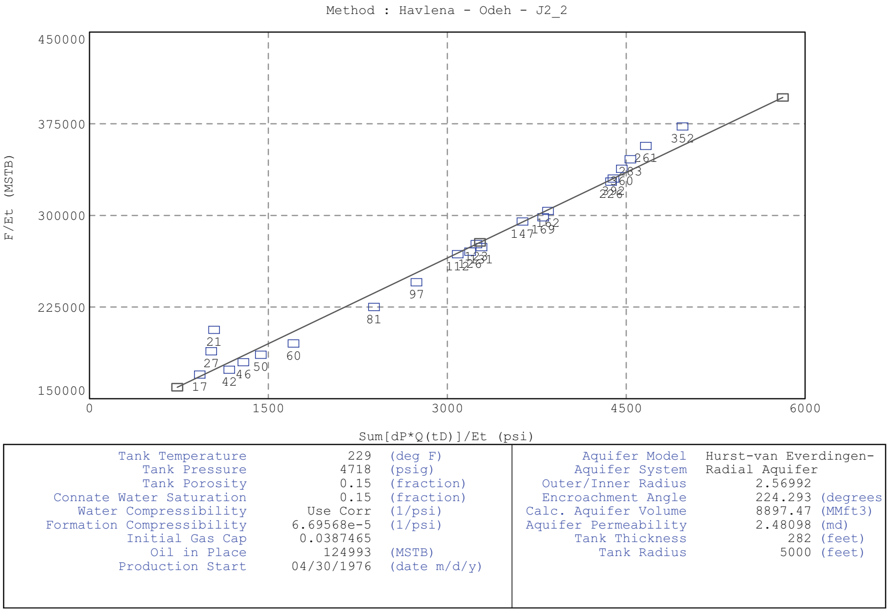
Table 6.5 Summary of Input Data for Ugua J2 Reservoir Aquifer Model and Transmissibility

J3 Reservoir		
Paramerter	**Value**	**Source**
Aquifer permeability	35	Regression in MBAL
Encroachment angle (deg.)	139	Fault polygon
Reservoir radius (m)	3576	Estimation from seismic map
Outer/inner radius (ratio)	3.93	Estimation from seismic map
Reservoir thickness(m)	96.7	Logs
Transmissibility(Rb/day*cp/psi)	4.76925	Regression in MBAL

Inference from Analysis

Inferences from the Material Balance analysis of the Ugua J2 reservoir are as follows:

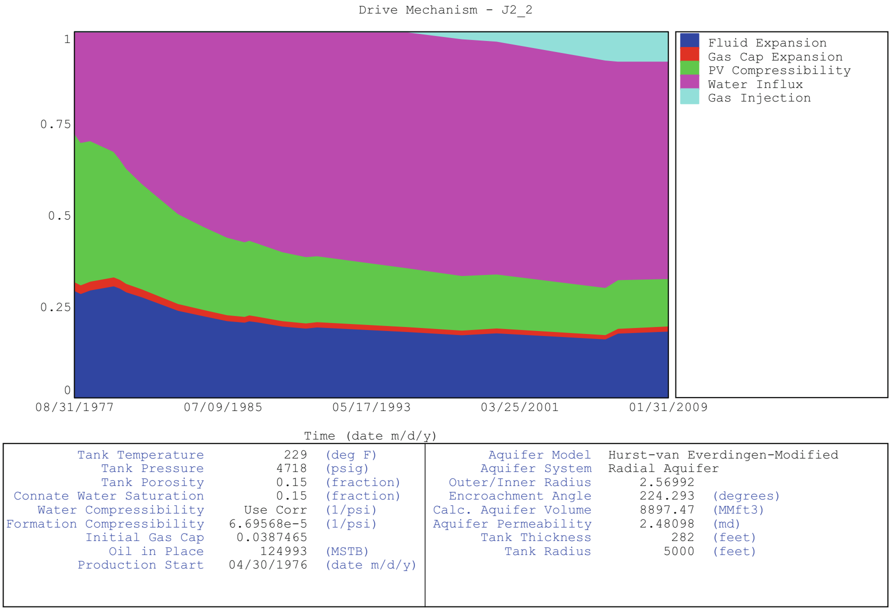
- The OOIP is 125.006MMstb.
- The most likely aquifer model is the Hurst-Van Everdingen Modified radial aquifer.



Parameter	Value	Unit
Tank Temperature	229	(deg F)
Tank Pressure	4718	(psig)
Tank Porosity	0.15	(fraction)
Connate Water Saturation	0.15	(fraction)
Water Compressibility	Use Corr	(1/psi)
Formation Compressibility	6.69568e-5	(1/psi)
Initial Gas Cap	0.0387465	
Oil in Place	124993	(MSTB)
Production Start	04/30/1976	(date m/d/y)

Parameter	Value	Unit
Aquifer Model	Hurst-van Everdingen-	
Aquifer System	Radial Aquifer	
Outer/Inner Radius	2.56992	
Encroachment Angle	224.293	(degrees
Calc. Aquifer Volume	8897.47	(MMft3)
Aquifer Permeability	2.48098	(md)
Tank Thickness	282	(feet)
Tank Radius	5000	(feet)

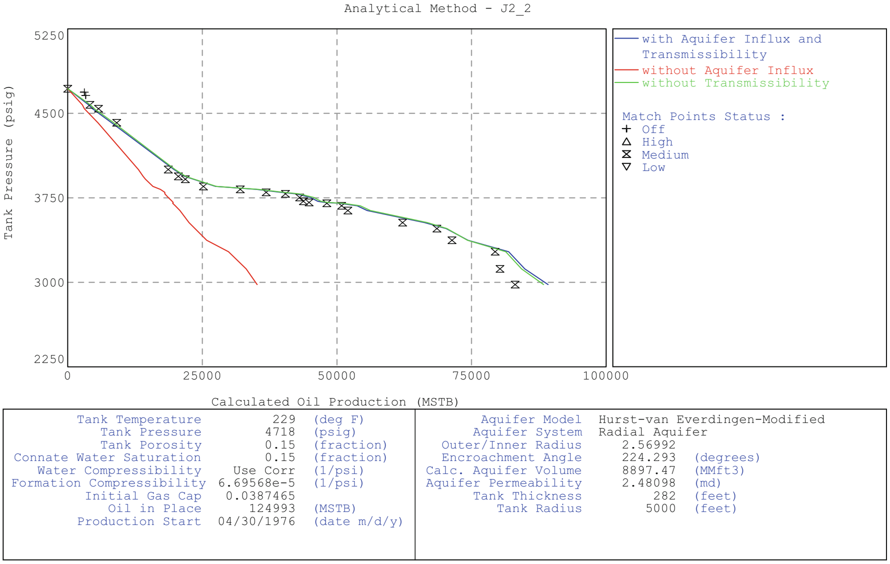
Fig. 6.4 J2 Reservoir Graphical Diagnostic Plot



Parameter	Value	Unit
Tank Temperature	229	(deg F)
Tank Pressure	4718	(psig)
Tank Porosity	0.15	(fraction)
Connate Water Saturation	0.15	(fraction)
Water Compressibility	Use Corr	(1/psi)
Formation Compressibility	6.69568e-5	(1/psi)
Initial Gas Cap	0.0387465	
Oil in Place	124993	(MSTB)
Production Start	04/30/1976	(date m/d/y)

Parameter	Value	Unit
Aquifer Model	Hurst-van Everdingen-Modified	
Aquifer System	Radial Aquifer	
Outer/Inner Radius	2.56992	
Encroachment Angle	224.293	(degrees)
Calc. Aquifer Volume	8897.47	(MMft3)
Aquifer Permeability	2.48098	(md)
Tank Thickness	282	(feet)
Tank Radius	5000	(feet)

Fig. 6.5 J2 Reservoir Energy Plot



Tank Temperature	229	(deg F)
Tank Pressure	4718	(psig)
Tank Porosity	0.15	(fraction)
Connate Water Saturation	0.15	(fraction)
Water Compressibility	Use Corr	(1/psi)
Formation Compressibility	6.69568e-5	(1/psi)
Initial Gas Cap	0.0387465	
Oil in Place	124993	(MSTB)
Production Start	04/30/1976	(date m/d/y)

Aquifer Model	Hurst-van Everdingen-Modified	
Aquifer System	Radial Aquifer	
Outer/Inner Radius	2.56992	
Encroachment Angle	224.293	(degrees)
Calc. Aquifer Volume	8897.47	(MMft3)
Aquifer Permeability	2.48098	(md)
Tank Thickness	282	(feet)
Tank Radius	5000	(feet)

Fig. 6.6 J2 Reservoir Analytical Plot

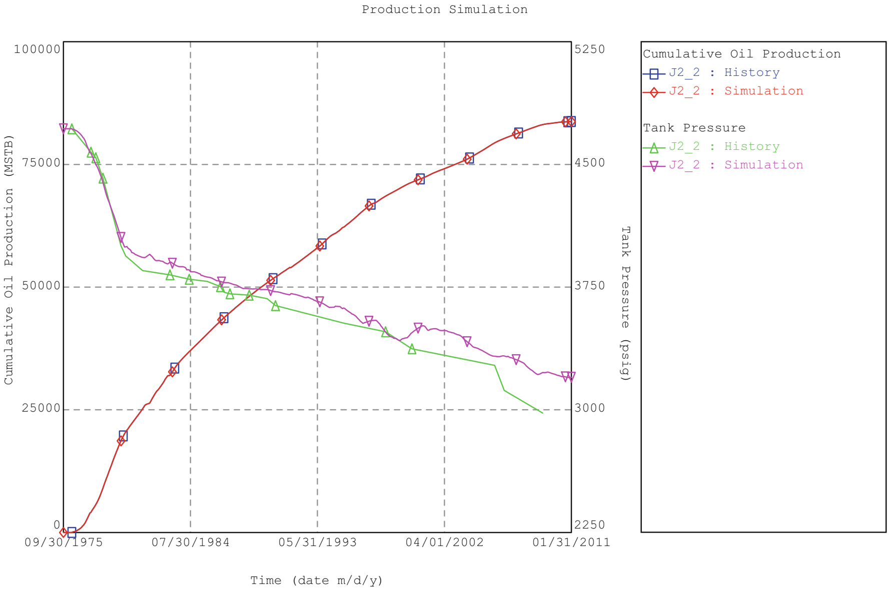
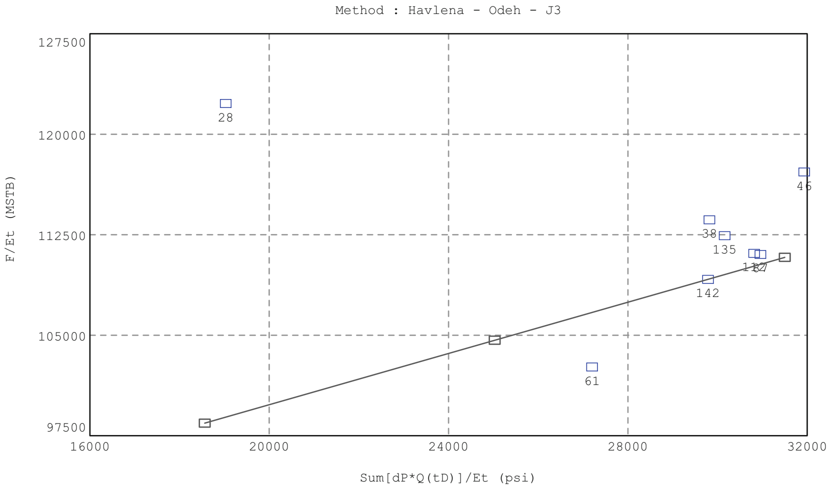
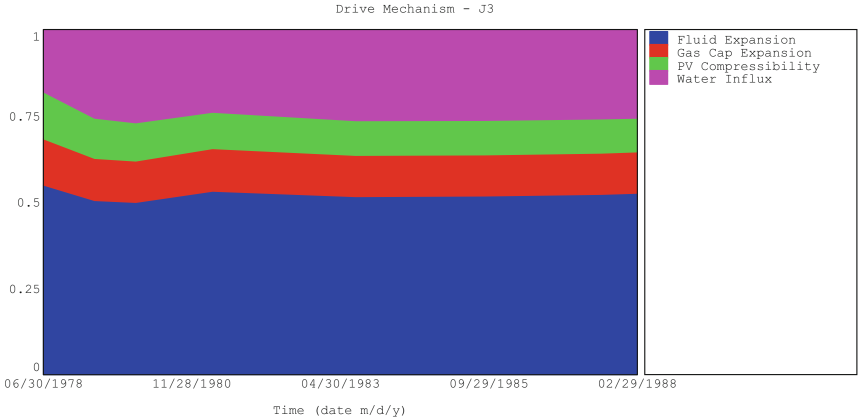


Fig. 6.7 J2 Reservoir Pressure History Match Plot



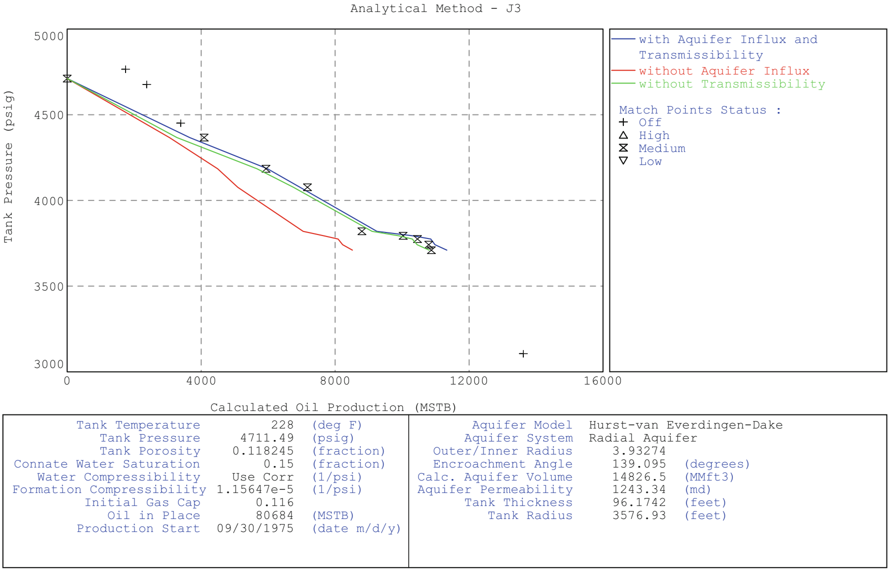
Tank Temperature	228	(deg F)	Aquifer Model	Hurst-van Everdingen-Dake	
Tank Pressure	4711.49	(psig)	Aquifer System	Radial Aquifer	
Tank Porosity	0.118245	(fraction)	Outer/Inner Radius	3.93274	
Connate Water Saturation	0.15	(fraction)	Encroachment Angle	139.095	(degrees)
Water Compressibility	Use Corr	(1/psi)	Calc. Aquifer Volume	14826.5	(MMft3)
Formation Compressibility	1.15647e-5	(1/psi)	Aquifer Permeability	1243.34	(md)
Initial Gas Cap	0.116		Tank Thickness	96.1742	(feet)
Oil in Place	80684	(MSTB)	Tank Radius	3576.93	(feet)
Production Start	09/30/1975	(date m/d/y)			

Fig. 6.8 J3 Reservoir Graphical Plot



Tank Temperature	228	(deg F)	Aquifer Model	Hurst-van Everdingen-Dake	
Tank Pressure	4711.49	(psig)	Aquifer System	Radial Aquifer	
Tank Porosity	0.118245	(fraction)	Outer/Inner Radius	3.93274	
Connate Water Saturation	0.15	(fraction)	Encroachment Angle	139.095	(degrees)
Water Compressibility	Use Corr	(1/psi)	Calc. Aquifer Volume	14826.5	(MMft3)
Formation Compressibility	1.15647e-5	(1/psi)	Aquifer Permeability	1243.34	(md)
Initial Gas Cap	0.116		Tank Thickness	96.1742	(feet)
Oil in Place	80684	(MSTB)	Tank Radius	3576.93	(feet)
Production Start	09/30/1975	(date m/d/y)			

Fig. 6.9 J3 Reservoir Energy Plot



Tank Temperature	228	(deg F)	Aquifer Model	Hurst-van Everdingen-Dake	
Tank Pressure	4711.49	(psig)	Aquifer System	Radial Aquifer	
Tank Porosity	0.118245	(fraction)	Outer/Inner Radius	3.93274	
Connate Water Saturation	0.15	(fraction)	Encroachment Angle	139.095	(degrees)
Water Compressibility	Use Corr	(1/psi)	Calc. Aquifer Volume	14826.5	(MMft3)
Formation Compressibility	1.15647e-5	(1/psi)	Aquifer Permeability	1243.34	(md)
Initial Gas Cap	0.116		Tank Thickness	96.1742	(feet)
Oil in Place	80684	(MSTB)	Tank Radius	3576.93	(feet)
Production Start	09/30/1975	(date m/d/y)			

Fig. 6.10 J3 Reservoir Analytical Plot

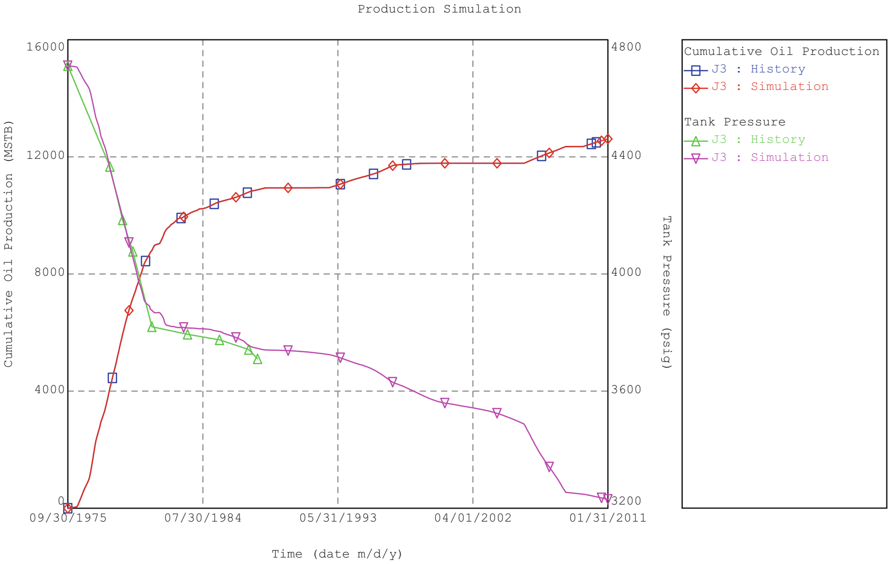


Fig. 6.11 J3 Reservoir Analytical Plot

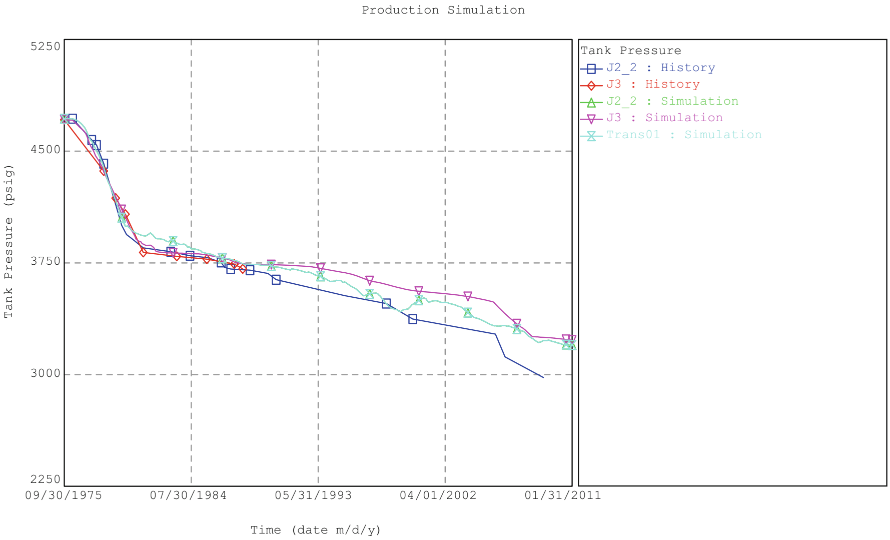


Fig. 6.12 J3 and J3 Reservoirs Pressure Plot with a transmissibility

- The reservoir is supported by a combination drive of water influx, fluid expansion, and gas cap expansion mechanisms.

Inferences from the Material Balance analysis of the Ugua J3 reservoir are as follows:

- The OOIP is 80.689MMstb.
- The most likely aquifer model is the Hurst-Van Everdingen-Dake radial aquifer.
- The reservoir is supported by a combination drive of fluid expansion and water influx with a minimal gas cap expansion mechanisms.

There is communication between J2 and J3 as can be seen from the combined history match pressure and transmissibility plot of Fig. 6.12.

6.8 Conclusion

From the hydraulic communication check performed as shown in Fig. 6.3, we suspect communication between J2 and J3 reservoirs, hence multi-tank material balance analysis approach linked with transmissibility was adopted to model the reservoirs. The results obtained shall be used in the full field Ugua reservoir simulation study and the oil initially in place volume will be validated with the static and dynamic models. The Hurst-Van Everdingen radial aquifer model was selected as the most likely case.

The summary of the results from the material balance analysis of the Ugua J2 and J3 reservoir levels is depicted in Table 6.6.

Table 6.6 Ugua J2 and J3 Material Balance Results

Reservoir Level	OIIP (MMstb)	GIIP (Bscf)	Available Drive Mechanism	Likely Aquifer
J2	125.006	42.72	Combination drive	Hurst-Van Everdigen-modified
J3	80.689	68.7	Combination drive	Hurst-Van Everdigen-Dake

Exercises

Ex 6.1 The data in the table below represent a data from a saturated oil reservoir without an active water drive. Confirm if the reservoir is actually undergoing volumetric depletion

Time (yrs)	N_p (MMstb)	Bg (cuft/scf)	B_o (rb/stb)	R_s (cuft/stb)	R_p (cuft/stb)
0	0	0.00433	1.5533	719.9045	701.7525
1	1.9891	0.00446	1.5440	702.4075	795.3195
2	7.0973	0.00466	1.5306	676.9572	860.8164
3	10.7186	0.00489	1.5168	650.6649	926.3133
4	18.5518	0.00525	1.4969	612.9574	954.3834
5	22.8154	0.00549	1.4854	591.0627	985.6700
6	28.0537	0.00581	1.4509	555.0749	1008.657
7	31.0359	0.00611	1.4201	525.0909	1041.227
8	34.5123	0.00641	1.3957	503.1039	1059.677

Ex 6.2 Check if the reservoir with data given in the tables below representing reservoir and production history of the field is aquifer supported

P_i	2560 psi	m	0.08	R_{si}	600 scf/stb
B_{oi}	1.316 rb/stb	P_b	2560 psi	B_w	1.05 rb/stb

Date	N_p (MMstb)	B_g (cuft/scf)	B_o (rb/stb)	R_s (cuft/stb)	R_p (cuft/stb)
Aug-93	21.456	0.00129	1.301	542	905
Aug-94	31.871	0.00138	1.278	498	898
Aug-95	41.871	0.00142	1.272	483	763
Aug-96	55.843	0.00148	1.267	473	659
Aug-97	67.78	0.00157	1.259	461	576
Aug-98	80.758	0.00161	1.254	452	518

Ex 6.3 Given a saturated reservoir with bubble point pressure as 4100 psia and based on the geological information provided, the gascap size was determine as 0.45 but this value is not certain. From the PVT data and the historic production provided in the table below, calculate the stock tank oil initially in place, free gas volume and the correct value for the gascap size.

Pressure (psia)	Np (MMSTB)	Rp (scf/STB)	Bo (rb/STB)	Rs (scf/STB)	Bg (rb/STB)
4100	0.000	0	1.3887	536	0.000895
3887	4.627	1260	1.3712	501	0.000947
3702	8.298	1272	1.3572	473	0.000988
3517	12.447	1392	1.3459	446	0.001039
3332	16.178	1482	1.3351	421	0.001101
3147	20.421	1518	1.3238	394	0.001163
2962	24.935	1560	1.3122	370	0.001235

Ex 6.4 Given the following data of Level GT oil reservoir in Ugbomro:

Connate water saturation	23%
Bubble point pressure	2650 psia
STOIIP	12.89 MMSTB

Pressure (psia)	Bo (rb/STB	Bg (rb/STB)	Rs (scf/STB)	Uo (cp)	Uo (cp	GOR (scf/STB)
2650	1.3814	0.000895	680	0.956	0.018	680
2180	1.3791	0.000947	574	1.236	0.0165	1480
1825	1.3572	0.000988	528	1.492	0.0152	2100

The cumulative gas-oil ratio at 1825 psia was recorded at 950 scf/STB. Calculate
The oil saturation at 1825 psia
The volume of free gas in the reservoir at 1825 psia
The relative permeability ratio (Kg/Ko) at 1825 psia

Ex 6.5 Determine the original gas-in-place and ultimate recovery at an abandonment pressure of 500 psia for the following reservoir.

Gas specific gravity

= 0.70
Reservoir temperature

= 150 °F
Original reservoir pressure
= 3600 psia
Abandonment reservoir pressure
= 500 psia

Production and pressure history as shown in the following table.

Gp (MMscf)	P (psia)	z	P/z (psia)
0	3600	0.8351	4310.861
640	3360	0.8204	4095.563
1550	3060	0.8187	3737.633
3250	2484	0.8134	3053.848

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Chapter 7
Decline Curve Analysis

Learning Objectives:

Upon completion of this chapter, students/readers should be able to:

- Describe the build-up, plateau and decline stage of hydrocarbon production
- Describe the application of decline curves
- Understand the causes of production decline
- Reservoir factors that affect the Decline Rate
- Operating conditions that influence the Decline Rate
- Describe the various types of decline curves
- Identify the decline model of any field from historical data
- Determine the model parameter
- Derive the appropriate equations of the different types of decline model
- Forecast future production of a field
- Determine the abandonment time and rate of a field
- Determine the cumulative production of a field

Nomenclature

Parameter	*Symbol*	*unit*
Initial oil or gas production rate	q_i	*bbl/yr or bbl/month or bbl/day or stb/day & scf/day*
Oil or gas flow rate at current time	q_t	*stb/day or scf/day*
Abandonment rate	q_a	*stb/day or scf/dayx*
Cumulative oil produced	N_p	*stb*
Cumulative gas produced	G_p	*scf*
Time	t	*yr or month or day*
Abandonment time	t_a	*yr or month or day*
Constant	k	–

(continued)

S. Okotie, B. Ikporo, *Reservoir Engineering*,
https://doi.org/10.1007/978-3-030-02393-5_7

Parameter	*Symbol*	*unit*
Exponent	n	–
Nominal or continuous or initial decline	D_i	*per day or month or year*
Arps' decline-curve exponent	b	–
Effective decline rate	$\boldsymbol{D_i'}$	*per day or month or year*

7.1 Introduction

Globally, the oil and gas production profiles differ considerably. When a field starts production, it builds up to a plateau state, and every operator will want to remain in this stage for a very long period of time if possible. But in reality, it is practically not possible, because, at a point in the life of the field, the production rate will eventually decline to a point at which it no longer produces profitable amounts of hydrocarbon as shown in Fig. 7.1. In some fields, the production build-up rate starts in the first few years, most fields' profiles have flat top and the length of the flat top depends on reservoir productivity.

Some fields have long producing lives depending upon the development plan of the field and reservoir characteristics such as the reservoir, drive mechanism. Wells in water-drive and gas-cap drive reservoirs often produce at a near constant rate until the encroaching water or expanding gas cap reaches the well, thereby causing a sudden decline in oil production. Wells in gas solution drive and oil expansion drive reservoirs have exponential or hyperbolic declines: rapid declines at first, then leveling off.

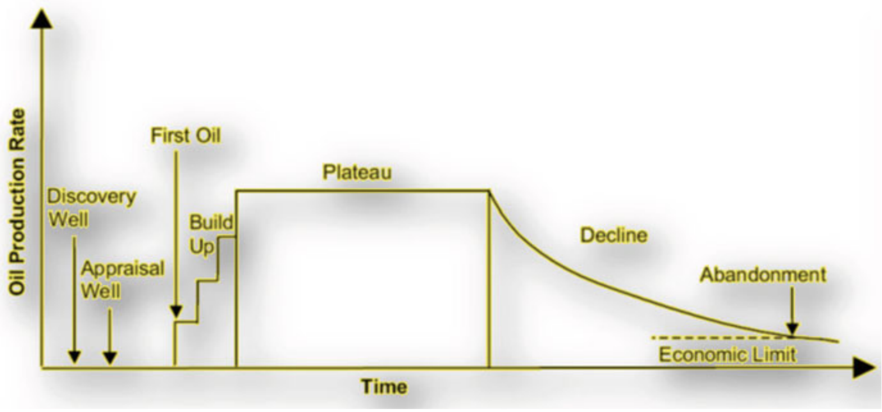


Fig. 7.1 A theoretical production curve, describing the various stages of maturity. (Source: Robelius (2007))

Therefore, decline curve analysis can be defined as a graphical procedure used for analyzing the rates of declining production and also a means of predicting future oil well or gas well production based on past production history. Production decline curve analysis is a traditional means of identifying well production problems and predicting well performance and life based on measured oil or gas well production. Today, several computer software have been built to perform this task and prior to the availability of computers, decline curve analysis was performed by hand on semi-log plot paper. Several authors (Rodriguez & Cinco-Ley (1993), Mikael (2009), Duong (1989) have developed new models or approach for production decline analysis. Agarwal et al. (1998) combined type curve and decline curve analysis concepts to analyse production data. Doublet et al. (1994), applied the material balance time for a field using decline curve analysis.

Furthermore, as stated by Thompson and Wright (1985), decline curve is one of the oldest methods of predicting oil reserves with the following advantages:

- They use data which is easy to obtain
- They are easy to plot
- They yield results on a time basis, and
- They are easy to analyze.

7.2 Application of Decline Curves

- Production decline curve illustrates the amount of oil and gas produced per unit of time.
- If the factors affecting the rate of production remaining constant, the curve will be fairly regular, and, if projected, can give the future production of the well with an assumption that the factors that controlled production in the past will continue to do so in future.
- The above knowledge is used to ascertain the value of a property and proper depletion and depreciation charges may be made on the books of the operating company.
- The analysis of the production decline curve is employed to determine the value in oil and gas wells economics.
- Identify well production problems
- Decline curves are used to forecast oil and gas production for the reservoir and on per well basis and field life span.
- Decline curves are also used to predict oil and gas reserves; this can be used as a control on the volumetric reserves calculated from log analysis results and geological contouring of field boundaries.
- It is often used to estimate the recovery factor by comparing ultimate recovery with original oil in place or gas in place calculations

7.3 Causes of Production Decline

- Changes in bottom hole pressure (BHP), gas-oil ratio (GOR), water-oil ratio (WOR), Condition in drilling area
- Changes in Productivity Index (PI)
- Changes in efficiency of vertical & horizontal flow mechanism or changes in equipment for lifting fluid.
- Loss of wells

7.4 Reservoir Factors that Affect the Decline Rate

- Pressure depletion
- Number of producing wells
- Reservoir drive mechanism
- Reservoir characteristics
- Saturation changes and
- Relative permeability.

7.5 Operating Conditions that Influence the Decline Rate

- Separator pressure
- Tubing size
- Choke setting
- Workovers
- Compression
- Operating hours, and
- Artificial lift.

As long as the above conditions do not change, the trend in decline can be analyzed and extrapolated to forecast future well performance. If these conditions are altered, for example; through a well workover, the decline rate determined during pre-workover will not be applicable to the post-workover period.

7.6 Types of Decline Curves

Arps (1945) proposed that the “curvature” in the production-rate-versus-time curve can be expressed mathematically by a member of the hyperbolic family of equations. Arps recognized the following three types of rate-decline behavior:

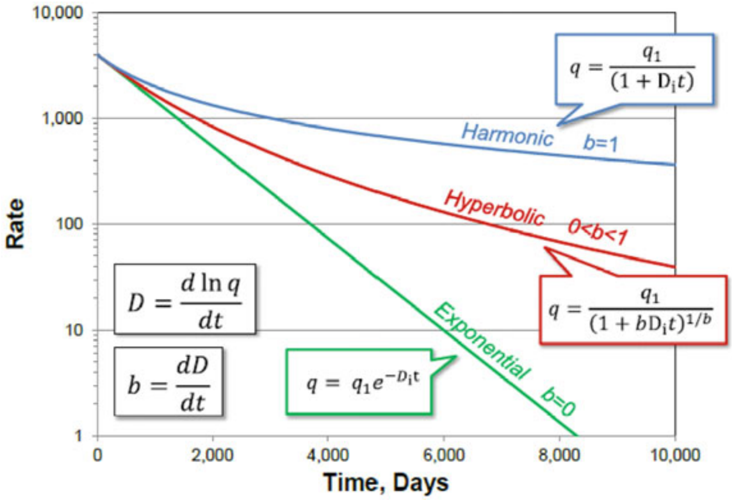


Fig. 7.2 Arps' three types of decline and their formulas on a semi-log plot after Arps (1945)

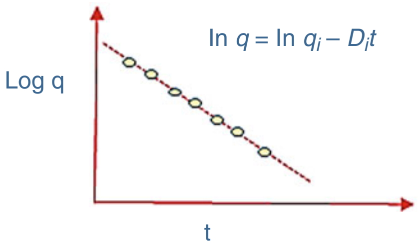
- Exponential decline
- Harmonic decline
- Hyperbolic decline

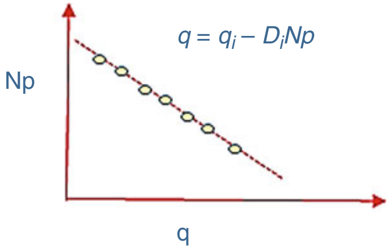
Arps introduces equations for each type and used the concept of loss-ratio and its derivative to derive the equations. The three declines have b values ranging from 0 to 1. Where $b = 0$ represents the exponential decline, $0 < b < 1$ represents the hyperbolic decline, and $b = 1$ represents the harmonic decline (Fig. 7.2).

The plots of production data such as log(q) versus t; q versus Np; log(q) versus log(t); Np versus log(q) are used to identify a representative decline model.

7.6.1 Identification of Exponential Decline

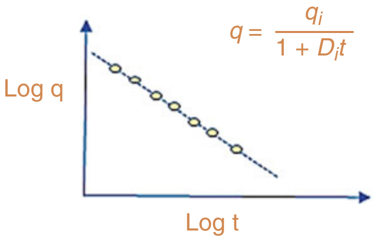
If the plot of log(q) versus t OR q versus Np shows a straight line (see figures below) and in accordance with the respective equations, the decline data follow an exponential decline model.

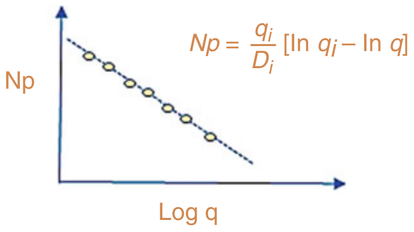




7.6.2 Identification of Harmonic Decline

If the plot of log(q) versus log(t) OR Np versus log(q) shows a straight line (see figures below) and is in accordance with the respective equations, the decline data follow a harmonic decline model.

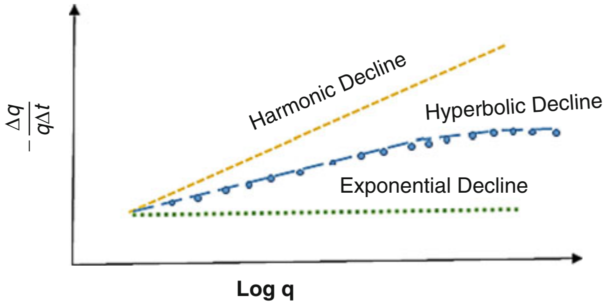




7.6.3 Identification of Hyperbolic Decline

- If no straight line is seen in these plots above, the model may be hyperbolic decline model
- A plot of the relative decline rate vs the flow rate has to be plotted to ascertain the model in accordance with the equation below

$$\frac{1}{q}\frac{dq}{dt} = -D_i q^b$$



7.7 Mathematical Expressions for the Various Types of Decline Curves

The three models are related through the following relative decline rate equation (Arps 1945):

$$\frac{1}{q}\frac{dq}{dt} = -kq^b$$

7.7.1 Exponential (Constant Percent) Decline

The model parameter is given as:

$$D_i = -\frac{1}{q}\frac{dq}{dt} = kq^b$$

When b = 0

$$D_i = -\frac{1}{q}\frac{dq}{dt} = kq^0$$

$$D_i = kq^0 = k = constant$$

Therefore, the decline rate is

$$D_i = -\frac{1}{q}\frac{dq}{dt} = \frac{1}{(t_{i+1} - t_i)}\ln\frac{q_i}{q_{i+1}}$$

The elapse time between two different rates is given as:

$$\ln(q_1) = \ln(q_i) - dt_1$$

$$\ln(q_2) = \ln(q_i) - dt_2$$

$$t_{i+1} - t_i = \frac{1}{D_i}\ln\frac{q_1}{q_2}$$

The abandonment time of a field is given as

$$t_a = \frac{1}{D_i} \ln \frac{q_i}{q_a}$$

Relationship of b at different times is:

$$b_a = 12b_m = 365b_d$$

The exponential production rate can be determined by integrating the decline rate's equation.

$$D_i \int_0^t dt = -\int_{q_i}^{q_t} \frac{dq}{q}$$

$$D_i[t]\begin{matrix} t \\ 0 \end{matrix} = -[\ln q]\begin{matrix} q_t \\ q_i \end{matrix}$$

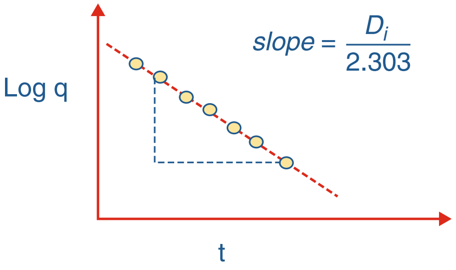
$$-D_i t = \ln q_t - \ln q_i = \ln \frac{q_t}{q_i}$$

$$e^{-D_i t} = \frac{q_t}{q_i}$$

$$q_t = q_i e^{-D_i t}$$

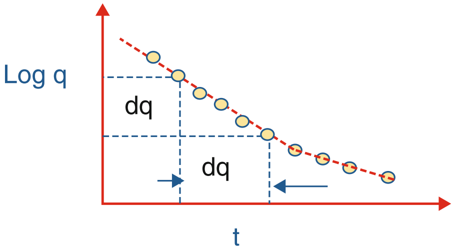
$$\log q_t = \log q_i - \frac{D_i}{2.303} t$$

Therefore, a plot of log q_t Vs t on a semi-log graph yield slope as $-\frac{D_i}{2.303}$.

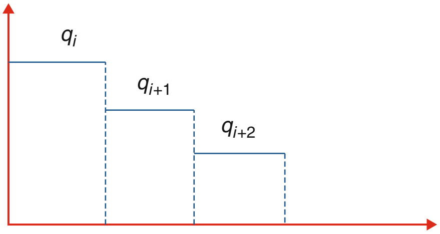


7.7.1.1 Relationship Between Nominal and Effective Decline Rate

The nominal decline rate (***D_i***) is defined as the negative slope of the curvature representing the natural logarithm of the production rate versus time.



Effective decline rate **(D_i')** is defined as the drop in production rate from initial rate to a current rate over a period of time divided by the production rate at the beginning of the period as shown in the figure below



Mathematically is given as:

$$D_i' = \frac{q_i - q_{i+1}}{q_i}$$

$$D_i' q_i = q_i - q_{i+1}$$

$$q_{i+1} = q_i(1 - D_i')$$

Comparing with the exponential production rate above, we have

$$e^{-D_i} = 1 - D_i'$$

Thus,

$$(1 - D_a') = (1 - D_m')^{12} = (1 - D_d')^{365}$$

7.7.1.2 Cumulative Production for Exponential Decline

The Integration of the production rate over time gives an expression for the cumulative oil production as:

$$N_p = \int_o^t q dt = \int_o^t q_i e^{-D_i t} dt$$

Let

$$u = D_i t \qquad \therefore \frac{du}{dt} = D_i$$

$$\rightarrow \quad dt = \frac{du}{D_i}$$

Substituting into the above equation gives

$$N_p = \int_o^t q_i e^{-D_i t} dt = \int_o^t q_i e^{-u} * \frac{du}{D_i} = \frac{q_i}{D_i} \int_o^t e^{-u} * du$$

$$N_p = -\frac{q_i}{D_i} e^{-u} \Big|_0^t \qquad but\ u = D_i t$$

$$N_p = -\frac{q_i}{D_i} e^{-D_i t} \Big|_0^t = -\frac{q_i}{D_i} \left[e^{-D_i t} - e^0 \right]$$

$$N_p = -\frac{q_i}{D_i} \left[e^{-D_i t} - 1 \right] = \frac{q_i}{D_i} \left[1 - e^{-D_i t} \right] = \frac{1}{D_i} \left[q_i - q_i e^{-D_i t} \right]$$

Recall

$$q_t = q_i e^{-D_i t}$$

$$\therefore \ \boldsymbol{N_p = \frac{q_i - q_t}{D_i}}$$

$$q_t = q_i - D_i N_p$$

The decline rate can also be calculated from cumulative production as

$$q_1 = q_i - D_i N_{p1}$$

$$q_2 = q_i - D_i N_{p2}$$

$$D_i = \frac{q_1 - q_2}{N_{p2} - N_{p1}}$$

7.7.1.3 Steps for Exponential Decline Curve Analysis

The following steps are taken for exponential decline analysis, for predicting future flow rates and recoverable reserves (Tarek, 2010):

- Plot flow rate vs. time on a semi-log plot (y-axis is logarithmic) and flow rate vs. cumulative production on a cartesian (arithmetic coordinate) scale.

- Allowing for the fact that the early time data may not be linear, fit a straight line through the linear portion of the data, and determine the decline rate "D" from the slope (−b/2.303) of the semi-log plot, or directly from the slope (D) of the rate-cumulative production plot.
- Extrapolate to $q = q_t$ to obtain the recoverable hydrocarbons.
- Extrapolate to any specified time or abandonment rate to obtain a rate forecast and the cumulative recoverable hydrocarbons to that point in time

7.7.2 *Harmonic Decline Rate*

When b = 1, the

$$\frac{1}{q}\frac{dq}{dt} = -D_i q^b$$

$$\frac{1}{q}\frac{dq}{dt} = -D_i q$$

$$\frac{1}{q^2}\frac{dq}{dt} = -D_i$$

Yields the differential equation for a harmonic decline model which when integrated gives:

$$\int_{q_i}^{q_t} \frac{1}{q^2} dq = \int_0^t -D_i dt$$

$$\int_{q_i}^{q_t} q^{-2} dq = -D_i \int_0^t dt$$

$$-q^{-1}\Big|_{q_i}^{q_t} = -D_i\left(t\big|_0^t\right)$$

$$-\left(q_t^{-1} - q_i^{-1}\right) = -D_i t$$

$$\left(q_t^{-1} - q_i^{-1}\right) = D_i t$$

$$\left(\frac{1}{q_t} - \frac{1}{q_i}\right) = D_i t$$

$$\left(\frac{q_i - q_t}{q_t q_i}\right) = D_i t$$

$$\left(\frac{q_i - q_t}{q_i}\right) = q_t D_i t$$

$$\left(1 - \frac{q_t}{q_i}\right) = q_t D_i t$$

$$1 = \frac{q_t}{q_i} + q_t D_i t = \frac{q_t}{q_i}(1 + D_i t)$$

$$q_t = \frac{q_i}{(1 + D_i t)}$$

$$1 + D_i t = \frac{q_i}{q_t}$$

Taking natural log of both sides

$$\ln(1 + D_i t) = \ln\left(\frac{q_i}{q_t}\right) = \ln q_i - \ln q_t$$

7.7.2.1 Cumulative Production for Harmonic Decline

The expression for the cumulative production for a harmonic decline is obtained by integration of the production rate. This is given by:

$$N_p = \int_o^t \left(\frac{q_i}{1 + D_i t}\right) dt$$

Let

$$u = 1 + D_i t \qquad\qquad \therefore \frac{du}{dt} = D_i$$

$$\rightarrow \quad dt = \frac{du}{D_i}$$

Therefore, the cumulative production can be re-written as:

$$N_p = \int_o^t \frac{q_i}{u} * \frac{du}{D_i} \quad = \quad \frac{q_i}{D_i} \int_o^t \frac{du}{u}$$

$$N_p = \frac{q_i}{D_i} \ln u \Big|_0^t$$

Recall

$$u = 1 + D_i t$$

$$N_p = \frac{q_i}{D_i} \ln (1 + D_i t) \Big|_0^t$$

$$N_p = \frac{q_i}{D_i} [\ln (1 + D_i t) - \ln (1 + D_i \{0\})]$$

$$\therefore \quad \boldsymbol{N_p = \frac{q_i}{D_i} \ln (1 + D_i t)}$$

By substitution of the above expression, we have:

$$N_p = \frac{q_i}{D_i} [\ln (q_i) - \ln (q_t)]$$

7.7.3 *Hyperbolic Decline*

The hyperbolic decline model is inferred when $0 < b < 1$

Hence the integration of

$$\frac{1}{q}\frac{dq}{dt} = -D_i q^b$$

Gives:

$$\int_{q_i}^{q_t} \frac{dq}{q^{1+b}} = -\int_0^t D_i dt$$

This result in

$$\boldsymbol{q_t = \frac{q_i}{(1 + bD_i t)^{1/b}}}$$

Or

$$\boldsymbol{q_t = \frac{q_i}{\left(1 + \frac{D_i}{a} t\right)^a}}$$

Where $a = 1/b$

7.7.3.1 Cumulative Production for Hyperbolic Decline

Expression for the cumulative production is obtained by integration (Table 7.1):

$$N_p = \int_o^t q dt = \int_o^t \frac{q_i}{\left(1 + \frac{D_i}{a} t\right)^a} dt$$

Table 7.1 Summary of Decline Model

Model	Rate (STB/D)	Cumulative Production (STB)	Time
Exponential	$q_t = q_i e^{-D_i t}$	$N_p = \frac{q_i - q_t}{D_i}$	$t = \frac{1}{D_i} \ln \frac{q_t}{q_t}$
Harmonic	$q_t = \frac{q_i}{(1 + D_i t)}$	$N_p = \frac{q_i}{D_i} \ln (1 + D_i t)$	$t = \frac{1}{D_i} \left(\frac{q_i}{q_t} - 1 \right)$
Hyperbolic	$q_t = \frac{q_i}{\left(1 + \frac{D_i}{a} t\right)^a}$	$N_p = \frac{q_i}{(1-b) D_i} \left\{ 1 - \left(\frac{q_t}{q_i} \right)^{1-b} \right\}$	$t = \frac{1}{0.5 D_i} \left(\sqrt{\frac{q_i}{q_t}} - 1 \right)$

Let

$$u = 1 + \frac{D_i}{a}t \qquad\qquad \therefore \frac{du}{dt} = \frac{D_i}{a}$$

$$\rightarrow \quad dt = \frac{a}{D_i}.du$$

$$N_p = \int_o^t \frac{q_i}{u^a}.\frac{a}{D_i}du = \frac{aq_i}{D_i}\int_o^t \frac{1}{u^a}.du = \frac{aq_i}{D_i}\int_o^t \frac{1}{u^a}.du = \frac{aq_i}{D_i}\int_o^t u^{-a}.du$$

$$N_p = \frac{aq_i}{D_i}.\left(\frac{1}{-a+1}\right).u^{-a+1}\Big|_0^t$$

$$N_p = \left(\frac{1}{(1-a)}.\frac{aq_i}{D_i}\right).\left(1 + \frac{D_i}{a}t\right)^{1-a}\Bigg|_0^t$$

$$N_p = \left(\frac{1}{(1-a)}.\frac{aq_i}{D_i}\right).\left\{\left(1 + \frac{D_i t}{a}\right)^{1-a} - 1\right\}$$

Multiplying through by (−1/−1), we have

$$N_p = \left(\frac{1}{(a-1)}.\frac{aq_i}{D_i}\right).\left\{1 - \left(1 + \frac{D_i t}{a}\right)^{1-a}\right\}$$

$$N_p = \frac{a}{(a-1)D_i}.\left\{q_i - q_i\left(1 + \frac{D_i t}{a}\right)^{1-a}\right\}$$

Recall that:

$$q_i = q_t\left(1 + \frac{D_i t}{a}\right)^a$$

$$N_p = \frac{a}{(a-1)D_i}.\left\{q_i - q_t\left(1 + \frac{D_i t}{a}\right)^a\left(1 + \frac{D_i t}{a}\right)^{1-a}\right\}$$

$$N_p = \frac{a}{(a-1)D_i}.\left\{q_i - q_t\left(1 + \frac{D_i t}{a}\right)\right\}$$

Recall also that:

$$a = \frac{1}{b}$$

$$N_p = \frac{{}^{1}\!/_{b}}{\left({}^{1}\!/_{b} - 1\right)D_i}.\{q_i - q_t(1 + bD_it)\}$$

$$N_p = \frac{1}{b\left(\frac{1-b}{b}\right)D_i}.\{q_i - q_t(1 + bD_it)\}$$

$$N_p = \frac{1}{(1-b)D_i}.\{q_i - q_t(1 + bD_it)\}$$

Multiply through by

$$\frac{q_i{}^b}{q_i{}^b}$$

Gives

$$N_p = \frac{q_i{}^b}{(1-b)D_i}.\left\{\frac{q_i}{q_i{}^b} - \frac{q_t}{q_i{}^b}(1 + bD_it)\right\}$$

$$N_p = \frac{q_i{}^b}{(1-b)D_i}.\left\{q_i{}^{1-b} - \frac{q_t}{q_i{}^b}(1 + bD_it)\right\}$$

Recall also that:

$$q_t = \frac{q_i}{\left(1 + \frac{D_i}{a}t\right)^a} = \frac{q_i}{(1 + bD_it)^{{}^{1}\!/_{b}}}$$

Thus, multiplying the powers by b, gives

$$q_i{}^b = q_t{}^b(1 + bD_it)$$

By substitution

$$N_p = \frac{q_i{}^b}{(1-b)D_i}.\left\{q_i{}^{1-b} - \frac{q_t(\cancel{1 + bD_it})}{q_t{}^b(\cancel{1 + bD_it})}\right\}$$

$$N_p = \frac{q_i{}^b}{(1-b)D_i}.\left\{q_i{}^{1-b} - q_t{}^{1-b}\right\}$$

$$N_p = \frac{q_i}{(1-b)D_i}\left\{1 - \left(\frac{q_t}{q_i}\right)^{1-b}\right\}$$

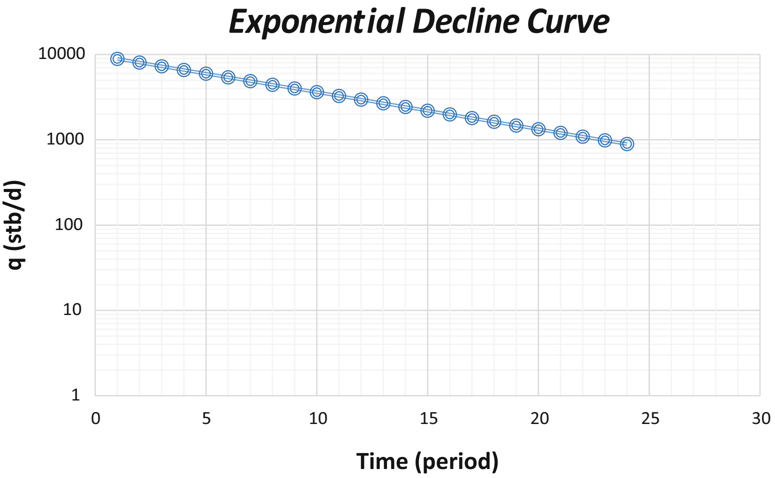
Example 7.1
An onshore field located at Okuatata as being on production for the past 2 years (24 months) given in the table below. As a production engineer hired by an operating company, you are required to perform the following tasks:

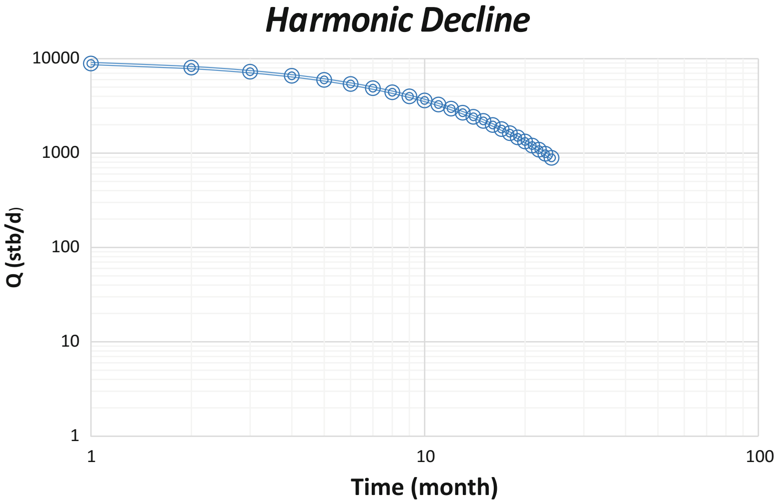
- Identify a suitable decline model
- Determine model parameters
- Project production rate until a marginal rate of 280 stb/day is reached.

Okuatata Field Production Data for 24 months

t (Month)	q (STB/D	t (Month)	q (STB/D
0	9100	13	2678.22
1	8892.18	14	2423.38
2	8045.93	15	2192.82
3	7280.31	16	1984.17
4	6587.44	17	1795.25
5	5960.59	18	1624.5
6	5393.36	19	1469.84
7	4880.12	20	1330.08
8	4415.74	21	1203.45
9	3995.47	22	1088.86
10	3615.28	23	985.322
11	3271.21	24	891.557
12	2959.91		

Solution
Based on the criteria stated above for decline curve model identification, the Okuatata oil field's production is undergoing an exponential decline as depicted in the plots below.





Model parameter is calculated as

Select points on the trend line:

$t_1 = 5$ months,

$q_1 = 5960.59$ STB/D

$t_2 = 15$ months,

$q_2 = 2192.82$ STB/D

$$D_i = -\frac{1}{q}\frac{dq}{dt} = \frac{1}{(t_{i+1} - t_i)} \ln \frac{q_i}{q_{i+1}}$$

$$D_i = \frac{1}{(t_2 - t_1)} \ln \frac{q_1}{q_2}$$

$$D_i = \frac{1}{(15 - 5)} \ln \left(\frac{5960.59}{2192.82} \right) = 0.0999 \, per \; month$$

The abandonment time of a field is given as

$$t_a = \frac{1}{D_i} \ln \frac{q_i}{q_a}$$

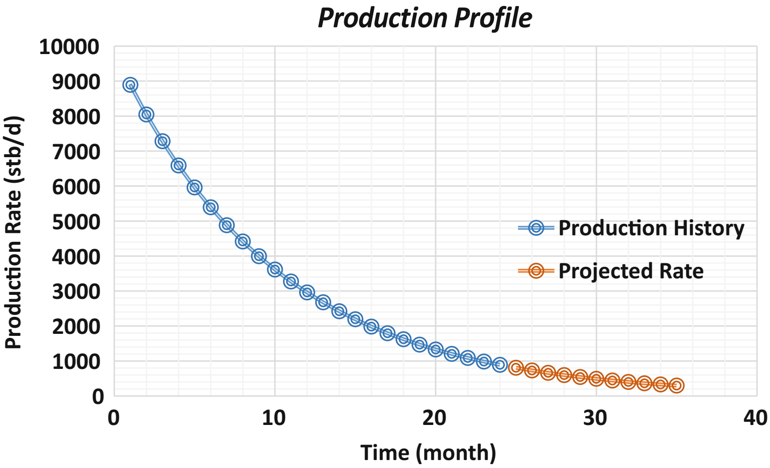
$$t_a = \frac{1}{0.0999} \ln \frac{9100}{280} = 34.8473 \, months$$

Applying the exponential decline rate equation, the projected rate profile is generated thus:

$$q_t = q_i e^{-D_i t}$$

25	806.795
26	730.091
27	660.68
28	597.868
29	541.027
30	489.591
31	443.044
32	400.923
33	362.806
34	328.314
35	297.1

The plot of Okuatata historic production and projected rates are given in the plot below



Example 7.2

A well that started production at a rate of 1100 stb/d has declined to 850 stb/d at the end of the first year. If the economic limit of the well is 25 stb/d, assuming exponential decline, calculate:

- The yearly and monthly effective decline rates
- The yearly and monthly continuous decline rates
- The life of the well
- The cumulative production

Solution

Yearly and monthly decline rates

$$q_t = q_i(1 - D_a{}')$$

$$q_i = 1100\ stb/d, \quad q_t = 850\ stb/d \quad q_a = 25\ stb/d$$

$$850 = 1100(1 - D_a{}')$$

$$1 - D_a{}' = \frac{850}{1100} = 0.7727$$

Therefore, the yearly effective decline rate is

$$D_a{}' = 1 - 0.7727 = 0.2272/yr \approx 22.7\%/yr$$

The monthly effective decline rate is given as:

$$(1 - D_m{}')^{12} = (1 - D_a{}')$$

$$1 - D_m{}' = (1 - D_a{}')^{1/12} = (1 - 0.2272)^{1/12=0.9787}$$

$$\therefore\ D_m{}' = 1 - 0.9787 = 0.021/month$$

Yearly and monthly continuous or nominal decline rates

$$e^{-D_a} = 1 - D_a{}' \qquad but\ D_a = 12D_m$$

Applying the exponential equation

$$q_t = q_i e^{-D_a t}$$

At the first year, t = 1

$$850 = 1100e^{-D_a(1)}$$

$$e^{-D_a} = \frac{850}{1100} = 0.7727$$

Take natural log of both sides

$$-D_a = \ln 0.7727 = -0.2579$$

$$\therefore D_a = 0.2579/yr$$

$$D_m = \frac{D_a}{12} = \frac{0.2579}{12} = 0.0215/month$$

The life of the well

Using 1 year as the unit of time. The rates should be converted to stb/yr. but since they will cancel out, we apply directly without conversion. The life of the well is calculated with respect to the abandonment rate of the well.

$$q_a = q_i e^{-D_a t}$$

$$25 = 1100e^{-0.2579 * t_a}$$

$$e^{-0.2579 * t_a} = \frac{25}{1100} = 0.02273$$

Take natural log of both sides

$$-0.2579 * t_a = \ln 0.02273 = -3.7841$$

$$t_a = \frac{-3.7841}{-0.2579} = 14.67\ yrs$$

The cumulative production is

$$N_p = \frac{q_i - q_t}{D_a}$$

The rates need to be converted to stb/yr.

$$q_i = 1100 \frac{stb}{d} * 365 \frac{days}{yr} = 401500\ stb/yr$$

$$q_a = 25 \frac{stb}{d} * 365 \frac{days}{yr} = 9125\ stb/yr$$

$$\therefore N_p = \frac{401500 - 9125}{0.2579} = 1521423.032\ stb$$

Example 7.3

Use the exponential (b = 0), hyperbolic (b = 0.5), and harmonic (b = 1) method to calculate the cumulative oil production and remaining life of Amassoma oil field whose current production rate over a period of 1 year is 10,000 B.P. to an estimated abandonment rate of 900 BOPD. The initial production of the field was 12,500 BOPD.

Solution
Exponential Decline

Calculate the decline rate

$$D_i = \frac{1}{t} \ln \frac{q_i}{q_t}$$

At time, t = 1 yr

$$D_i = \frac{1}{t} \ln \frac{q_i}{q_t} = \frac{1}{1} * \ln \left(\frac{12500}{10000} \right) = 0.2231/yr$$

The cumulative production is

$$N_p = \frac{q_i - q_t}{D_i}$$

At the economic limit of 900 BOPD

$$\therefore N_p = \frac{(10000 - 900)\frac{bbls}{d} * \frac{365days}{yr}}{0.2231 * \frac{1}{yr}} = 14887942.63\ bbls$$

Calculate the remaining life using the rate-time equation

$$t = \frac{1}{D_i} \ln \frac{q_t}{q_a} = \frac{1}{0.2231} * \ln\left(\frac{10000}{900}\right) = 10.79\ years$$

Hyperbolic Decline

$$q_t = \frac{q_i}{(1 + 0.5D_i t)^{1/0.5}} = \frac{q_i}{(1 + 0.5D_i t)^2} \qquad for\ b = 0.5$$

$$(1 + 0.5D_i t)^2 = \frac{q_i}{q_t}$$

$$1 + 0.5D_i t = \sqrt{\frac{q_i}{q_t}}$$

$$D_i = \frac{1}{0.5t}\left(\sqrt{\frac{q_i}{q_t}} - 1\right)$$

$$D_i = \frac{1}{0.5 * 1}\left(\sqrt{\frac{12500}{10000}} - 1\right) = 0.2361/yr$$

The cumulative oil production

$$N_p = \frac{q_i}{(1 - b)D_i}\left\{1 - \left(\frac{q_a}{q_i}\right)^{1-b}\right\}$$

At the economic limit of 900 BOPD

$$N_p = \frac{10000 * 365}{(1 - 0.5) * 0.2361}\left\{1 - \left(\frac{900 * 365}{10000 * 365}\right)^{1-0.5}\right\} = 21643371.45\ bbls$$

The remaining life of the well

$$t = \frac{1}{0.5D_i}\left(\sqrt{\frac{q_i}{q_t}} - 1\right) = \frac{1}{0.5*0.2361}\left(\sqrt{\frac{10000}{900}} - 1\right) = 19.77\ years$$

Harmonic Decline

$$1 + D_i t = \frac{q_i}{q_t} = \frac{12500}{10000} = 1.25$$

$$D_i t = 1.25 - 1 = 0.25$$

At t = 1 yr

$$D_i = 0.25$$

The cumulative oil production

$$N_p = \frac{q_i}{D_i}\ln(1 + D_i t)$$

At the economic limit of 900 BOPD

$$N_p = \frac{10000*365}{0.25}\ln(1 + 0.25*1) = 3257895.849\ bbls$$

The remaining life of the well

$$t = \frac{1}{D_i}\left(\frac{q_i}{q_a} - 1\right) = \frac{1}{0.25}\left(\frac{10000}{900} - 1\right) = 40.44\ years$$

The summary of the result from the models are given in the table below

Model	Decline Rate (per yr)	Cum. Production (bbls)	Time (yrs)
Exponential	0.2231	14887942.63	10.97
Harmonic	0.25	3257895.849	40.44
Hyperbolic	0.2361	21643371.45	19.77

Example 7.4

KC field located North-East of Cape field in the Niger Delta was discovered in 2014 with an initial oil in place of 458 MMstb. The field started production a year later with an initial oil production of 11,580 stb/day from KC1 well and after a year of exponential decline, the production rate decreased to 9400 stb/day. Predict the cumulative oil production at the end of the 14th year.

Solution

Calculate the decline rate

$$D_i = \frac{1}{t}\ln\frac{q_i}{q_t}$$

At time, t = 1 yr

$$D_i = \frac{1}{t}\ln\frac{q_i}{q_t} = \frac{1}{1} * \ln\left(\frac{11580}{9400}\right) = 0.2086/yr$$

At the end of the first year

$$q_1 = 9400\ stb/d$$

The cumulative production at the end of first year

$$N_{p1} = \frac{q_i - q_1}{D_i} = \frac{11580 - 9400}{0.2086} = 10067.1141\ stb$$

At the end of the second year

$$q_2 = q_1 e^{-D_i t} = 9400e^{-0.2086*1} = 7630.1667\ MMscf/d$$

The cumulative production at the end of second year

$$N_{p2} = \frac{q_1 - q_2}{D_i} = \frac{9400 - 7630.1667}{0.2086} = 8484.3400\ stb$$

$$Cum\ N_{p2} = 10067.1141 + 8484.3400 = 18551.4541\ stb$$

At the end of the third year

$$q_3 = q_2 e^{-D_i t} = 7630.1667e^{-0.2086*1} = 6193.5578\ MMscf/d$$

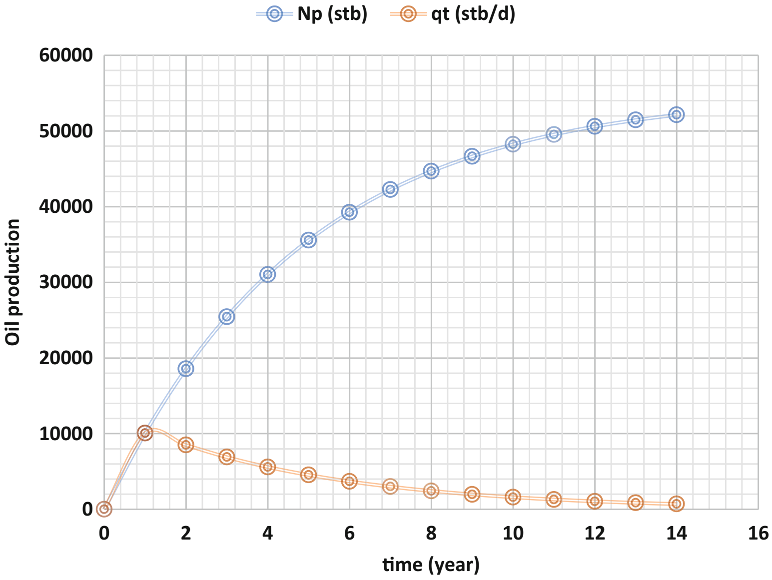
The cumulative production at the end of third year

$$N_{p3} = \frac{q_2 - q_3}{D_i} = \frac{7630.1667 - 6193.5578}{0.2086} = 6886.9073\ stb$$

$$Cum\ N_{p3} = 18551.4541 + 6886.9073 = 25438.3614\ stb$$

The table and figure below is a prediction to the 14th year of production using the approach above

Time (yr)	q_{end} (stb/yr)	Yearly Production (stb)	Cumulative Production stb)
0	11,500	–	–
1	9400	10067.1141	10067.1141
2	7630.1667	8484.3400	18551.4541
3	6193.5578	6886.9073	25438.3614
4	5027.4339	5590.2394	31028.6008
5	4080.8679	4537.7083	35566.3091
6	3312.5215	3683.3480	39249.6571
7	2688.8395	2989.8467	42239.5038
8	2182.5844	2426.9179	44666.4218
9	1771.6471	1969.9775	46636.3992
10	1438.0811	1599.0698	48235.4691
11	1167.3190	1297.9967	49533.4658
12	947.5360	1053.6097	50587.0756
13	769.1338	855.2359	51442.3115
14	624.3212	694.2120	52136.5235



Example 7.5

The result of the volumetric analysis carried out on Akpet gas field gave an estimated ultimate recoverable gas reserves as 30 MMMscf. The economic limit was estimated as 20 MMscf/month and a nominal decline rate of 0.03 per month. The allowable or restricted production rate given by the department of petroleum resources is 400 MMscf/month. Calculate the life of the well, the prorated (restricted) time and the yearly production performance of the well. Assume exponential decline.

Solution
The life of the well is

$$t_a = \frac{1}{D_i} \ln\left(\frac{q_r}{q_a}\right) = \frac{1}{0.03} \ln\left(\frac{400}{20}\right) = 99.86\, months \approx 100\, months = 8.32\, yrs$$

To calculate the restricted time, we can do this in two ways

Case 1: The Procedure Is
Cumulative gas production during the restricted rate

$$G_p = \frac{q_r - q_a}{D_i} = \frac{400 - 20}{0.03} = 12666.67\, MMscf$$

The reserve during the restricted rate is

$$= ultimate\ recoverable\ gas\ reserve - cum.gas\, production\ during\ the\ restriction$$

$$= 30000 - 12666.67 = 17333.33\, MMscf$$

The time during the restricted production is

$$t_a = \frac{Reserve\ during\ restriction}{restricted\ production} = \frac{17333.33}{400} = 43.33\, months$$

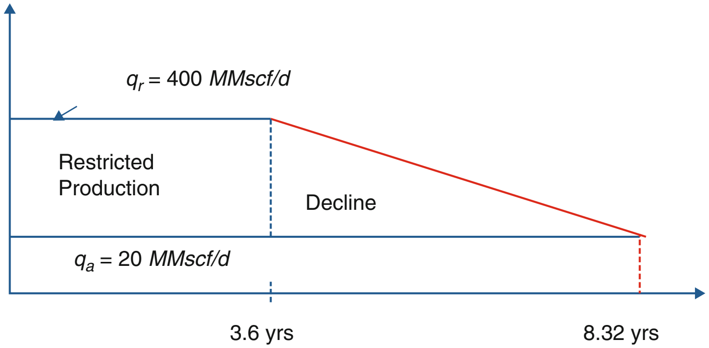
Case 2: The Procedure Is
The initial production rate is

$$q_i = G_p D_i + q_a = (0.03 * 30000) + 20 = 920\, MMscf/month$$

The cumulative gas production during the restricted period is

$$G_{pr} = \frac{q_i - q_r}{D_i} = \frac{920 - 400}{0.03} = 17333.33\, MMscf$$

$$t_a = \frac{Reserve\ during\ restriction}{restricted\ production} = \frac{17333.33}{400} = 43.33\, months = 3.6\, yrs$$



To prepare the production forecast, the restricted production period span for 3.6 years (three and half years). Therefore, in the first 3 years of production, the yearly production is given as:

$$G_{p1} = G_{p2} = G_{p3} = 400\frac{MMscf}{month} * 12\frac{months}{yr} = 4800\,MMscf/yr$$

The production in the fourth year is divided into 0.6 years (an equivalent of 7.2 months = 0.6 yrs. *12 month/yrs. = 4.8 months) at constant production plus 4.8 months of declining production.

For the constant 7.2 months straight line production (restricted production)

$$G_{pc} = 400\frac{MMscf}{month} * 7.2\frac{months}{yr} = 2880\,MMscf/yr$$

For the declining period of 4.8 months production

$$q_4 = q_r e^{-D_i t} = 400e^{-0.03*4.8} = 346.36\,MMscf/d$$

The cumulative production for the 4.8 months of decline

$$G_{pd} = \frac{q_r - q_4}{D_i} = \frac{400 - 346.36}{0.03} = 1788\,MMscf$$

Therefore, the total production for the fourth years

$$G_{p4} = 2880 + 1788 = 4668\,MMscf$$

At the end of the fifth year

$$q_5 = q_4 e^{-D_i t} = 346.36e^{-0.03*12} = 241.65\,MMscf/d$$

The cumulative production at the end of fifth year

$$G_{p5} = \frac{q_4 - q_5}{D_i} = \frac{346.36 - 241.65}{0.03} = 3490.3333\,MMscf$$

At the end of the sixth year

$$q_6 = q_5 e^{-D_i t} = 241.65 e^{-0.03*12} = 168.599\,MMscf/d$$

The cumulative production at the end of sixth year

$$G_{p6} = \frac{q_5 - q_6}{D_i} = \frac{241.65 - 168.599}{0.03} = 2435.033\,MMscf$$

At the end of the seventh year

$$q_7 = q_6 e^{-D_i t} = 168.599 e^{-0.03*12} = 117.63\,MMscf/d$$

The cumulative production at the end of seventh year

$$G_{p7} = \frac{q_6 - q_7}{D_i} = \frac{168.599 - 117.63}{0.03} = 1698.9667\,MMscf$$

At the end of the 8 year

$$q_8 = q_7 e^{-D_i t} = 117.632 e^{-0.03*12} = 82.072\,MMscf/d$$

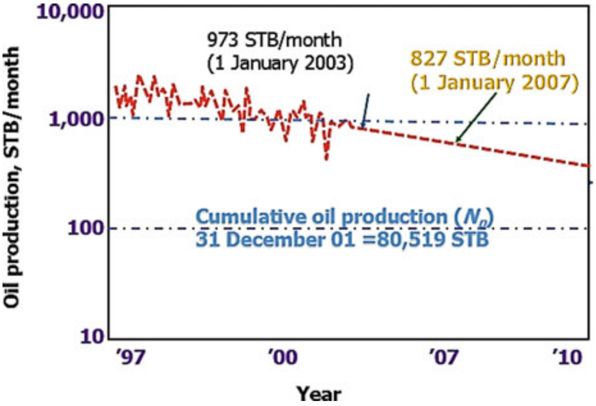
The cumulative production at the end of 8 year

$$G_{p8} = \frac{q_7 - q_8}{D_i} = \frac{117.63 - 82.072}{0.03} = 1185.2667\,MMscf$$

The summary of result is:

T (years)	q_i (MMscf/month)	q_{end} (MMscf/month)	Yearly Production (MMscf)	Cumulative Production (MMscf)
1	400	400	4800	4800
2	400	400	4800	9600
3	400	400	4800	14,400
4	400	346.36	4668	19,068
5	346.36	241.65	3490.333	22558.333
6	241.65	168.599	2435.033	24993.366
7	168.599	117.63	1698.9667	26692.3327
8	117.63	82.072	1185.2667	27877.5994

Example 7.6
The production trend of Okoso oil field is presented in the figure below. The estimated economic limit of this field is 190 stb/month.



Calculate

- Effective (*D'*) and nominal (*D*) decline rates
- Remaining reserves (N_P) from 1 January 2003 to the field's economic limit
- Time (*t*) to produce to economic limit
- Ultimate oil recovery (N_{ul}) to economic limit
- Production rate (q_{o5}) at end of year 2008

Solution
The effective rate is:

$$D_i' = \frac{q_i - q_{i+1}}{q_i} = \frac{973 - 827}{973} = 0.1501/yr = 15.01\%/yr$$

Nominal decline rate is:

$$e^{-D_i} = 1 - D_i'$$

$$D_i = -\ln(1 - D_i') = \ln(1 - 0.1501) = 0.1626/yr = 16.26\%/yr$$

The remaining reserve:

$$N_r = \frac{q_i - q_a}{D_i} = \frac{q_i - q_{EL}}{D_i} = \frac{(973 - 190)\ {}^{stb}/_{month} * {}^{12 months}/_{yr}}{0.1626/yr} = 57785.97\ stb$$

Abandonment time (time to economic limit)

$$t_a = \frac{1}{D_i} \ln\left(\frac{q_i}{q_{EL}}\right) = \frac{1}{0.1626} \ln\left(\frac{973}{190}\right) = 10.05\ yrs$$

The ultimate oil recovery to economic limit is

$$N_{UL} = N_p + N_r = 80519 + 57785.97 = 138304.97\ stb$$

The production rate at the end of 2008 is

$$q_{o5} = q_{04} e^{-D_i t}$$

$$q_{04} = 827\ stb/month$$

$$q_{o5} = 827 e^{-0.1626*12} = 117.52\ stb/month$$

Exercises

Ex 7.1 The production history of K35 field is given in the table below, calculate the following:

I. The decline model
II. The model parameters
III. Projected production rate until the end of the fifth year.

t (yr)	q (1000 stb/d)	t (yr)	q (1000 stb/d)
0.11	10.59	2.31	5.70
0.22	10.21	2.42	5.56
0.33	9.85	2.53	5.41
0.44	9.50	2.64	5.28
0.55	9.19	2.75	5.15
0.66	8.88	2.86	5.03
0.77	8.59	2.97	4.91
0.88	8.31	3.08	4.79
0.99	8.05	3.19	4.68
1.10	7.80	3.30	4.57
1.21	7.56	3.41	4.47
1.32	7.34	3.52	4.37
1.43	7.12	3.63	4.27
1.54	6.91	3.74	4.18
1.65	6.71	3.85	4.08
1.76	6.52	3.96	4.00
1.87	6.35	4.07	3.92

(continued)

t (yr)	q (1000 stb/d)	t (yr)	q (1000 stb/d)
1.98	6.17	4.18	3.84
2.09	6.01	4.29	3.75
2.20	5.85	4.40	3.67

Ex 7.2 The production history of K38 field is given in the table below, calculate the following:

I. The decline model
II. The model parameters
III. Projected production rate until the end of the fifth year.

t (yr)	q (1000 stb/d)	t (yr)	q (1000 stb/d)
0.11	10.59	2.31	5.70
0.22	10.21	2.42	5.56
0.33	9.85	2.53	5.41
0.44	9.50	2.64	5.28
0.55	9.19	2.75	5.15
0.66	8.88	2.86	5.03
0.77	8.59	2.97	4.91
0.88	8.31	3.08	4.79
0.99	8.05	3.19	4.68
1.10	7.80	3.30	4.57
1.21	7.56	3.41	4.47
1.32	7.34	3.52	4.37
1.43	7.12	3.63	4.27
1.54	6.91	3.74	4.18
1.65	6.71	3.85	4.08
1.76	6.52	3.96	4.00
1.87	6.35	4.07	3.92
1.98	6.17	4.18	3.84
2.09	6.01	4.29	3.75
2.20	5.85	4.40	3.67

Briefly explain how the following causes decline in production:

(i) Changes in bottom hole pressure (BHP)
(ii) Gas-oil ratio (GOR)
(iii) Water-oil ratio (WOR)

Briefly explain how the following reservoir factors affect Decline Rate

(i) Pressure depletion
(ii) Number of producing wells
(iii) Drive mechanism

(iv) Reservoir characteristics
(v) Saturation changes
(vi) Relative permeability.

Ex 7.3 Given that a well has declined from 950 stb/day to 780 stb/day during a one-month period, use the exponential decline model to determine the following

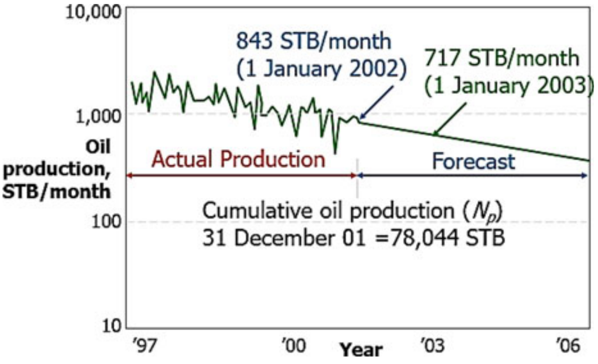
1. Predict the production rate after 11 more months
2. Calculate the amount of oil produced during the first year
3. Project the yearly production for the well for the next 5 years.

Ex 7.4 The volumetric calculations on a gas well show that the ultimate recoverable Reserves, Gpa, are 23 MMMscf of gas. By analogy with other wells in the area, the following data are assigned to the well:

- Exponential decline
- Allowable (restricted) production rate = 415 MMscf/month
- Economic limit = 22 MMscf/month
- Nominal decline rate = 0.038 month^{-1}

Calculate the yearly production performance of the well.

Ex 7.5



Calculate

- Effective (a) and nominal (d) decline rates
- Remaining reserves (N_P) from 1 January 2002 to EL of 200 STB/month
- Time (t) to produce from 1 January 2002 to EL
- Ultimate oil recovery (N_{ul}) to EL
- Production rate (q_{04}) at end of year 2004

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Chapter 8
Pressure Regimes and Fluid Contacts

Learning Objectives
Upon completion of this chapter, students/readers should be able to:

- Describe the various pressure regimes
- Write the mathematical expression for the different pressure regimes
- Know the range of the different fluids gradient
- Understand the causes of abnormal pressure
- Understand the various method for determining fluid contacts
- Calculate the average pressure of a reservoir with multiple wells using pressure-depth survey data
- Calculate gas-oil and oil-water contacts

Nomenclature

Parameter	*Symbol*	*Unit*
Oil, gas & water pressure	$P_{oil}, \ P_{gas} \ \& \ P_{water}$	*psia*
Depth	D	*ft*
Water saturation	S_w	–
Fluid gradient	$\frac{dP}{dD}$	*psi/ft*
Oil-water contact	OWC	*ft*
Gas-oil contact	GOC	*ft*
Initial reservoir pressure	P_i	*psia*
Current/average reservoir pressure	P	*psia*
Gas deviation/compressibility factor	z	–
Cumulative gas produced	G_p	*scf*
Gas initially in place	G	*scf*
Oil formation volume factor	B_o	*rb/stb*
Initial oil formation volume factor	B_{oi}	*rb/stb*

(continued)

S. Okotie, B. Ikporo, *Reservoir Engineering*,
https://doi.org/10.1007/978-3-030-02393-5_8

Parameter	*Symbol*	*Unit*
Cumulative oil produced	N_p	*stb*
Stock tank oil initially in place	N	*stb*
Effective oil compressibility	C_{oe}	$psia^{-1}$
Oil, gas & water compressibility	C_o, C_g & C_w	$psia^{-1}$

8.1 Introduction

The main source of energy during primary hydrocarbon recovery is the pressure of the reservoir. At any given time in the reservoir, the average reservoir pressure is an indication of how much gas, oil or water is remaining in the porous rock media. This represents the amount of the driving force available to push the remaining hydrocarbon out of the reservoir during a production sequence. Most reservoir systems are identified to be heterogeneous and it is worthy to note that the magnitude and variation of pressure across the reservoir is a paramount aspect in understanding the reservoir both in exploration and development (production) phases (Fig. 8.1).

Hydrocarbon reservoirs are discovered at some depths beneath the earth crust as a result of depositional process and thus, the pore pressure of a fluid is developed within a rock pore space due to physical, chemical and geologic processes through time over an area of sediments. There are three identified pressure regimes:

- Normal (relative to sea level and water table level, i.e. hydrostatic)
- Abnormal or overpressure (i.e. higher than hydrostatic)
- Subnormal or underpressure (i.e. lower than hydrostatic)

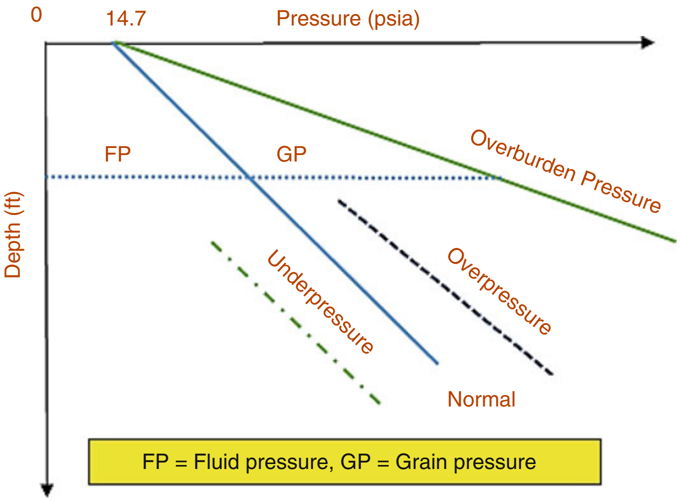


Fig. 8.1 Pressure regime

Fluid pressure regimes in hydrocarbon columns are dictated by the prevailing water pressure in the vicinity of the reservoir (Bradley 1987). In a perfectly normal pressure zone, the water pressure at any depth can be calculated as:

$$P_{water} = \left(\frac{dP}{dD}\right)_{water} D + 14.7 \qquad (psia)$$

Where $\left(\frac{dP}{dD}\right)_{water}$ = the water pressure gradient, which is dependent on the chemical composition (salinity), and for pure water has the value of 0.4335 psi/ft.

Contrary to the normal pressure zone, the abnormal hydrostatic pressure is encountered and can be defined by mathematical equation as:

$$P_{water} = \left(\frac{dP}{dD}\right)_{water} D + 14.7 + C \qquad (psia)$$

Where C is a constant that is positive if the water is overpressured and negative if underpressured (Dake, 1978).

8.2 Pressure Regime of Different Fluids

$$P_{water} = \left(\frac{dP}{dD}\right)_{water} D + 14.7$$

$$P_{oil} = \left(\frac{dP}{dD}\right)_{oil} D + C_o$$

$$P_{gas} = \left(\frac{dP}{dD}\right)_{gas} D + C_g$$

Typical values of pressure gradient for the different fluids are:

$$\left(\frac{dP}{dD}\right)_{water} = 0.45\, psi/ft$$

$$\left(\frac{dP}{dD}\right)_{oil} = 0.35\, psi/ft$$

$$\left(\frac{dP}{dD}\right)_{gas} = 0.08\, psi/ft$$

8.3 Some Causes of Abnormal Pressure

- **Incomplete compaction of sediments**

Fluids in sediments have not escaped and are still helping to support the overburden.

- **Aquifers in Mountainous Regions**

Aquifer recharge is at higher elevation than drilling rig location.

- **Charged shallow reservoirs due to nearby underground blowout.**
- **Large structures**
- **Tectonic movements**

Abnormally high pore pressures may result from local and regional tectonics. The movement of the earth's crustal plates, faulting, folding, lateral sliding and slipping, squeezing caused by down dropped of fault blocks, diapiric salt and/or shale movements, earthquakes, etc. can affect formation pore pressures.

Due to the movement of sedimentary rocks after lithification, changes can occur in the skeletal rock structure and interstitial fluids. A fault may vertically displace a fluid bearing layer and either create new conduits for migration of fluids giving rise to pressure changes or create up-dip barriers giving rise to isolation of fluids and preservation of the original pressure at the time of tectonic movement.

When crossing faults, it is possible to go from normal pressure to abnormally high pressure in a short interval. Also, thick, impermeable layers of shale (or salt) restrict the movement of water. Below such layers abnormal pressure may be found. High pressure occurs at the upper end of the reservoir and the hydrostatic pressure gradient is lower in gas or oil than in water.

8.4 Fluid Contacts

In the volumetric estimation of a field's reserve, the initial location of the fluid contacts and also for the field development, the current fluid contacts are very critical factor for adequate evaluation of the hydrocarbon prospect. Typically, the position of fluid contacts are first determined within control wells and then extrapolated to other parts of the field. Once initial fluid contact elevations in control wells are determined, the contacts in other parts of the reservoir can be estimated. Initial fluid contacts within most reservoirs having a high degree of continuity are almost horizontal, so the reservoir fluid contact elevations are those of the control wells.

Estimation of the depths of the fluid contacts, gas/water contact (GWC), oil/water contact (OWC), and gas/oil contact (GOC) can be made by equating the pressures of the fluids at the said contact. Such that at GOC, the pressure of the gas is equal to the pressure of the oil and the same concept holds for OWC.

Mathematically, at GOC:

$$P_{oil} = P_{gas}$$

Therefore,

$$\left(\frac{dP}{dD}\right)_{oil} D + 14.7 + C_o = \left(\frac{dP}{dD}\right)_{gas} D + C_g$$

8.4.1 Methods of Determining Initial Fluid Contacts

8.4.1.1 Fluid Sampling Methods

This is a direct measurement of fluid contact such as: Production tests, drill stem tests, repeat formation tester (RFT) tests (Schlumberger, 1989). These methods have some limitation which are:

- Rarely closely spaced, so contacts must be interpolated
- Problems with filtrate recovery on DST and RFT
- Coring, degassing, etc. may lead to anomalous recoveries

8.4.1.2 Saturation Estimation from Wireline Logs

It is the estimation of fluid contacts from the changes in fluid saturations or mobility with depth, it is low cost and accurate in simple lithologies and rapid high resolution but have limitations as:

- Unreliable in complex lithologies or low resistivity sands
- Saturation must be calibrated to production

8.4.1.3 Estimation from Conventional and Sidewall Cores

Estimates fluid contacts from the changes in fluid saturation with depth which can be related to petrophysical properties. It can estimates saturation for complex lithologies (Core Laboratories, 2002). The limitations are:

- Usually not continuously cored, so saturation profile is not as complete
- Saturation measurements may not be accurate

8.4.1.4 Pressure Methods

There are basically two types of pressure methods: the pressure profiles from repeat formation tester and pressure profiles from reservoir tests, production tests and drill stem tests.

8.4.1.5 Pressure Profiles from Repeat Formation Tester

It estimates free water surface from inflections in pressure versus depth curve.

8.4.1.6 Pressure Profiles from Reservoir Tests, Production Tests and Drill Stem Tests

It estimates free water surface from pressures and fluid density measurements which makes use of widely available pressure data.

Both pressure techniques are pose with limitations such as:

- Data usually require correction
- Only useful for thick hydrocarbon columns
- Most reliable for gas contacts, Requires many pressure measurements for profile, Requires accurate pressures

Example 8.1

The result of an RFT tests conducted on an appraisal well in a field located in the Niger Delta region is presented in the table below. Determine the types of hydrocarbons present and find the fluid contacts

Depth TVD (ft)	Formation Pressure (psia)
12,893	6375
12,966	6381
12,986	6382
13,128	6422
13,166	6435
13,249	6465
13,308	6484
13,448	6547
13,458	6551
13,500	6570
13,532	6587

Solution

A plot of depth versus pressure is represented in the figure below.

The gas gradient is:

$$\left(\frac{dP}{dD}\right)_{gas} = \frac{\Delta P}{\Delta D} = \frac{P_2 - P_1}{D_2 - D_{1.}}$$

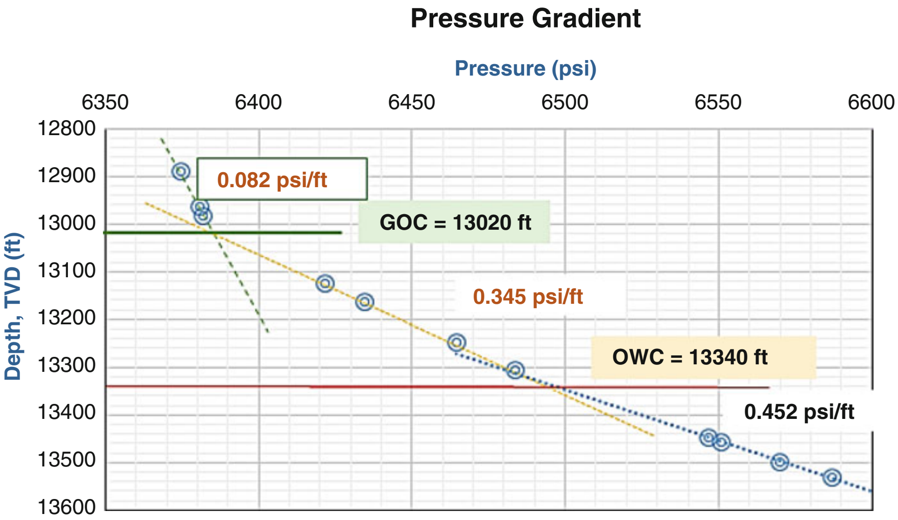
$$\left(\frac{dP}{dD}\right)_{gas} = \frac{6381 - 6375}{12966 - 12893} = 0.082\ psia/ft$$

The oil gradient is:

$$\left(\frac{dP}{dD}\right)_{oil} = \frac{6484 - 6435}{13308 - 13166} = 0.345\ psia/ft$$

The water gradient is:

$$\left(\frac{dP}{dD}\right)_{water} = \frac{6570 - 6551}{13500 - 13458} = 0.452\ psia/ft$$



Example 8.2

A pressure survey was carried out on a well that penetrates through the gas zone in a reservoir at FUPRE. The result of test 1 recorded a pressure of 4450 psia at 9825 ft with fluid gradient of 0.35 psi/ft while test 2 at 9500 ft recorded a pressure of 4180 psia with fluid gradient of 0.11 psi/ft Calculate:

- Estimate the fluid contacts (GOC & OWC) in the reservoir
- The thickness of the oil column
- Calculate the pressures at GOC and OWC respectively

Hint: take the water gradient as 0.445 psi/ft and atmospheric pressure as 14.69 psia

Solution

From test 1

$$P_{oil} = \left(\frac{dP}{dD}\right)_{oil} D + C_o$$

$$4450 = 0.35 * 9825 + C_o$$

$$C_o = 4450 - 3438.75 = 1011.25\, psia$$

$$\therefore \;\; P_{oil} = 0.35D + 1011.25$$

From test 2

$$P_{gas} = \left(\frac{dP}{dD}\right)_{gas} D + C_o$$

$$4180 = 0.11 * 9500 + C_g$$

$$C_g = 4180 - 1045 = 3135\, psia$$

$$\therefore \;\; P_{gas} = 0.11D + 3135$$

Recall: **at GOC**

$$P_{oil} = P_{gas}$$

$$0.35D + 1011.25 = 0.11D + 3135$$

$$0.35D - 0.11D = 3135 - 1011.25$$

$$0.24D = 2123.75$$

$$\therefore \;\; D = GOC = \frac{2123.75}{0.24} = 8848.96\, ft$$

The water pressure is

$$P_{water} = 0.445D + 14.69$$

At OWC

$$P_{oil} = P_{water}$$

$$0.35D + 1011.25 = 0.445D + 14.69$$

$$1011.25 - 14.69 \quad = 0.445D - 0.35D$$

$$0.095D = 996.56$$

$$D = OWC = \frac{996.56}{0.095} = 10490.11\,ft$$

The thickness of the oil column is

$$= OWC - GOC$$

$$= 10490.11 - 8848.96 = 1641.15\,ft$$

The pressures at the fluid contacts are:

$$P@GOC = (0.35 * 8848.96) + 1011.25 = 4108.39\,psia$$

$$P@OWC = (0.445 * 10490.11) + 14.69 = 4682.79\,psia$$

8.5 Estimate the Average Pressure from Several Wells in a Reservoir

When dealing with oil, the average reservoir pressure is only calculated with material balance when the reservoir is undersaturated (i.e when the reservoir pressure is above the bubble point pressure). Average reservoir pressure can be estimated in two different ways but are not covered in this book (see well test analysis textbooks for details).

- By measuring the long-term buildup pressure in a bounded reservoir. The buildup pressure eventually builds up to the average reservoir pressure over a long enough period of time. Note that this time depends on the reservoir size and permeability (*k*) (i.e. hydraulic diffusivity).
- Calculating it from the material balance equation (MBE) is given below

For a gas well

$$\frac{\bar{P}}{\bar{z}} = \frac{P_i}{z_i}\left[1 - \frac{G_p}{G}\right]$$

For oil well

$$\bar{P} = P_i - \frac{N_p B_o}{B_{oi} C_{oe} N}$$

Where the effective oil compressibility is

$$C_{oe} = \frac{C_o S_o + S_{wi} C_w + C_f}{1 - S_{wi}}$$

Example 8.3

An engineer uses pressure-depth survey data to calculate the average pressure value of a reservoir but discovers that all ten wells clearly indicate two distinct reservoirs. Using 9650 ftss as Datum depth and the survey results listed below, calculate the average pressure of the reservoir.

Well A = 3774 psig at 9520 ftss	Well F = 3678 psig at 9545 ftss
Well B = 3815 psig at 9700 ftss	Well G = 3744 psig at 9815 ftss
Well C = 3699 psig at 9620 ftss	Well H = 3779 psig at 9510 ftss
Well D = 3718 psig at 9710 ftss	Well I = 3749 psig at 9820 ftss
Well E = 3761 psig at 9845 ftss	Well J = 3703 psig at 9630 ftss

Take gas gradient = 0.09 psi/ft, oil gradient = 0.32 psi/ft, water gradient = 0.434 psi/ft, GOC = 9530 ft and OWC = 9815.

Solution

The pressure recorded at each well is referred to the datum depth.

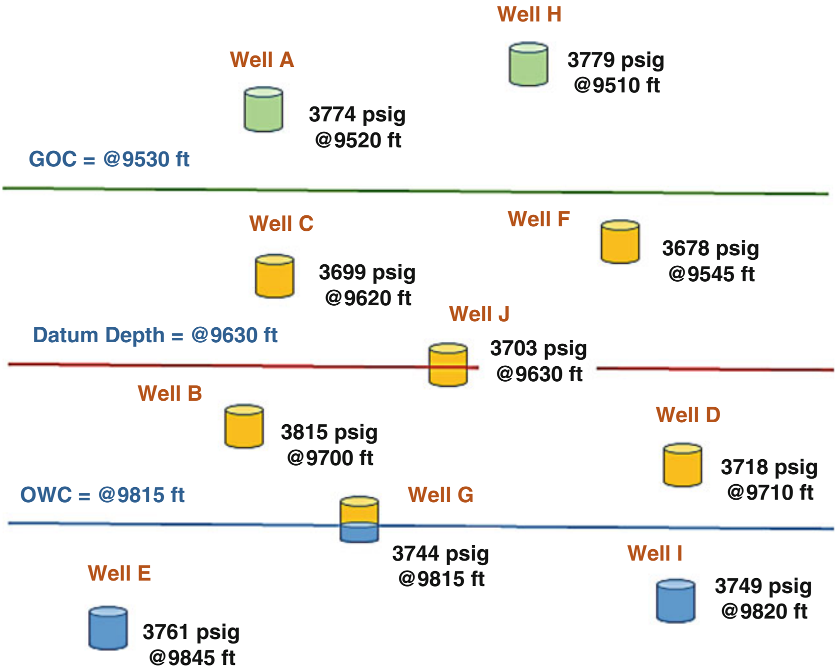
All wells above the datum depth

The pressure recorded is added to the pressure due to the column of fluids (gas, oil & water) in the reservoir. Mathematically it is given as:

$$P = P_{guage} + \left(\frac{dP}{dD}\right)_{fluid} D$$

All wells below the datum

$$P = P_{guage} - \left(\frac{dP}{dD}\right)_{fluid} D$$



Well A

It passes 10 ft through the gas zone and 110 ft in the oil zone, thus

$$P_A = P_{guage} + \left(\frac{dP}{dD}\right)_{gas} D + \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_A@9530ft = (3774 + 14.7) + (0.09*10) + (0.32*110) = 3824.8\ psia$$

Well B

It passes through 70 ft in the oil zone

$$P_B = P_{guage} - \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_B@9700ft = (3815 + 14.7) - 0.32*70 = 3807.3\ psia$$

Well C

It passes through 10 ft in the oil zone, thus

$$P_C = P_{guage} + \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_C@9620 = (3699 + 14.7) + 0.32*10 = 3716.9\,psia$$

Well D
It passes through 80 ft in the oil zone, thus

$$P_D = P_{guage} - \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_D@9710ft = (3718 + 14.7) - 0.32*80 = 3707.1\,psia$$

Well E
It passes 30 ft through the water zone and 185 ft in the oil zone, thus

$$P_E = P_{guage} - \left(\frac{dP}{dD}\right)_{water} D - \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_E@9845ft = (3761 + 14.7) - (0.434*30) - (0.32*185) = 3703.48\,psia$$

Well F
It passes through 85 ft in the oil zone, thus

$$P_F = P_{guage} + \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_F@9545ft = (3678 + 14.7) + 0.32*85 = 3719.9\,psia$$

Well G
It is at the OWC, thus

$$P_G@9815ft = P_{guage} = 3744 + 14.7 = 3758.7\,psia$$

Well H
It passes 20 ft through the gas zone and 100 ft in the oil zone, thus

$$P_H = P_{guage} + \left(\frac{dP}{dD}\right)_{gas} D + \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_H@9510ft = (3779 + 14.7) + 0.09*20 + 0.32*100 = 3827.5\,psia$$

Well I
It passes 5 ft through the water zone and 185 ft in the oil zone, thus

$$P_H = P_{guage} - \left(\frac{dP}{dD}\right)_{water} D - \left(\frac{dP}{dD}\right)_{oil} D$$

$$P_H@9820ft = (3749 + 14.7) - 0.434*5 - 0.32*185 = 3702.33\,psia$$

Well J
It is at the datum depth, thus

$$P_J@9630ft = P_{guage} = 3703 + 14.7 = 3717.7\,psia$$

The average reservoir pressure is

$$\bar{P} = \frac{1}{n}\sum_{i=1}^{n} P_i$$

Where n = total number of wells, P_i = i^{th} well pressure and $\bar{P}$ average reservoir pressure

$$\bar{P} = \frac{1}{10}[3824.8 + 3807.3 + 3716.9 + 3707.1 + 3703.48 + 3719.9$$
$$+3758.7 + 3827.5 + 3702.33 + 3717.7] = \mathbf{3748.571}\,\boldsymbol{psia}$$

Exercises

Ex 8.1 An exploratory well penetrates a reservoir near the top of the oil column. Logs run in the well clearly located the gas-oil contact at 5200 ft also DST test conducted on this well and sample analysis of the fluid sample collected from the same well gave reservoir pressure of 2402 psia at 5250 ft and oil gradient of 0.35 psi/ft the depth of the oil-water contact is uncertain because it could not be confirmed by logs.

- Determine the probable oil-water contact
- What is the pressure at the crest of the reservoir?

Ex 8.2 Calculate the average reservoir pressure at the Datum depth of 8750 ftss for the following fluid pressure gradients, given that the GOC and OWC are at 8700 ftss and 8800 ftss respectively:

$$\left(\frac{dP}{dD}\right)_{gas} = 0.08\,psi/ft, \quad \left(\frac{dP}{dD}\right)_{oil} = 0.269\,psi/ft, \quad \left(\frac{dP}{dD}\right)_{water} = 0.434\,psi/ft$$

Well A = 3685 psig at 8690 ftss
Well B = 3716 psig at 8800 ftss
Well C = 3725 psig at 8820 ftss
Well D = 3689 psig at 8710 ftss
Well E = 3713 psig at 8790 ftss

Ex 8.3 A well penetrates a reservoir near the top of a fluid column. The GOC has been detected by logs but the OWC. An oil sample was taken at 7890 ft TVD with a pore pressure of 3080 psig recorded. The field water gradient is 0.445 psi/ft, oil gradient is 0.347 psi/ft find the OWC.

Ex 8.4 The result of an RFT tests conducted on an appraisal well in a field located in the Niger Delta region is presented in the table below. Determine the types of hydrocarbons present and find the fluid contact.

Depth TVD (ft)	Formation Pressure (psia)
11,200	4648
11,300	4656
11,450	4664
11,500	4672
11,600	4730
11,700	4745
11,820	4778
11,900	4810

Ex 8.5 The result of an RFT tests conducted on an appraisal well in a field located in the Niger Delta region is presented in the table below. Determine the types of hydrocarbons present and find the fluid contacts

Depth TVD (ft)	Formation Pressure (psia)
11,762	5816
11,829	5821
11,847	5822
11,977	5859
12,011	5871
12,087	5898
12,141	5915
12,269	5973
12,278	5976
12,316	5994
12,345	6009

Ex 8.6 A pressure survey was carried out on a well that penetrates through the gas zone in a reservoir at FUPRE. The result of test 1 recorded a pressure of 3830 psia at 9525 ft with fluid gradient of 0.352 psi/ft while test 2 at 9200 ft recorded a pressure of 3560 psia with fluid gradient of 0.118 psi/ft. Calculate:

- Estimate the fluid contacts (GOC & OWC) in the reservoir
- During history match, it was observed that the fluid contacts given by the geologists were wrong which was traceable to wrong fluid gradient. After careful analysis, it was observed that the oil gradient is 0.341 psi/ft recomputed the fluid contacts and estimate the absolute relative error.
- The thickness of the oil column
- Calculate the pressures at GOC and OWC respectively

Hint: take the water gradient as 0.445 psi/ft and atmospheric pressure as 14.69 psia

Ex 8.7 An exploratory well penetrates a reservoir near the top of the oil column. Logs run in the well clearly located the gas-oil contact at 5200 ft also DST test conducted on this well and sample analysis of the fluid sample collected from the same well gave reservoir pressure of 2402 psia at 5250 ft and oil gradient of 0.35 psi/ft the depth of the oil-water contact is uncertain because it could not be confirmed by logs.

- Determine the probable oil-water contact
- What is the pressure at the crest of the reservoir?

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Chapter 9
Inflow Performance Relationship

Learning Objectives

Upon completion of this chapter, students/readers should be able to:

- Understand the concept of inflow performance relationship
- Describe the factors that affect inflow performance relationship
- Describe the steps in constructing a straight line inflow performance relationship
- Describe the steps in constructing Vogel's inflow performance relationship
- Describe other methods of constructing inflow performance relationship
- Perform some basic calculations on inflow performance relationship

Nomenclature

Parameter	Symbol	Unit
Oil rate	Q_o	*stb/d*
Productivity index	j	*stb/d/psi*
Average reservoir pressure	$\overline{P_r}$	*psi*
Bottomhole flowing pressure	P_{wf}	*psi*
Maximum oil rate	$Q_{o,\ max}$	*stb/d*
Bubble point pressure	P_b	*psi*
Absolute open flow potential	AOF	*stb/d*
Oil viscosity	μ_o	*cp*
Oil formation volume factor	B_o	*rb/stb*
Skin factor	s	-
Drainage radius	r_e	*ft*
Wellbore radius	r_w	*ft*

S. Okotie, B. Ikporo, *Reservoir Engineering*,
https://doi.org/10.1007/978-3-030-02393-5_9

9.1 Introduction

Subsurface production of hydrocarbon has to do with the movement of fluid from the reservoir through the wellbore to the wellhead. This fluid movement is divided into two as depicted in Fig. 9.1.

The flow of fluids (hydrocarbons) from the reservoir rock to the wellbore is termed the inflow. The inflow performance represents fluid production behavior of a well's flowing pressure and production rate. This differs from one well to another especially in heterogeneous reservoirs. The Inflow Performance Relationship (IPR) for a well is the relationship between the flow rate of the well (q), average reservoir pressure (P_e) and the flowing pressure of the well (P_{wf}). In single phase flow, this relationship is a straight line but when gas is moving in the reservoir, at a pressure below the bubble point, this is not a linear relationship.

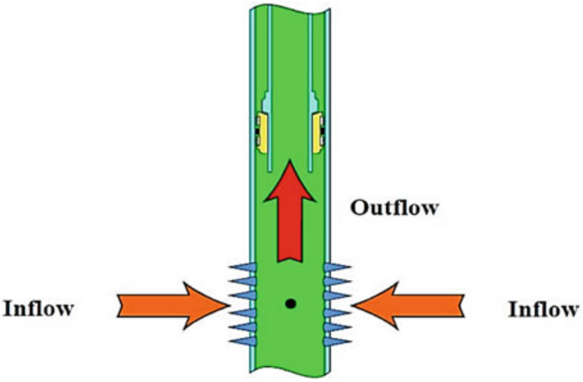
A well starts flowing if the flowing pressure exceeds the backpressure that the producing fluid exerts on the formation as it moves through the production system. When this condition holds, the well attains its absolute flow potential.

The backpressure or bottomhole pressure has the following components:

- Hydrostatic pressure of the producing fluid column
- Friction pressure caused by fluid movement through the tubing, wellhead and surface equipment
- Kinetic or potential losses due to diameter restrictions, pipe bends or elevation changes.

The IPR is often required for estimating well capacity, designing well completion, designing tubing string, optimizing well production, nodal analysis calculations, and designing artificial lift.

Fig. 9.1 Subsurface production



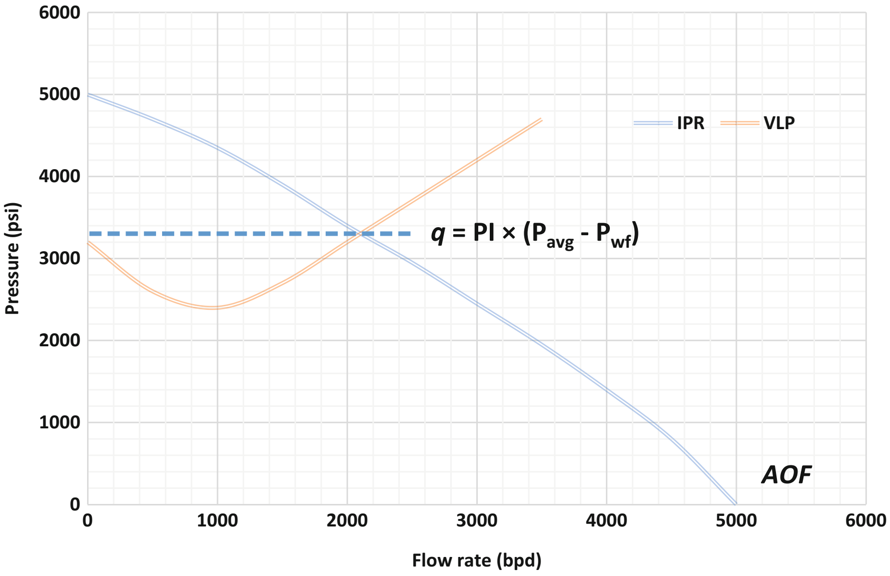


Fig. 9.2 Inflow performance relationship

The performance is commonly defined in terms of a plot of surface production rate (stb/d) versus flowing bottomhole pressure (P_{wf} in psi) on cartesian coordinate (Fig. 9.2). Maximum rate of flow occurs when P_{wf} is zero. This maximum rate is called absolute open flow and referred to as AOF. The following textbooks and articles where consulted to have the authors idea on the subject: Craft et al. (1991), Lyons & Plisga (2005), Dake (1978), Tarek (2010), Guo B, Ghalambor A (2005), Lea et al. (2008), Lee & Wattenbarger (1996), Al-Hussainy (1966), Bendakhlia & Aziz (1989), Giger et al. (1984) & Golan & Whitson (1986).

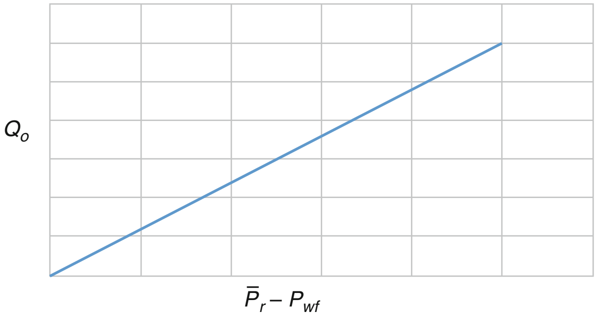
9.2 Factors Affecting IPR

Factors influencing the shape of the IPR are the pressure drop, viscosity, formation volume factor, skin and relative permeability across the reservoir.

There are several existing empirical correlations developed for IPR. This are:

9.3 Straight Line IPR Model

When the flow rate is plotted against the pressure drop, it gives a straight line from the origin with slope as the productivity index as shown in the figure below.



For a constant productivity index (j), the flow equation is given as:

$$Q_o = J\left(\overline{P_r} - P_{wf}\right)$$

The flow rates under different regimes are presented in Chap. 1 above.

When the well flowing pressure is zero, the corresponding rate is the AOF given as:

$$Q_o = J\overline{P_r}$$

9.3.1 Steps for Construction of Straight Line IPR

Step 1: Obtain a stabilize flow test data
Step 2: Determine the well productivity
Step 3: Assume different pressure value to zero in a tabular form
Step 4: Calculate the rate corresponding to the assume pressure
Step 5: Make a plot of rate versus pressure

9.4 Wiggins's Method IPR Model

Wiggins (1993) developed the following generalized empirical three phase IPR similar to Vogel's correlation based on his developed analytical model in 1991:

For Oil

$$Q_o = Q_{o,\ max}\left\{1 - 0.519167\left(\frac{P_{wf}}{\overline{\overline{P_r}}}\right) - 0.481092\left(\frac{P_{wf}}{\overline{\overline{P_r}}}\right)^2\right\}$$

For Water

$$Q_w = Q_{w,\ max}\left\{1 - 0.72\left(\frac{P_{wf}}{\overline{P_r}}\right) - 0.28\left(\frac{P_{wf}}{\overline{P_r}}\right)^2\right\}$$

9.5 Klins and Majcher IPR Model

Based on Vogel's work, Klins and Majcher (1992) developed the following IPR that takes into account the change in bubble-point pressure and reservoir pressure.

$$Q_o = Q_{o,\ max}\left\{1 - 0.295\left(\frac{P_{wf}}{\overline{P_r}}\right) - 0.705\left(\frac{P_{wf}}{\overline{P_r}}\right)^N\right\}$$

Where N is given as:

$$N = \left(0.28 + 0.72\frac{\overline{P_r}}{P_b}\right)(1.235 + 0.001P_b)$$

9.6 Standing's Method

The model developed by Standing (1970) to predict future inflow performance relationship of a well as a function of reservoir pressure was an extension of Vogel's model (1968).

$$Q_o = Q_{o,\ max}\left(1 - \frac{P_{wf}}{\overline{P_r}}\right)\left(1 - 0.8\left[\frac{P_{wf}}{\overline{P_r}}\right]\right)$$

Standing presented the future IPR as:

$$Q_o = \left\{\frac{J_f{}^* \langle\overline{P_r}\rangle_f}{1.8}\right\}\left\{1 - 0.2\left(\frac{P_{wf}}{\langle\overline{P_r}\rangle_f}\right) - 0.8\left(\frac{P_{wf}}{\langle\overline{P_r}\rangle_f}\right)^2\right\}$$

Where

$$J_f{}^* = J_p{}^* \frac{\left[\frac{k_{ro}}{\mu_o B_o}\right]_f}{\left[\frac{k_{ro}}{\mu_o B_o}\right]_p}$$

And

$$J_p{}^* = 1.8\left\{\frac{Q_{o,\ max}}{\overline{P_r}}\right\}$$

9.7 Vogel's Method

$$Q_o = Q_{o,\ max}\left\{1 - 0.2\left(P_{wf}\overline{P_r}\right) - 0.8\left(P_{wf}\overline{P_r}\right)^2\right\}$$

9.7.1 Steps for Construction of Vogel's IPR

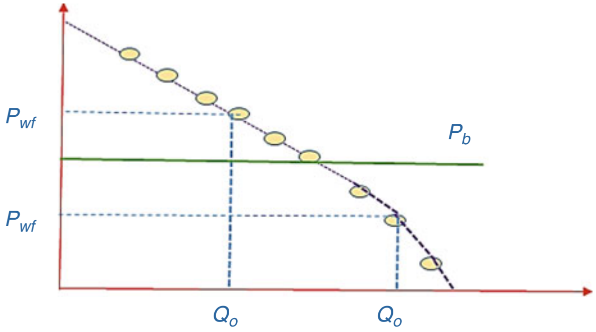
The same procedure is applicable to other models

Step 1: Obtain a stabilize flow test data
Step 2: Determine the maximum flow rate

$$Q_{o,\ max} = \frac{Q_{o, Test}}{\left\{1 - 0.2\left(\frac{P_{wf, Test}}{\overline{P_r}}\right) - 0.8\left(\frac{P_{wf, Test}}{\overline{P_r}}\right)^2\right\}}$$

Step 3: Assume different pressure value to zero in a tabular form
Step 4: Calculate the rate corresponding to the assume pressure
Step 5: Make a plot of rate versus pressure

Vogel presented IPR model for undersaturated and saturated oil reservoirs as depicted in the figure below.



9.7.2 Undersaturated Oil Reservoir

An undersaturated reservoir is a system whose pressure is greater than the bubble point pressure of the reservoir fluid. For the fact that the pressure of the reservoir is greater than the bubble point pressure does not mean that as production increases for a period of time, the pressure will not go below the bubble point pressure. Hence, careful evaluation will lead to a right decision and vice versa.

Since the reservoirs are tested regularly, it means that the stabilized test can be conducted below or above the bubble point pressure. Thus, for:

Case: pressure above bubble point
From stabilized test data point, the productivity index is:

$$J = \frac{Q_{o,Test}}{\overline{P_r} - P_{wf,Test}}$$

The inflow performance relationship can be generated with at different pressures

$$Q_o = J\left(\overline{P_r} - P_{wf}\right)$$

And when the reservoir pressure during production goes below the bubble point pressure, the IPR is generated as:

$$Q_o = Q_{ob} + \frac{JP_b}{1.8}\left\{1 - 0.2\left(\frac{P_{wf}}{P_b}\right) - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right\}$$

Where the bubble point oil flow rate is

$$Q_{ob} = J\left(\overline{P_r} - P_b\right)$$

The maximum oil flow rate is given

$$Q_{o,max} = Q_{ob} + \frac{JP_b}{1.8}$$

Case: pressure below bubble point

From stabilized test data point, the productivity index is:

$$J = \frac{Q_{o,Test}}{\left(\overline{P_r} - P_b\right) + \frac{P_b}{1.8}\left\{1 - 0.2\left(\frac{P_{wf,Test}}{P_b}\right) - 0.8\left(\frac{P_{wf,Test}}{P_b}\right)^2\right\}}$$

Then generate the IPR below the bubble point pressure as:

$$Q_o = Q_{ob} + \frac{JP_b}{1.8}\left\{1 - 0.2\left(\frac{P_{wf}}{P_b}\right) - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right\}$$

OR

$$Q_o = j\left[\left(\overline{P_r} - P_b\right) + \frac{P_b}{1.8}\left\{1 - 0.2\left(\frac{P_{wf,Test}}{P_b}\right) - 0.8\left(\frac{P_{wf,Test}}{P_b}\right)^2\right\}\right]$$

9.7.3 Vogel IPR Model for Saturated Oil Reservoirs

This is a reservoir whose pressure is below the bubble point pressure of the fluid. In this case, we calculate the maximum oil flow rate from the stabilized test and then generate the IPR model. Mathematically

$$Q_{o,\ max} = \frac{Q_{o,Test}}{\left\{1 - 0.2\left(\frac{P_{wf,Test}}{\overline{P_r}}\right) - 0.8\left(\frac{P_{wf,Test}}{\overline{P_r}}\right)^2\right\}}$$

$$Q_o = Q_{o,\ max}\left\{1 - 0.2\left(\frac{P_{wf}}{\overline{P_r}}\right) - 0.8\left(\frac{P_{wf}}{\overline{P_r}}\right)^2\right\}$$

9.8 Fetkovich's Model

According to Tarek (2010), the model developed by Fetkovich in 1973 for undersaturated and saturated region, was an expansion of Muskat and Evinger (1942) model derived from pseudosteady-state flow equation to observe the IPR nonlinear flow behavior.

9.8.1 Undersaturated Fetkovich IPR Model

$$Q_o = \frac{0.00708kh}{\mu_o B_o \left\{ \ln\left(\frac{r_e}{r_w}\right) - 7.5 + S \right\}} \left(\overline{P_r} - P_{wf}\right)$$

9.8.2 Saturated Fetkovich IPR Model

$$Q_o = \frac{0.00708kh}{(\mu_o B_o)_{P_b} \left\{ \ln\left(\frac{r_e}{r_w}\right) - 7.5 + S \right\}} \left(\frac{1}{2P_b}\right) \left(\overline{P_r}^2 - P_{wf}{}^2\right)$$

To account for turbulent flow in oil wells, Fetkovich introduced an exponent (n) and a performance coefficient (C) calculated graphically in the pressure square model given by:

$$Q_o = C\left[P_r{}^{-2} - P_{wf}{}^2\right]^n$$

While Klins and Clark (1993), derived a mathematical correlation for calculation the exponent and performance coefficient

9.9 Cheng Horizontal IPR Model

Cheng (1990) presented a form of Vogel's equation for horizontal wells that is based on the results of a numerical simulator. The proposed expression has the following form:

$$Q_o = Q_{o,\ max}\left\{1 + 0.2055\left(\frac{P_{wf}}{\overline{P_r}}\right) - 1.1818\left(\frac{P_{wf}}{\overline{P_r}}\right)^2\right\}$$

Example 9.1
An undersaturated oil reservoir at Uqwa with bubble point pressure of 2100 psi was shut-in for a pressure build up test which was conducted for 18 h and average pressure obtained was 2750 psi. For a proper production allocation, a flow test was conducted on well J6. The result shows that it is capable of producing at a stabilized flow rate of 165 STB/day and a bottom-hole flowing pressure of 2380 psi. Calculate the following using straight line and Vogel's method:

- Well J6 productivity index
- The AOF
- Generate the IPR of the well

Solution
Well productivity index j for both methods are:
Straight line method

$$J = \frac{Q_{o,Test}}{\overline{P_r} - P_{wf,Test}} = \frac{165}{2750 - 2380} = 0.4459\ STB/day/psi$$

For Vogel's method, the same formula is applied when the pressure of the stabilized test is above the bubble point pressure of the reservoir fluid.

The absolute open flow potential (AOF)
Straight line

$$Q_{o,max} = AOF = J\overline{P_r}$$

$$AOF = 0.4459 * 2750 = 1226.23\ STB/day$$

Vogel's method

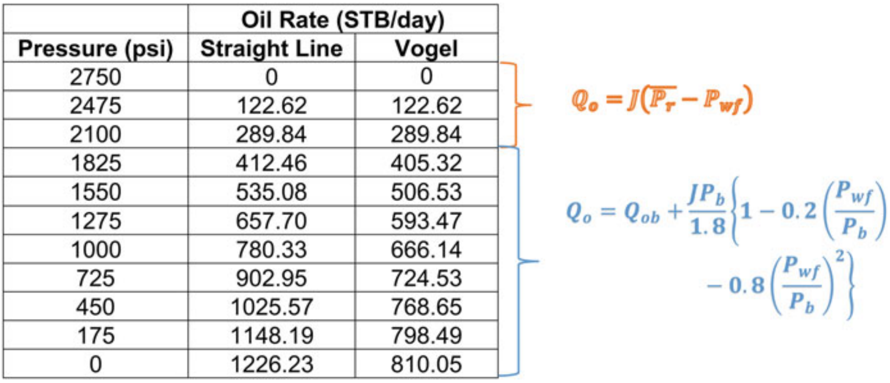
$$AOF = Q_{o,max} = j\left(\left(\overline{P_r} - P_b\right) + \frac{P_b}{1.8} \right)$$

$$AOF = 0.4459 \left((2750 - 2100) + \frac{2100}{1.8} \right) = 810.05\ STB/day$$

Generating IPR for both models

$$Q_{ob} = J\left(\overline{P_r} - P_b\right) = 0.4459(2750 - 2100) = 289.84\ STB/day$$

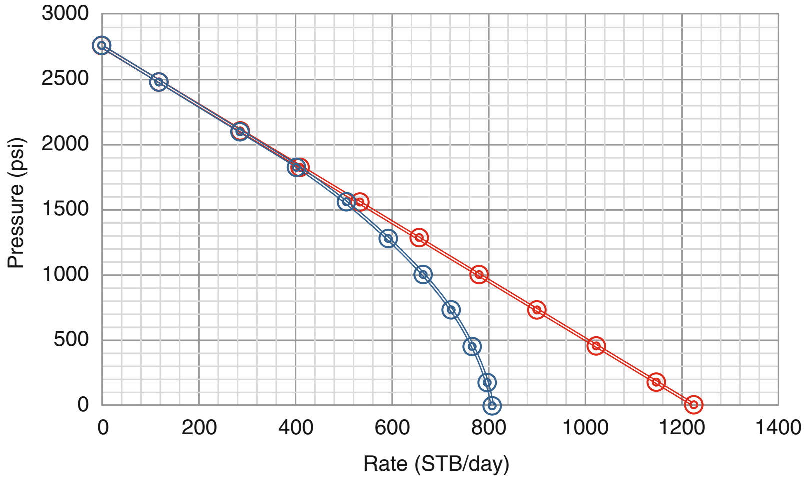
The IPR for the straight line is generated using $Q_o = J\left(\overline{P_r} - P_{wf}\right)$ while Vogel's IPR formulae are designated by the side of the table below.



	Oil Rate (STB/day)	
Pressure (psi)	Straight Line	Vogel
2750	0	0
2475	122.62	122.62
2100	289.84	289.84
1825	412.46	405.32
1550	535.08	506.53
1275	657.70	593.47
1000	780.33	666.14
725	902.95	724.53
450	1025.57	768.65
175	1148.19	798.49
0	1226.23	810.05

$$Q_o = J\left(\overline{P_r} - P_{wf}\right)$$

$$Q_o = Q_{ob} + \frac{JP_b}{1.8}\left\{1 - 0.2\left(\frac{P_{wf}}{P_b}\right) - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right\}$$



Example 9.2

Apply the information given in Example 9.1 for a case where the stabilized rate is 320 STB/day at a pressure of 1950 psi to calculate the IPR using Vogel's method.

Vogel's method

$$J = \frac{Q_{o,Test}}{\left(\overline{P_r} - P_b\right) + \frac{P_b}{1.8}\left\{1 - 0.2\left(\frac{P_{wf,Test}}{P_b}\right) - 0.8\left(\frac{P_{wf,Test}}{P_b}\right)^2\right\}}$$

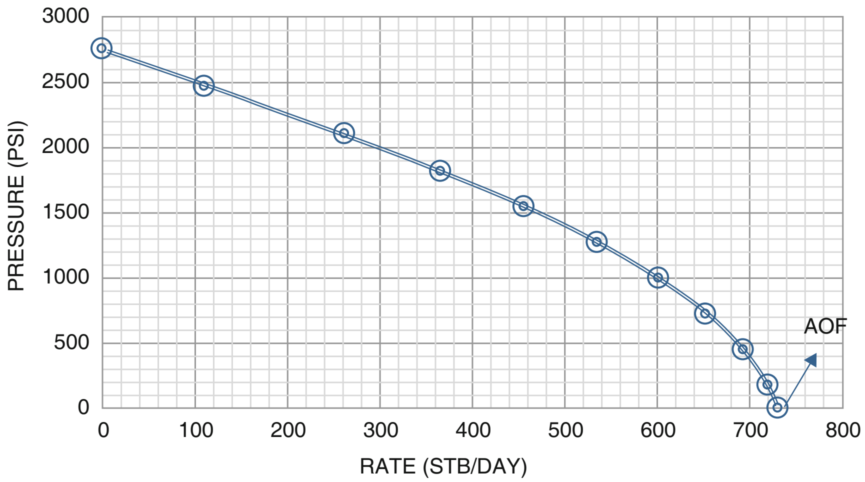
$$J = \frac{320}{(2750 - 2100) + \frac{2100}{1.8}\left\{1 - 0.2\left(\frac{1950}{2100}\right) - 0.8\left(\frac{1950}{2100}\right)^2\right\}} = 0.4024\ STB/day/psi$$

$$Q_{ob} = J\left(\overline{P_r} - P_b\right) = 0.4024(2750 - 2100) = 261.56\ STB/day$$

The IPR is generated for pressure below the bubble point pressure using the equation below:

$$Q_o = j\left[\left(\overline{P_r} - P_b\right) + \frac{P_b}{1.8}\left\{1 - 0.2\left(\frac{P_{wf}}{\overline{P_r}}\right) - 0.8\left(\frac{P_{wf}}{\overline{P_r}}\right)^2\right\}\right]$$

Pressure (psi)	Qo (STB/day
2750	0
2475	110.66
2100	261.56
1825	365.78
1550	457.12
1275	535.58
1000	601.15
725	653.85
450	693.66
175	720.59
0	731.03



9.10 How Do We Improve the Productivity Index?

This can be done by altering the parameters in the flow equation. Thus, for the well productivity or inflow performance to be improved, we need to carry out any of the following:

- Acid stimulation to remove skin
- Increasing the effective permeability around the wellbore
- Reduction in fluid viscosity
- Reduction in the formation volume factor
- Increasing the well penetration

A case study of an improvement to IPR curve of a well
Well k35 result for before and after stimulation

Parameter	Pre-stimulation	Post-stimulation
Rate [stb/d]	1000	1180
total skin S_T	31.19	14.14
damage skin	24.3	7.25
other skin	6.89	6.89
ΔP due to damage skin	47.45969262	13.92378842
ΔP due to total skin [psi]	60.9163709	27.15618873
productivity index [stb/d/psi]	16.41594838	43.452342
increase in production [stb]	180	

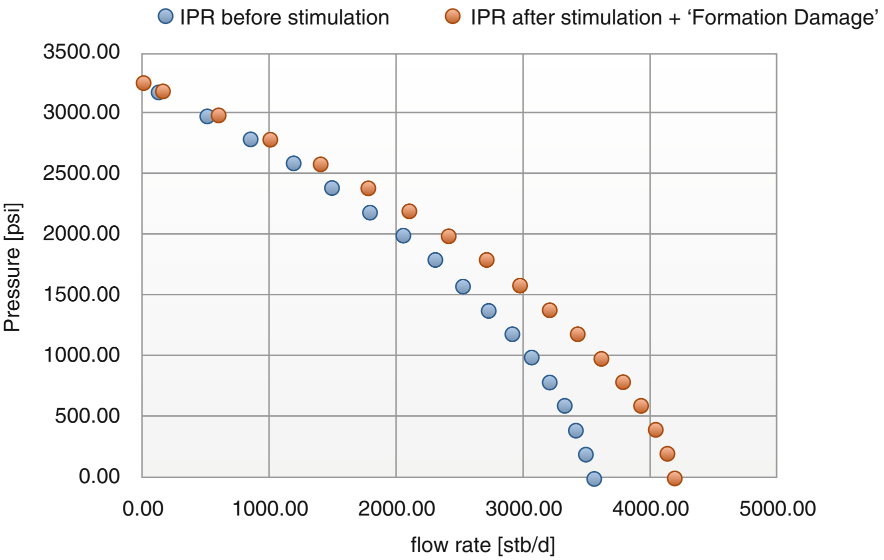
The result from the pressure transient analysis conducted on well k35 indicates that the well was damage during the drilling operation and as such a stimulation job which yields a success gave an increase in flow rate of 180stb/d and productivity index of 27.0364 stb/d/psi. In addition, there was a large reduction in the value of skin due to damage of this well. We will say at this point that the stimulation job is justified as a success. The inflow relationship is tabulated and plotted below.

IPR for well k35 before and after stimulation

Before stimulation		After stimulation	
press [psi]	rate[stb/d]	press [psi]	rate[stb/d]
3251.23	0.00	3251.23	0
3184.42	129.88	3184.42	153.2605
2985.63	502.18	2985.63	592.5703
2786.84	853.28	2786.84	1006.867
2588.05	1183.18	2588.05	1396.151
2389.26	1491.88	2389.26	1760.423
2190.47	1779.39	2190.47	2099.681
1991.68	2045.70	1991.68	2413.927
1792.89	2290.81	1792.89	2703.16
1594.10	2514.73	1594.1	2967.38

(continued)

Before stimulation		After stimulation	
press [psi]	rate[stb/d]	press [psi]	rate[stb/d]
1395.31	2717.45	1395.31	3206.588
1196.52	2898.97	1196.52	3420.782
997.73	3059.29	997.73	3609.964
798.94	3198.42	798.94	3774.133
600.15	3316.35	600.15	3913.29
401.36	3413.08	401.36	4027.433
202.57	3488.61	202.57	4116.564
3.78	3542.95	3.78	4180.682
0.00	3543.78	0	4181.658



Exercises

Ex 9.1 An oil well is flowing at a rate of 420 STB/day under steady state conditions. The wellbore flowing pressure is 2750 psia. The reservoir thickness is 28 f. and permeability of 62 mD. The wellbore and reservoir radii are 0.325 f. and 700 f. respectively. A result from well test conducted on the well shows that it was damaged with skin of 2.87. PVT report gave the oil FVF as 1.356 bbl/STB and oil viscosity of 2.108 cp. Calculate:

- The reservoir pressure
- The absolute open flow potential
- The productivity index

Ex 9.2 The following reservoir and flow-test data are available on an oil well:

Average reservoir pressure	3560 psia
Bubble point pressure	2600 psia
Flow bottom hole pressure from flow test	2930 psia
Flow rate from flow test	300 STB/day

Generate the IPR data of the well.

Ex 9.3 An oil well is producing from an undersaturated reservoir that is characterized by a bubble-point pressure of 2500 psig. The current average reservoir pressure is 3750 psig. Available flow test data show that the well produced 379 STB/day at a stabilized Pwf of 3050 psig. Construct the current IPR data by using:

- Vogel's correlation
- Wiggins' method
- Klins and Majcher method
- Standing method

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Chapter 10
History Matching

Learning Objectives

- Define history matching
- Identify the known parameters to match and the unknown parameters to tune
- Understand history match plan
- Understand the key variables to consider when conducting history matching
- Perform simple history matching on pressure and saturation
- Describe some problems associated with history matching
- Describe the types of history matching

10.1 History Matching

The update of a model to fit the actual performance is known as history matching. Clearly speaking, developing a model that cannot accurately predict the past performance of a reservoir within a reasonable engineering tolerance of error is not a good tool for predicting the future of the same reservoir. To history match a given field data with material balance equation, we have to state clearly the known parameters to match and the unknown parameters to tune to get the field historical production data with minimum tolerance of error and these parameters are given in Table 10.1.

Besides, one of the paramount roles of a reservoir engineer is to forecast the future production rates from a specific well or a given reservoir. From history, engineers have formulated several techniques to estimate hydrocarbon reserves and future performance. The approaches start from volumetric, material balance, decline curve analysis techniques to sophisticated reservoir simulators. Whatever approach

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Table 10.1 History match and prediction parameters

	Known parameters	
History matching	Parameter	Symbol
	Production data	Np, Gp, Wp and Rp
	Hydrocarbon properties	Boi, Bo, Bg, Bgi, Rsi, Rs
	Reservoir properties	Sw, cw, cf, m
	Pressure drop	ΔP
	Unknown parameters	
	Reserves	N
	Water influx	We
Prediction	Reserves, water influx, hydrocarbon properties, reservoir properties	

taken by the engineers to predict production rates and reservoir performance predictions whether simple or complex method used relies on the history match.

The general approach by the engineer whose production history is already available, is to determine the rates for the given period of production. The value calculated is use to validate the actual rates and if there is an agreement, the rate is assumed to be correct. Thus, it is then used to predict the future production rates. On the contrary, if there is no agreement between the calculated and the actual rates, the calculation is repeated by modifying some of the key parameters. This process of matching the computed rate with the actual observed rate is called history matching.

It therefore implies that history matching is a process of adjusting key properties of the reservoir model to fit or match the actual historic data. It helps to identify the weaknesses in the available data, improves the reservoir description and forms basis for the future performance predictions. One of these parameters that is vital in history matching, is the aquifer parameters that are not always known. Hence, modification of one or several of these parameters to obtain an acceptable match within reasonable engineering tolerance of error or engineering accuracy is history matching (Donnez 2010). Therefore, to complete this chapter, the following textbooks and articles were reviewed: Aziz & Settary (1980), Crichlow (1977), Kelkar & Godofredo (2002), Chavent et al. (1973), Chen et al. (1973), Harris (1975), Hirasaki (1973), Warner et al. (1979), Watkins et al. (1992).

10.2 History Matching Plan

The validity of a model should be approach in two phases: pressure match and saturation match (oil, gas and water rates). The pressure and saturation phases matche, follows different pattern depending on purpose (experience of the individual carrying out the study). The simulation follows the same basic steps for the two phases. These steps include:

- Gather data
- Prepare analysis tools
- Identify key wells/tank

- Interpret reservoir behavior from observed data
- Run model
- Compare model results to observed data
- Adjust models parameters

10.3 Mechanics of History Matching

There are several parameters that are varied either singly or collectively to minimize the differences between the observed data and those calculated data by the simulator. Modifications are usually made on the following areas as presented by Crichlow (1977):

- Rock data modifications (permeability, porosity, thickness & saturations)
- Fluid data modifications (compressibility, PVT data & viscosity)
- Relative permeability data
- Shift in relative permeability curve (shift in critical saturation data)
- Individual well completion data (skin effect & bottom hole flowing pressure)

The two fundamental processes which are controllable in history matching are as follows:

1. The quantity of fluid in the system at any time and its distribution within the reservoir, and
2. The movement of fluid within the system under existing potential gradients (Crichlow 1977).

The manipulation of these two processes enables the engineer to modify any of the earlier-mentioned parameters which are criteria to history matching. It is mandatory that these modifications of the data reflect good engineering judgment and be within reasonable limits of conditions existing in that area. History matching is actually an act and time consuming. This implies that the total time spent on history matching depends largely on the expertise of the engineer and his familiarity with the particular reservoir. Here are some of the key variables to consider when conducting history matching:

- Porosity (local)
- Water Saturation (Global)
- Permeability (Local)
- Gross Thickness (Local)
- Net Thickness (Local)
- kv/kh Ratio (Global · · · Local?)
- Transmissibility (x/y/z/) (Local)
- Aquifer Connectivity and Size (Regional)
- Pore Volume (Local)
- Fluid Properties (Global)
- Rock Compressibility (Global)

- Relative Permeability (Global -regional with Justification)
- Capillary Pressure (Global -regional with justification)
- Mobile Oil Volume (Global or Local?)
- Datum Pressure (Global)
- Original Fluid Contact (Global)
- Well Inflow Parameters (Local)

10.4 Quantification of the Variables Level of Uncertainty

The following variables are often considered to be determinate (**low uncertainty**):

- Porosity
- Gross thickness
- Net thickness
- Structure (reservoir top/bottom/extent)
- Fluid properties
- Rock compressibility
- Capillary pressure
- Datum pressure
- Original fluid contact
- Production rates

The following variables are often considered to be indeterminate (**high uncertainty**):

- Pore volume
- Permeability
- Transmissibility
- Kv/Kh ratio
- Rel. perm. curves
- Aquifer properties
- Mobile oil volumes
- Well inflow parameters

10.5 Pressure Match

Here are two proposed option for pressure match

Option 1

- Run the model under reservoir voidage control
- Examine the overall pressure levels
- Adjust the pore volume/aquifer properties to match overall pressure

- Match the well pressures
- Modify local PVs/aquifers to match overall pressures
- Modify local transmissibility to match pressure gradient

Key elements	Adjusted parameters
Total voidage	Rate constraints
Pressure level	Total compressibility, thickness, porosity, water influx
Pressure shape	Permeability
Individual wells	Total compressibility, thickness, porosity, water influx

Option 2

- Check/Initialization
- Run simulation model
- Adjust Kx for well which cannot meet target rates
- Adjust pore volume and compressibility to match pressure change with time
- Adjust Kv and Tz to capture vertical pressure gradient
- Adjust Kv and Tz to meet areal pressure
- Adjust Tx and Ty at the faults
- Adjust PI's to meet production allocations
- Iterate

10.6 Saturation Match

Option 1

- Normally attempted once pressures matched
- Most important parameters are relative permeability curves and permeabilities
- Try to explain the reasons for the deviations and act accordingly
- Changes to relative permeability tables should affect the model globally
- Changes to permeabilities should have some physical justification
- Consider the use of well pseudos
- Assumed layer KH allocations may be incorrect (check PLTs, etc.)

Option 2

- Check/Initialization Model
- Run simulation model
- Check overall model water/gas movement(process physics)
- Adjust relative permeability
- Introduce and adjust well's relative permeabilities (Krs) to match individual well performance
- Adjust PI's to match production allocation
- Add or delete completion layers to account for channeling, leaking plugs
- Iterate

10.7 Well PI Match

- Not usually matched until pressures and saturations are matched, unless BHP affects production rates
- Must be matched before using model in prediction mode
- Match FBHP data by modifying KH, skin or PI directly

10.8 Problems with History Matching

- Non uniqueness of accepted match
- Lack of reliable field data
- Available data may be limited
- Errors in simulator can cause a correct set of parameters to yield incorrect result.

10.9 Review Data Affecting STOIIP

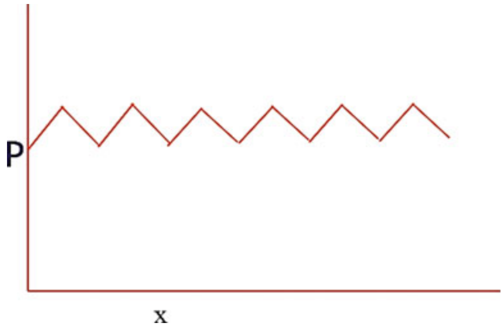
Verify that the value of STOIIP calculated by the model is in line with estimated values by volumetric calculations and material balance. If the calculated value is too high/low, this is normally due to errors of the following type:

- High/low porosity values (data entry format error)
- Misplace fluid contacts (gas-oil and/or water-oil)
- Inclusion/exclusion of grid blocks that belong or not to the reservoir model.
- High/low values in the capillary pressure curves.
- Errors in net sand thickness.

10.9.1 Problems and Likely Modifications

- **Localised high pressure area and localised low pressure area.**
  - **Remedies:**
    - Modify k to allow case of flow from high pressure region to low pressure region
    - Reduce oil in high pressure region by changing ϕ or h or So or all of them.
    - If rock data are varied, there may be need for redigitizing.
- **Generally high pressure in the whole system**

- **Remedy:**
  - Reduce oil in place by reducing porosity in the whole system.
- **Discontinuous pressure distribution**



Remedy: increase k to smoothen effect

- **Model runs out of fluid**

  Remedy:
  - Increase initial fluid saturation. Fluid contacts may be varied.
- **No noticeable drawdown in pressure even after considerable withdrawal.**

  Remedy:
  - Error in compressibility entered.
- **S_w increase without any injection or influx of water.**

  Remedy:
  - Increase rock compressibility used.
- **Problem with matching GOR, WOR**

  Remedy:
  - Modify relative permeability

If simulated GOR > observed GOR, reduce K_{rg} vale in the simulator. The reverse is true.

If free gas starts flowing early, increase critical gas sat. The reverse is also the case.

After everything has been done, observed pressures and production are greater than the model.

Cause:

- Reservoir getting energy from region not defined for example, fluid influx

Remedy:

- Redefine area and model or include aquifer if observed water cut is increasing.

10.10 Methods of History Matching

The method adopted for matching a field's historic data depends on the engineer in question. History matching has been improved from manual turning of some parameters to a more sophisticated computer aided tool. Today, some engineers still use manual turning which work well for them rather than the computer aided history matching.

10.10.1 Manual History Matching

During manual history matching, changing one or two parameters manually by trial-and error can be tedious and inconsistent with the geological models. To make the parameters best fit with the simulated and observed data gives considerable uncertainties and does not have the reliability for a longer period.

10.10.2 Automated History Matching

Automated history matching is much faster and requires fewer simulation runs than manual history matching. It includes a large number of different parameters and tackles a large number of wells without problems. In manual history matching, one or two parameters are varied at a time and it would require preliminary analysis first for tackling the wells.

Besides, automatic history matching could give more reliable results in the case of complex lithology conditions with considerable heterogeneity. The basic process in automatic history matching is to start from an initial parameter guess and then improve it by integrating field data in an automatic loop. In this case, parameter changes are done by computer programming to minimize the function to show differences between simulated and observed data. This is called objective function that includes both model mismatch and data mismatch parts.

10.10.3 Classification of Automatic History Matching

- Deterministic Algorithm
- Stochastic Algorithm

10.10.3.1 Deterministic Algorithm

Deterministic algorithms use traditional optimization approaches and obtain one local optimum reservoir model within the number of simulation iteration constraints. In implementation, the gradient of the objective function is calculated and the direction of the optimization search is then determined (Liang 2007). The gradient based algorithms minimize the difference between the observed and simulated measurements which is called the minimization of the objective function that considered the following loop:

- To run the flow simulator for the complete history matching period,
- To evaluate the cost function,
- To update the static parameters and go back to the first step.

The following are the list of several algorithms that are commonly used for the basis of gradient based algorithms (Landa 1979; Liang 2007):

- **Gradient based algorithms:**
  - Steepest Descent
  - Gauss-Newton (GN)
  - Levenberg-Marquardt
  - Singular Value Decomposition
  - Particle Swarm Optimization
  - Conjugate Gradient
  - Quasi-Newton
  - Limited Memory Broyden Fletcher Goldfarb Shanno (LBFGS)
  - Gradual Deformation

10.10.3.2 Stochastic Algorithm

The stochastic algorithm takes considerable amounts of computational time compared to a deterministic algorithm, but due to the rapid development of computer memory and computation speed, stochastic algorithms are receiving more and more attention.

Stochastic algorithms have three main direct advantages:

- The stochastic approach generates a number of equal probable reservoir models and therefore is more suitable to non-unique history matching problems,
- It is straight-forward to quantify the uncertainty of performance forecasting by using these equal probable model,
- Stochastic algorithms theoretically reach the global optimum.

The following are list of several algorithms that are commonly used on the basis of non-gradient based stochastic algorithms (Landa 1979; Liang 2007):

- **Non-gradient based algorithms**:
  - Simulated Annealing
  - Genetic Algorithm
  - Polytope
  - Scatter & Tabu Searches
  - Neighborhood
  - Kalman Filter

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Chapter 11
Reservoir Performance Prediction

Learning Objectives

Upon completion of this chapter, students/readers should be able to:

- Understand the concept of reservoir performance prediction
- Describe the various prediction methods
- Derive instantaneous gas-oil ratio
- Understand the derivatives of some of the methods of prediction
- Describe the step by step approach of the various prediction method
- Perform prediction calculations

Nomenclature

Parameter	Symbol	Unit
Initial gas formation volume factor	β_{gi}	*cuft/scf*
Gas formation volume factor	β_g	*cuft/scf*
Cumulative water influx	W_e	*bbl*
Cumulative water produced	W_p	*bbll*
Cumulative gas produced	G_p	*scf*
Cumulative oil produced	N_p	*Stb*
Stock tank oil initially In place	N	*stb*
Stock tank gas initially in place	G	*scf*
Initial solution gas-oil ratio	R_{si}	*scf/stb*
Solution gas-oil ratio	R_s	*scf/stb*
Cumulative produced gas-oil ratio	R_p	*Scf/stb*
Bottom hole (wellbore) flowing pressure	P_{wf}	*psia*
Initial reservoir pressure	P_i	*psia*
Oil formation volume factor	β_o	*rb/stb*
Initial oil formation volume factor	β_{oi}	*rb/stb*

(continued)

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Parameter	Symbol	Unit
Water formation volume factor	β_w	*rb/stb*
Gas formation volume factor	β_g	*cuft/scf*
Initial gas formation volume factor	β_{gi}	*cuft/scf*
Reservoir temperature	T	$^\circ R$
Total fluid compressibility	C_t	$psia^{-1}$
Oil isothermal compressibility	C_o	$psia^{-1}$
Effective oil isothermal compressibility	C_{oe}	$psia^{-1}$
Water & rock compressibility	C_w & C_r	$psia^{-1}$
Gas deviation factor at depletion pressure	z	–
Gas/oil sand volume ratio or gas cap size	m	–
Connate & initial water saturation	S_{wi} & S_{wc}	*– or %*
Residual gas saturation to water displacement	S_{grw}	*– or %*
Residual oil-water saturation	S_{orw}	–
Pore volume of water-invaded zone	PV_{water}	ft^3
Reservoir pore volume	PV	ft^3
Flow rate	q	*stb/d*
Oil & gas viscosity	μ_o & μ_g	*cp*
Formation permeability	k	*mD*
Reservoir thickness	h	*ft*
Area of reservoir	A	*acres*
Wellbore radius	r_w	*ft*
Recovery factor	RF	*%*
Pressure drop	ΔP	*psi*
Initial & current gas expansion factor	E_i & E	*scf/cuft*
Oil & gas relative permeability	k_{ro} & k_{rg}	–
Oil & gas saturation	S_o & S_g	*– or %*
Pore volume	V_p	*cuft*

11.1 Introduction

Some of the roles of Reservoir Engineers are to estimate reserve, field development planning which requires detailed understanding of the reservoir characteristics and production operations optimization and more importantly; to develop a mathematical model that will adequately depict the physical processes occurring in the reservoir such that the outcome of any action can be predicted within reasonable engineering tolerance of errors. Muskat (1945) stated that one of the functions of reservoir engineers is to predict the past performance of a reservoir which is still in the future. Therefore, whether the concept of the engineer is wrong or right, stupid or clever, honest or dishonest, the reservoir is always right.

We have to bear in mind that reservoirs rarely perform as predicted and as such, reservoir engineering model has to be updated in line with the production behaviour. Thus, an accurate prediction of the future production rates under various operating conditions, apply the primary requirement for the oil and gas reservoirs feasibility evaluation and performance optimization. The conventional method of utilizing deliverability and material balance equations to predict the production performance of these reservoirs cannot be utilized often when the complete reservoir data are lacking.

Reservoir performance prediction is an iterative process. it requires that a convergence criterion must be met after a satisfactory history match is achieved, to be executed in a short period of time, for a proper optimization of future reservoir management planning of a field. There are basically four methods of reservoir performance prediction applying material balance concept and not a numerical approach where the reservoir is divided into grid blocks. These are:

- Tracy method
- Muskat method
- Tarner method
- Schilthuis method

All the techniques used to predict the future performance of a reservoir are based on combination of appropriate MBE with the instantaneous GOR using the proper saturation equation. The calculations are repeated at a series of assumed reservoir pressure drops. These calculations are usually based on stock-tank barrel of oil-in-place at the bubble-point pressure. Above the bubble point pressure, the cumulative oil produced is calculated directly from the material balance equations as presented in Craft & Hawkins (1991), Dake (1978), Tarek (2010), Cole (1969), Cosse (1993), Economides et al. (1994) & Hawkins (1955). The MBE for undersaturated reservoir are expressed below.

11.1.1 *For Undersaturated Reservoir* (**P > P$_b$**) *with No Water Influx*

That is above the bubble point; the assumptions made are:

$$m = 0, W_e = 0, R_{si} = R_s = R_p, \quad G_p = NR_p, W_{inj} = G_{inj} = 0, K_{rg} = 0, W_p = W_e = 0$$

(*because there is no free gas in the formation*); From the general material balance equation, cancelling out all the assumed parameters gives

$$\begin{aligned} N_p\{B_o + \cancel{[R_p - R_s]B_g}\} + \cancel{W_pB_w} &= \cancel{mNB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]} + N\{[B_o - B_{oi}] - \cancel{[R_{si} - R_s]B_g}\} \\ &+ [1 + \cancel{m}]\frac{NB_{oi}}{1 - S_{wc}}[C_r + C_{wc}S_{wc}]\Delta P + \cancel{W_eB_w + G_{inj}B_{ginj} + W_{inj}B_w} \end{aligned}$$

It implies that

$$N = \frac{N_p B_o}{(B_o - B_{oi}) + B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P}$$

From Hawkin's equation, the isothermal compressibility of oil C_o, can be expressed as:

$$C_o = -\frac{1}{B_o}\left(\frac{\partial B_o}{\partial P}\right)_T = -\frac{1}{B_{oi}}\left(\frac{B_o - B_{oi}}{P_i - P}\right)$$

$$B_o - B_{oi} = -C_o B_{oi}(P - P_i) = C_o B_{oi}(P_i - P)$$

Put these two equations into the N equation gives:

$$N = \frac{N_p B_o}{C_o B_{oi}(P_i - P) + B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P}$$

$$N = \frac{N_p B_o}{\Delta P B_{oi}\left(C_o + \frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)}$$

$$N = \frac{N_p B_o}{B_{oi}\Delta P\left(\frac{C_o(1 - S_{wi}) + S_{wi}C_w + C_f}{1 - S_{wi}}\right)}$$

$$N = \frac{N_p B_o}{B_{oi}\Delta P\left(\frac{C_o S_o + S_{wi}C_w + C_f}{1 - S_{wi}}\right)}$$

Expressing the isothermal compressibility in terms of effective compressibility, C_{oe}. thus;

$$C_{oe} = \frac{C_o S_o + S_{wi}C_w + C_f}{1 - S_{wi}}$$

$$N = \frac{N_p B_o}{B_{oi}\Delta P C_{oe}} = \frac{N_p B_o}{B_{oi}C_{oe}(P_i - P)}$$

Therefore, the pressure at any time, is given as

$$P = P_i - \frac{N_p B_o}{B_{oi}C_{oe}N}$$

The cumulative oil produced in the undersaturated region can be calculated directly from the equation given as:

$$N_p = \frac{NB_{oi}C_{oe}(P_i - P)}{B_o}$$

Where

$$G_p = N_pR_{si}$$

11.1.2 Undersaturated Reservoir with Water Drive

Assumptions: $W_{inj} = G_{inj} = 0,\ m = 0$

$$N = \frac{N_pB_o + W_pB_w - W_eB_w}{(B_o - B_{oi}) + B_{oi}\left(\frac{S_{wi}C_w + C_f}{1 - S_{wi}}\right)\Delta P}$$

In terms of effective oil compressibility

$$N = \frac{N_pB_o + W_pB_w - W_eB_w}{B_{oi}C_{oe}(P_i - P)}$$

$$N_p = \frac{N\,B_{oi}C_{oe}(P_i - P) + \left(W_eB_w - W_pB_w\right)}{B_o}$$

Where

$$G_p = N_pR_{si}$$

In applying the above methods of prediction for saturated reservoirs, we require some additional information to match the previous field production data in order to predict the future. Such relations are the instantaneous gas-oil ratio (GOR), equation relating the cumulative GOR to the instantaneous GOR and the equation that relates saturation to cumulative oil produced.

On the contrary, despite the fact that the material balance equation is a tool used by the reservoir engineers, there are some aspects which were not put into consideration when performing prediction performance. These are:

- The contribution of the individual well’s production rate
- The actual number of wells producing from the reservoir
- The positions of these wells in the reservoir are not considered since it is assume to be a tank model
- The time it will take to deplete the reservoir to an abandonment pressure
- Does not see faults in the reservoir if there is any and the variation in rock and fluid properties.

11.1.3 Instantaneous Gas- Oil Ratio

Instantaneous gas-oil ratio at any time, R is defined as the ratio of the standard cubic feet of gas produced to the stock tank barrel of oil produced at that same instant of time and reservoir pressure. The gas production comes from solution gas and free gas in the reservoir which has come out of the solution (Tarek, 2010).

Instantaneous producing GOR is given mathematically as

$$R = \frac{Gas\,producing\ rate\ (scf/day)}{oil\ producing\ rate\ (stb/day)}$$

$$R_p = \frac{Cumulative\ gas\,produced,\ \ SCF}{Cumulative\ oil\ produced,\ \ STB/day}$$

Free gas at surface condition is given as:

$$= \frac{q_g}{B_g}$$

Solution gas is

$$= Q_o R_s$$

The total gas production rate

$$Q_g = \frac{q_g}{B_g} + Q_o R_s$$

Oil production rate is

$$Q_o = \frac{q_o}{B_o}$$

Thus,

$$R = \frac{Q_g}{Q_o} = \frac{\frac{q_g}{B_g} + Q_o R_s}{\frac{q_o}{B_o}}$$

Since $Q_o = \frac{q_o}{B_o}$

$$R = \frac{\frac{q_g}{B_g} + \frac{q_o}{B_0} R_s}{\frac{q_o}{B_o}}$$

Thus,

$$R = \frac{{}^{q_g}/_{q_g}}{{}^{q_o}/_{B_o}} + R_s$$

$$q_g = \frac{2\pi k_{rg} h \Delta P}{\mu_g \ln \frac{r_e}{r_w}}$$

$$q_o = \frac{2\pi k_{ro} h \Delta P}{\mu_o \ln \frac{r_e}{r_w}}$$

$$R = \frac{\frac{2\pi k_{rg} h \Delta P}{\mu_g \ln \frac{r_e}{r_w}}}{\frac{2\pi k_{ro} h \Delta P}{\mu_o \ln \frac{r_e}{r_w}}} + R_s$$

$$R = 5.615 \frac{B_o k_{rg} \mu_o}{B_g k_{ro} \mu_g} + R_s$$

The instantaneous GOR can be used to history match relative permeability. Thus, re-arranging the above equation gives:

$$\frac{k_{rg}}{k_{ro}} = (R - R_s) \frac{B_g \mu_g}{B_o \mu_o}$$

11.2 Muskat's Prediction Method

In 1945, Muskat developed a method for reservoir performance prediction at any stage of pressure depletion by expressing the material balance equation for a depletion-drive reservoir in differential form as derived below.

The oil pore volume (original volume of oil in the reservoir) is given as:

$$V_p S_{oi} = N B_{oi}$$

$$N = \frac{V_p S_{oi}}{B_{oi}} \qquad where \quad S_{oi} = 1 - S_{wc}$$

At any given pressure, the oil remaining in the reservoir at stock tank barrels is given as:

$$N_r = \frac{V_p S_o}{B_o}$$

Differentiate this equation with respect to pressure, assuming the pore volume remains constant. Then, we have

$$\frac{dN_r}{dP} = V_p \left[\frac{1}{B_o} \frac{dS_o}{dP} - \frac{S_o}{B_o{}^2} \frac{dB_o}{dP} \right]$$

The dissolved gas in the reservoir at any pressure is given by:

$$G_{dis} = \frac{V_p S_o}{B_o} R_{so}$$

While the free gas in the reservoir at the same pressure is given by:

$$G_{free} = \frac{V_p S_g}{B_g} = \frac{V_p (1 - S_o - S_w)}{B_g}$$

Therefore, the total gas remaining in the reservoir at standard cubic feet is the summation of the free and dissolved gases given as:

$$G_r = \frac{V_p S_o}{B_o} R_{so} + \frac{V_p (1 - S_o - S_w)}{B_g}$$

Differentiating the remaining gas volume with respect to pressure gives:

$$\frac{dG_r}{dP} = V_p \left[\frac{S_o}{B_o} \frac{dR_{so}}{dP} - \frac{R_{so} S_o}{B_o{}^2} \frac{dB_o}{dP} + \frac{R_{so}}{B_o} \frac{dS_o}{dP} - \frac{1}{B_g} \frac{dS_o}{dP} - \frac{(1 - S_o - S_w)}{B_g{}^2} \frac{dB_g}{dP} \right]$$

The current or producing gas-oil ratio is given as

$$R = \frac{\frac{dG_r}{dP}}{\frac{dN_r}{dP}} = \frac{\cancel{V_p} \left[\frac{S_o}{B_o} \frac{dR_{so}}{dP} - \frac{R_{so} S_o}{B_o{}^2} \frac{dB_o}{dP} + \frac{R_{so}}{B_o} \frac{dS_o}{dP} - \frac{1}{B_g} \frac{dS_o}{dP} - \frac{(1 - S_o - S_w)}{B_g{}^2} \frac{dB_g}{dP} \right]}{\cancel{V_p} \left[\frac{1}{B_o} \frac{dS_o}{dP} - \frac{S_o}{B_o{}^2} \frac{dB_o}{dP} \right]}$$

From material balance equation, the producing GOR is given as:

$$R = \frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g} + R_{so}$$

Therefore,

$$\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g} + R_{so} = \frac{\frac{S_o}{B_o}\frac{dR_{so}}{dP} - \frac{R_{so}S_o}{{B_o}^2}\frac{dB_o}{dP} + \frac{R_{so}}{B_o}\frac{dS_o}{dP} - \frac{1}{B_g}\frac{dS_o}{dP} - \frac{(1-S_o-S_w)}{{B_g}^2}\frac{dB_g}{dP}}{\frac{1}{B_o}\frac{dS_o}{dP} - \frac{S_o}{{B_o}^2}\frac{dB_o}{dP}}$$

$$\left[\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g} + R_{so}\right] * \left\{\frac{1}{B_o}\frac{dS_o}{dP} - \frac{S_o}{{B_o}^2}\frac{dB_o}{dP}\right\}$$
$$= \frac{S_o}{B_o}\frac{dR_{so}}{dP} - \frac{R_{so}S_o}{{B_o}^2}\frac{dB_o}{dP} + \frac{R_{so}}{B_o}\frac{dS_o}{dP} - \frac{1}{B_g}\frac{dS_o}{dP} - \frac{(1-S_o-S_w)}{{B_g}^2}\frac{dB_g}{dP}$$

Expanding gives

$$\left\{\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g}\frac{1}{B_o} + \frac{R_{so}}{B_o}\right\}\frac{dS_o}{dP} - \left\{\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g}\frac{S_o}{{B_o}^2}\right\}\frac{dB_o}{dP} - \left\{\frac{R_{so}S_o}{{B_o}^2}\right\}\frac{dB_o}{dP}$$
$$= \left\{\frac{S_o}{B_o}\right\}\frac{dR_{so}}{dP} - \left\{\frac{R_{so}S_o}{{B_o}^2}\right\}\frac{dB_o}{dP} + \left\{\frac{R_{so}}{B_o} - \frac{1}{B_g}\right\}\frac{dS_o}{dP} - \left\{\frac{(1-S_o-S_w)}{{B_g}^2}\right\}\frac{dB_g}{dP}$$

$$\left\{\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g}\frac{1}{B_o} + \frac{R_{so}}{B_o}\right\}\frac{dS_o}{dP} - \left\{\frac{R_{so}}{B_o} - \frac{1}{B_g}\right\}\frac{dS_o}{dP}$$
$$= \left\{\frac{S_o}{B_o}\right\}\frac{dR_{so}}{dP} - \left\{\frac{R_{so}S_o}{{B_o}^2}\right\}\frac{dB_o}{dP} - \left\{\frac{(1-S_o-S_w)}{{B_g}^2}\right\}\frac{dB_g}{dP} + \left\{\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g}\frac{S_o}{{B_o}^2}\right\}\frac{dB_o}{dP}$$
$$+ \left\{\frac{R_{so}S_o}{{B_o}^2}\right\}\frac{dB_o}{dP}$$

$$\left\{\frac{\cancel{B_o}k_{rg}\mu_o}{B_g k_{ro}\mu_g}\frac{1}{\cancel{B_o}} + \cancel{\frac{R_{so}}{B_o}} - \cancel{\frac{R_{so}}{B_o}} + \frac{1}{B_g}\right\}\frac{dS_o}{dP}$$
$$= \left\{\frac{S_o}{B_o}\right\}\frac{dR_{so}}{dP} \cancel{-} \left\{\cancel{\frac{R_{so}S_o}{{B_o}^2}}\right\}\cancel{\frac{dB_o}{dP}} - \left\{\frac{(1-S_o-S_w)}{{B_g}^2}\right\}\frac{dB_g}{dP} + \left\{\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g}\frac{S_o}{{B_o}^2}\right\}\frac{dB_o}{dP}$$
$$\cancel{+}\left\{\cancel{\frac{R_{so}S_o}{{B_o}^2}}\right\}\cancel{\frac{dB_o}{dP}}$$

$$\rightarrow \qquad \frac{dS_o}{dP} = \frac{\left\{\frac{S_o}{B_o}\right\}\frac{dR_{so}}{dP} - \left\{\frac{(1-S_o-S_w)}{{B_g}^2}\right\}\frac{dB_g}{dP} + \left\{\frac{B_o k_{rg}\mu_o}{B_g k_{ro}\mu_g}\frac{S_o}{{B_o}^2}\right\}\frac{dB_o}{dP}}{\frac{1}{B_g} + \frac{k_{rg}\mu_o}{B_g k_{ro}\mu_g}}$$

Multiple the above expression by ${}^{B_g}/_{B_g}$ gives

$$\therefore \quad \frac{dS_o}{dP} = \frac{\left\{\frac{S_o B_g}{B_o}\right\}\frac{dR_{so}}{dP} - \left\{\frac{(1-S_o-S_w)}{B_g}\right\}\frac{dB_g}{dP} + \left\{\frac{k_{rg}\mu_o}{k_{ro}\mu_g}\frac{S_o}{B_o}\right\}\frac{dB_o}{dP}}{1 + \frac{k_{rg}\mu_o}{k_{ro}\mu_g}}$$

Craft et al. (1991) simplified this equation above with expressions of group symbols as a function of as:

$$X(p) = \left\{\frac{B_g}{B_o}\right\}\frac{dR_{so}}{dP}$$

$$Y(p) = \left\{\frac{1}{B_o} * \frac{\mu_o}{\mu_g}\right\}\frac{dB_o}{dP}$$

$$Z(p) = \left\{\frac{1}{B_g}\right\}\frac{dB_g}{dP}$$

The increment saturation form using the pressure group is:

$$\frac{\Delta S_o}{\Delta P} = \frac{S_o X(p) + \left\{\frac{k_{rg}}{k_{ro}} S_o\right\} Y(p) - (1 - S_o - S_w) Z(p)}{1 + \frac{k_{rg}\mu_o}{k_{ro}\mu_g}}$$

Where

$$\Delta S_o = (S_o)_{i-1} - (S_o)_i$$

$$\Delta P = P_{i-1} - P_i$$

The pressure groups $X(p)$, $Y(p)$ & $Z(p)$ can be determine from reservoir fluid properties given above. The values of the derivatives $\left(\frac{dR_{so}}{dP}, \frac{dB_o}{dP} \,\&\, \frac{dB_g}{dP}\right)$ attached to each pressure group is obtained from a graphically plot of R_{so}, B_o & B_g *versus pressure* respectively. To be more accurate, determination of $\frac{dB_g}{dP}$ is obtained when ${}^{1}/_{B_g}$ is plotted versus pressure. The expression is given as:

$$\frac{d\left({}^{1}\!/_{B_g}\right)}{dP} = -\left\{\frac{1}{{B_g}^2}\right\}\frac{dB_g}{dP}$$

$$\frac{dB_g}{dP} = -{B_g}^2\frac{d\left({}^{1}\!/_{B_g}\right)}{dP}$$

$$Z(p) = \frac{1}{B_g}\left\{-{B_g}^2\frac{d\left({}^{1}\!/_{B_g}\right)}{dP}\right\} = -B_g\frac{d\left({}^{1}\!/_{B_g}\right)}{dP}$$

Muskat's Prediction Algorithm

At any given pressure, Craft et al. (1991) developed the following algorithm for solving Muskat's equation:

Step 1: Obtain relative permeability data at corresponding saturation values and then make a plot of k_{rg}/k_{ro} versus saturation.

Step 2: Make a plot of fluid properties $\{R_s$, B_o and $(1/B_g)\}$ versus pressure and determine the slope of each plot at selected pressures, i.e., dB_o/dp, dR_s/dp, and $d(1/B_g)/dp$.

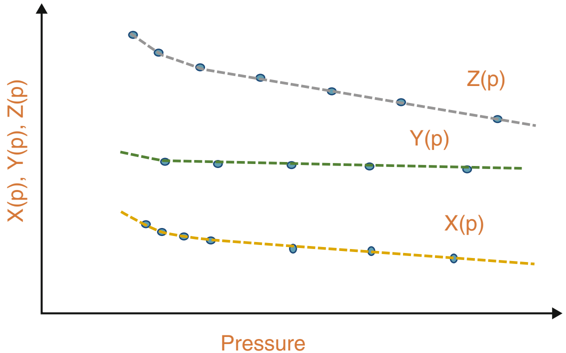
Step 3: Calculate the pressure dependent terms X(p), Y(p), and Z(p) that correspond to the selected pressures in Step 2.

$$X(p) = \left\{\frac{B_g}{B_o}\right\}\frac{dR_{so}}{dP}$$

$$Y(p) = \left\{\frac{1}{B_o}*\frac{\mu_o}{\mu_g}\right\}\frac{dB_o}{dP}$$

$$Z(p) = \left\{\frac{1}{B_g}\right\}\frac{dB_g}{dP} = -B_g\frac{d\left({}^{1}\!/_{B_g}\right)}{dP}$$

Step 4: Plot the pressure dependent terms as a function of pressure, as illustrated in the figure below.



Step 5: Graphically determine the values of X(p), Y(p), and Z(p) that correspond to the pressure P.

Step 6: Solve for (ΔS_o /ΔP) by using the oil saturation $(S_o)_{i-1}$ at the beginning of the pressure drop interval P_{i-1}.

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{i-1} = \frac{(S_o)_{i-1}X(P_{i-1}) + \left\{\frac{k_{rg}}{k_{ro}}(S_o)_{i-1}\right\}Y(P_{i-1}) - \left(1-(S_o)_{i-1}-S_{wi}\right)Z(P_{i-1})}{1+\left[\frac{k_{rg}\mu_o}{k_{ro}\mu_g}\right]_{i-1}}$$

Step 7: Determine the oil saturation S_o at the average reservoir pressure P, from:

$$(S_o)_i = (S_o)_{i-1} - [P_{i-1} - P_i]\left(\frac{\Delta S_o}{\Delta P}\right)_{i-1}$$

Step 8: Using the S_o from Step 7 and the pressure P, recalculate (ΔS_o/ΔP)

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{i} = \frac{(S_o)_{i}X(P_{i}) + \left\{\frac{k_{rg}}{k_{ro}}(S_o)_{i}\right\}Y(P_{i}) - \left(1-(S_o)_{i}-S_{wi}\right)Z(P_{i})}{1+\left[\frac{k_{rg}\mu_o}{k_{ro}\mu_g}\right]_{i}}$$

Step 9: Calculate the average value for (ΔS_o/ΔP) from the two values obtained in Steps 6 and 8.

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg} = \frac{\left(\frac{\Delta S_o}{\Delta P}\right)_{i-1} + \left(\frac{\Delta S_o}{\Delta P}\right)_{i}}{2}$$

Step 10: Using $\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg}$, solve for the oil saturation S_o from:

$$(S_o)_i = (S_o)_{i-1} - [P_{i-1} - P_i]\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg}$$

Note that the value of $(S_o)_i$ becomes $(S_o)_{i-1}$ for the next pressure drop interval.

Step 11: Calculate gas saturation $(S_g)_i$ by:

$$\left(S_g\right)_i = 1 - (S_o)_i - S_{wi}$$

Step 12: Using the saturation equation given below

$$S_o = (1 - S_{wi})\left[1 - \frac{N_p}{N}\right]\left(\frac{B_o}{B_{oi}}\right)$$

To solve for the cumulative oil production.

$$N_p = N\left[1 - \left\{\frac{S_o}{(1 - S_{wi})}\right\}\left\{\frac{B_{oi}}{B_o}\right\}\right]$$

Step 13: Calculate k_{rg}/k_{ro} at the selected pressure, P_i
Step 14: Calculate the instantaneous GOR at the selected pressure, P_i

$$R^{New} = \left[\frac{B_o k_{rg} \mu_o}{B_g k_{ro} \mu_g}\right]_i + (R_{so})_i$$

Step 15: Calculate the average GOR

$$R_{avg} = \frac{R_i + R^{New}}{2}$$

Step 16: Calculate the cumulative gas production by using N_p from step 12 and step 15

$$G_p = R_{avg} N_p$$

Step 17: Repeat Steps 5 through 13 for all pressure drops of interest.

Example 11.1
Given a saturated oil reservoir located at Amassoma oil field in Bayelsa State with no gas cap; whose initial pressure is 3620 psia and reservoir temperature of 220 °F. The initial (connate) water saturation is 0.195 and from volumetric analysis, the STOIIP was estimate as 45 MMSTB. There is no aquifer influx. The PVT data is given in the table below.

Pressure (psia)	B_o (bbl/STB)	R_{so} (SCF/STB)	B_g (bbl/SCF)	Oil vis (cp)	Gas vis (cp)
3620	1.5235	858	0.001091	0.7564	0.0239
3335	1.4879	796	0.001202	0.8355	0.0233
3045	1.4533	734	0.001332	0.9223	0.0227
2755	1.4187	672	0.001499	1.0199	0.0222
2465	1.3841	610	0.001700	1.1253	0.0216
2175	1.3496	549	0.001961	1.2431	0.0211
1885	1.3140	487	0.002296	1.3749	0.0205
1595	1.2794	425	0.002762	1.5206	0.0199

In this field, there is no relative permeability data available. Hence, the correlation below is used to generate the relative permeability curve.

$$\frac{k_{rg}}{k_{ro}} = 0.000149e^{12.57S_g}$$

Calculate the cumulative oil and gas production at 3335 psia using the Muskat method

Solution
Muskat Method

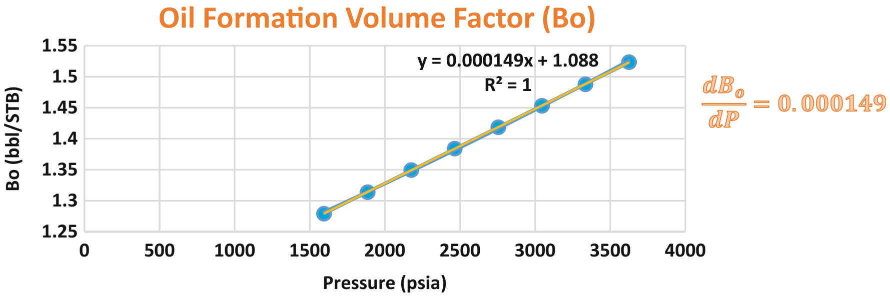
Step 1: Obtain relative permeability data at corresponding saturation values and then make a plot of k_{rg}/k_{ro} versus saturation.

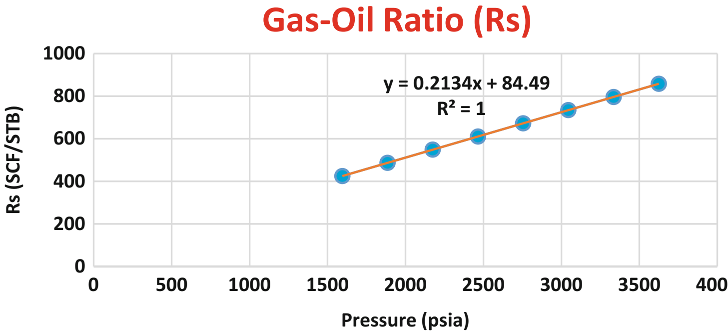
Since no relative permeability data was given, the correlation is used to obtain the relative permeability ratio which is given as:

$$\frac{k_{rg}}{k_{ro}} = 0.000149e^{12.57S_g}$$

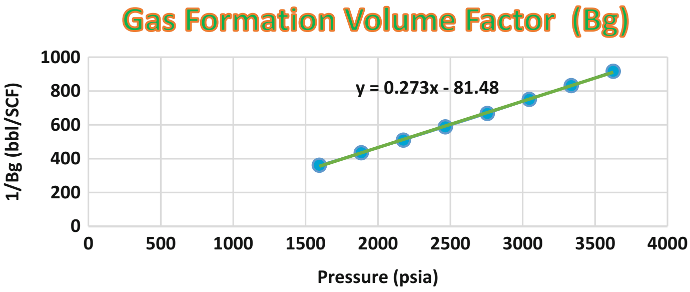
Step 2: Make a plot of fluid properties $\{R_s, B_o \text{ and } (1/B_g)\}$ versus pressure and determine the slope of each plot at the selected pressures, i.e., dB_o/dp, dR_s/dp, and $d(1/B_g)/dp$.

The fluid properties are plotted versus pressure in the figures below





$$\frac{dR_{so}}{dP} = 0.2134$$



$$\frac{d\left({}^{1}/_{B_g}\right)}{dP} = 0.2737$$

Step 3: Calculate the pressure dependent terms X(p), Y(p), and Z(p) that correspond to the selected pressures in Step 2.

At P = 3620 psia

$$X(p) = \left\{\frac{B_g}{B_o}\right\}\frac{dR_{so}}{dP} = \left\{\frac{0.001091}{1.5235}\right\}*0.2134 = 0.000153$$

$$Y(p) = \left\{\frac{1}{B_o}*\frac{\mu_o}{\mu_g}\right\}\frac{dB_o}{dP} = \left\{\frac{1}{1.5235}*\frac{0.7564}{0.0239}\right\}*0.000149 = 0.003095$$

$$Z(p) = \left\{\frac{1}{B_g}\right\}\frac{dB_g}{dP} = -B_g\frac{d\left({}^{1}/_{B_g}\right)}{dP} = 0.001091*0.2737 = 0.000298$$

At P = 3335 psia

$$X(p) = \left\{\frac{B_g}{B_o}\right\}\frac{dR_{so}}{dP} = \left\{\frac{0.001202}{1.4879}\right\}*0.2134 = 0.000172$$

$$Y(p) = \left\{\frac{1}{B_o} * \frac{\mu_o}{\mu_g}\right\} \frac{dB_o}{dP} = \left\{\frac{1}{1.4879} * \frac{0.8355}{0.0233}\right\} * 0.000149 = 0.003591$$

$$Z(p) = \left\{\frac{1}{B_g}\right\} \frac{dB_g}{dP} = -B_g \frac{d\left(^1/_{B_g}\right)}{dP} = 0.001202 * 0.2737 = 0.000329$$

It is represented in a tabular form as:

Pressure (psia)	X (p)	Y (p)	Z (p)
3620	0.000153	0.003095	0.000298
3335	0.000172	0.003591	0.000329

Step 4: Solve for ($\Delta S_o / \Delta P$) by using the oil saturation $(S_o)_{i-1}$ at 3620 psia

$$S_{wi} = 0.195$$

$$(S_o)_{3650\ psia} = 1 - 0.195 = 0.805$$

$$\left(\frac{\Delta S_o}{\Delta P}\right)_i = \frac{(S_o)_i X(P_i) + \left\{\frac{k_{rg}}{k_{ro}}(S_o)_i\right\} Y(P_i) - \left(1 - (S_o)_i - S_{wi}\right) Z(P_i)}{1 + \left[\frac{k_{rg}\mu_o}{k_{ro}\mu_g}\right]_i}$$

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{3620} = \frac{(S_o)_{3650} X(3650) + \left\{\frac{k_{rg}}{k_{ro}}(S_o)_{3650}\right\} Y(P_{3650}) - \left(1 - (S_o)_{3650} - S_{wi}\right) Z(P_{3650})}{1 + \left[\frac{k_{rg}\mu_o}{k_{ro}\mu_g}\right]_{3620}}$$

Since no free gas initially in place

$$\frac{k_{rg}}{k_{ro}} = 0.000149 e^{12.57 S_g}$$

$$\frac{k_{rg}}{k_{ro}} = 0$$

$$\therefore \left(\frac{\Delta S_o}{\Delta P}\right)_{3620} = \frac{0.805 * (0.000153) + 0 + 0}{1 + 0} = 0.000123$$

Step 5: Determine the oil saturation S_o at 3335 psia

$$(S_o)_i = (S_o)_{i-1} - [P_{i-1} - P_i] \left(\frac{\Delta S_o}{\Delta P}\right)_{i-1}$$

$$(S_o)_{3335} = (S_o)_{3620} - [P_{3620} - P_{3335}]\left(\frac{\Delta S_o}{\Delta P}\right)_{3620}$$

$$(S_o)_{3335} = 0.805 - [3620 - 3335]*0.000123 = 0.7699$$

$$\left(S_g\right)_i = 1 - (S_o)_i - S_{wi}$$

$$\left(S_g\right)_{3335} = 1 - 0.7699 - 0.195 = 0.0351$$

Step 6: Using the S_o from Step 5 and the pressure P, recalculate ($\Delta S_o/\Delta$P)

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{3335} = \frac{\begin{array}{c}(S_o)_{3335}X(P_{3335}) + \left\{\dfrac{k_{rg}}{k_{ro}}(S_o)_{3335}\right\}Y(P_{3335}) \\ -\left(1 - (S_o)_{3335} - S_{wi}\right)Z(P_{3335})\end{array}}{1 + \left[\frac{k_{rg}\mu_o}{k_{ro}\mu_g}\right]_{3335}}$$

The relative permeability ratio is

$$\frac{k_{rg}}{k_{ro}} = 0.000149e^{(12.57*0.0351)} = 0.000232$$

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{3335} = \frac{\begin{array}{c}(0.7699*0.000172) + (\{0.000232*0.7699\}*0.003591) \\ -(1 - 0.7699 - 0.195)*0.000329\end{array}}{1 + (0.000232)*\frac{0.8355}{0.0233}}$$

$$\therefore \left(\frac{\Delta S_o}{\Delta P}\right)_{3335} = 0.0001205$$

Step 7: Calculate the average value for ($\Delta S_o/\Delta$P) from the two values obtained in Steps 4 and 6.

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg} = \frac{\left(\frac{\Delta S_o}{\Delta P}\right)_{i-1} + \left(\frac{\Delta S_o}{\Delta P}\right)_i}{2}$$

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg} = \frac{\left(\frac{\Delta S_o}{\Delta P}\right)_{3620} + \left(\frac{\Delta S_o}{\Delta P}\right)_{3335}}{2}$$

$$\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg} = \frac{0.000123 + 0.0001205}{2} = 0.0001218$$

Step 8: Using $\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg}$, solve for the oil saturation S_o from:

$$(S_o)_{3335} = (S_o)_{3620} - [P_{3620} - P_{3335}]\left(\frac{\Delta S_o}{\Delta P}\right)_{Avg}$$

$$(S_o)_{3335} = 0.805 - [3620 - 3335]*0.0001218 = 0.7703$$

Step 9: Calculate gas saturation $(S_g)_i$ by:

$$\left(S_g\right)_{3335} = 1 - 0.7703 - 0.195 = 0.0347$$

Step 10: Calculate the cumulative oil production

$$N_p = N\left[1 - \left\{\frac{S_o}{(1 - S_{wi})}\right\}\left\{\frac{B_{oi}}{B_o}\right\}\right]$$

$$N_p = 45 \times 10^6\left[1 - \left\{\frac{0.7703}{(1 - 0.195)}\right\}\left\{\frac{1.5235}{1.4879}\right\}\right] = 909477.451\ STB$$

Step 11: Calculate k_{rg}/k_{ro} at 3335 psia as calculated above

$$\frac{k_{rg}}{k_{ro}} = 0.000232$$

Step 12: Calculate the instantaneous GOR at the 3335 psia

$$R^{New} = \left[\frac{B_o k_{rg} \mu_o}{B_g k_{ro} \mu_g}\right]_i + (R_{so})_i$$

$$R^{New} = 0.000232*\left(\frac{1.4879}{0.001202}\right)\left(\frac{0.8355}{0.0233}\right) + 796 = 806.2979\ scf/STB$$

Step 13: Calculate the average GOR

$$R_{avg} = \frac{R_{3335} + R^{New}}{2}$$

$$R_{avg} = \frac{796 + 806.2979}{2} = 801.1489\ scf/STB$$

Step 14: Calculate the cumulative gas production by using N_p from step 10 and step 13

$$G_p = R_{avg}N_p$$

$$G_p = 801.1489 * 909477.451 = 728626859.4\ scf = 728.6269\ MMscf$$

11.3 Tarner's Prediction Method

Tarner (1944) suggested an iterative technique for predicting cumulative oil production Np and cumulative gas production Gp as a function of reservoir pressure. The method is based on solving the MBE and the instantaneous GOR equation simultaneously for a given reservoir pressure drop from a known pressure P_{i-1} to an assumed (new) pressure P_i. It is accordingly assumed that the cumulative oil and gas production has increased from known values of $(N_p)_{i-1}$ and $(G_p)_{i-1}$at reservoir pressure P_{i-1} to future values of $(N_p)_i$ and $(G_p)_i$ at the assumed pressure P_i. To simplify the description of the proposed iterative procedure, the stepwise calculation is illustrated for a volumetric saturated oil reservoir; however, this method can be used to predict the volumetric behavior of reservoirs under different driving mechanisms.

Tarner's method was preferred to Tracy and Muskat because of the differential form of expressing each parameter of the material balance equation by Tracy. Also, Tarner and Muskat method use iterative approach in the prediction until a convergence is reached.

Furthermore, a first approach of the Cumulative Oil Production is needed before the calculation is performed; a second value of this variable is calculated through the equation that defines the Cumulative Gas Production, as an average of two different moments in the production life of the reservoir; this expression, as we will see, is a function of the Instantaneous Gas Oil Rate, then we need also to calculate this value in advance from an equation derived from Darcy's law, this is a very important relationship since it is strongly affected by the relative permeability ratio between oil and gas. Finally, both values are compared, if the difference is within certain predefined tolerance, our first estimate of the Cumulative Oil Production will be considered essentially right, otherwise the entire process is repeated until the desire level of accuracy is reached (Tarner 1944).

Tarner's Prediction Algorithm

Step 1: Select a future reservoir pressure P_i below the initial (current) reservoir pressure P_{i-1} and obtain the necessary PVT data. Assume that the cumulative oil production has increased from $(N_p)_{i-1}$ *to* $(N_p)_i$. It should be pointed out that $(N_p)_{i-1}$ and $(G_p)_{i-1}$ are set equal to zero at the bubble-point pressure (initial reservoir pressure).

Step 2: Estimate or guess the cumulative oil production $(N_p)_i$ at P_i.

Step 3: Calculate the cumulative gas production $(G_p)_i$ by rearranging the MBE to give:

$$(G_p)_{MBE,\ i} = \mathrm{N}\left(\{(\mathrm{R_{si}})_{i-1} - (\mathrm{R_s})_i\} - \left\{\frac{(\mathrm{B_{oi}})_{i-1} - (\mathrm{B_o})_i}{(\mathrm{B_g})_i}\right\}\right)$$
$$- (N_p)_i\left\{\frac{\mathrm{B_o}}{\mathrm{B_g}} - \mathrm{R_s}\right\}_i$$

Step 4: Calculate the oil and gas saturations $\{(\mathrm{S_o})_i$ *and* $(\mathrm{S_g})_i\}$ at the assumed cumulative oil production $(N_p)_i$ and the selected reservoir pressure P_i by applying Equations

$$(S_o)_i = (1 - S_{wi})\left[1 - \frac{(N_p)_i}{N}\right]\left(\frac{(\mathrm{B_o})_i}{(\mathrm{B_{oi}})_{i-1}}\right)$$
$$(S_g)_i = 1 - (S_o)_i - S_w$$

Step 5: Using the available relative permeability data, determine the relative permeability ratio krg/kro that corresponds to the gas saturation at P_i and compute the instantaneous GOR (R_i) at P_i as:

$$\mathrm{R}_i = (\mathrm{R_{so}})_i + \left(\frac{\mathrm{K_{rg}}}{\mathrm{K_{ro}}}\right)_i\left(\frac{\mu_o \mathrm{B_o}}{\mu_g \mathrm{B_g}}\right)_i$$

It should be noted that all the PVT data in the expression must be evaluated at the assumed reservoir pressure P_i.

Step 6: Calculate again the cumulative gas production $(G_p)_i$ at P_i given as

$$(G_p)_{GOR,\ i} = (G_p)_{i-1} + \left[\frac{\mathrm{R}_{i-1} + \mathrm{R}_i}{2}\right]\left[(N_p)_i - (N_p)_{i-1}\right]$$

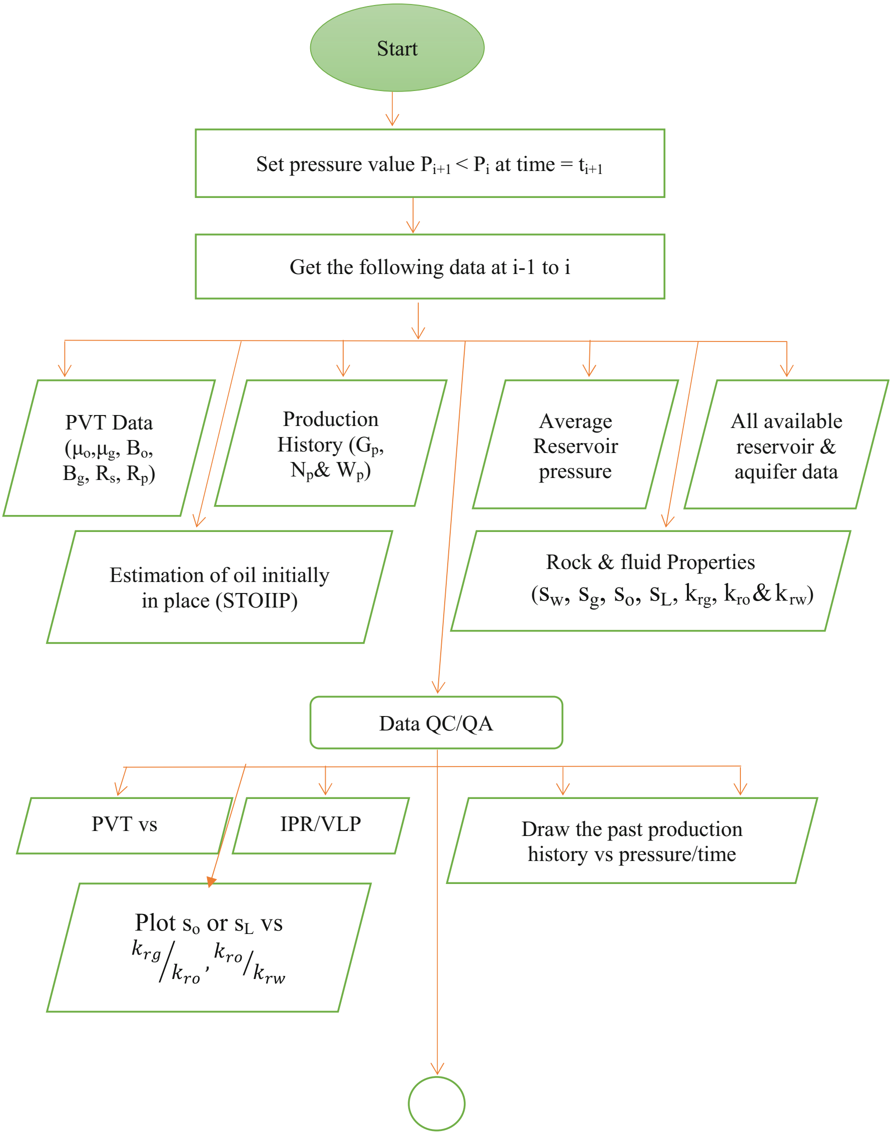
In which R_{i-1} represents the instantaneous GOR at P_{i-1}. If P_{i-1} represents the initial reservoir pressure, then set $\mathrm{R}_{i-1} = \mathrm{R}_{si}$.

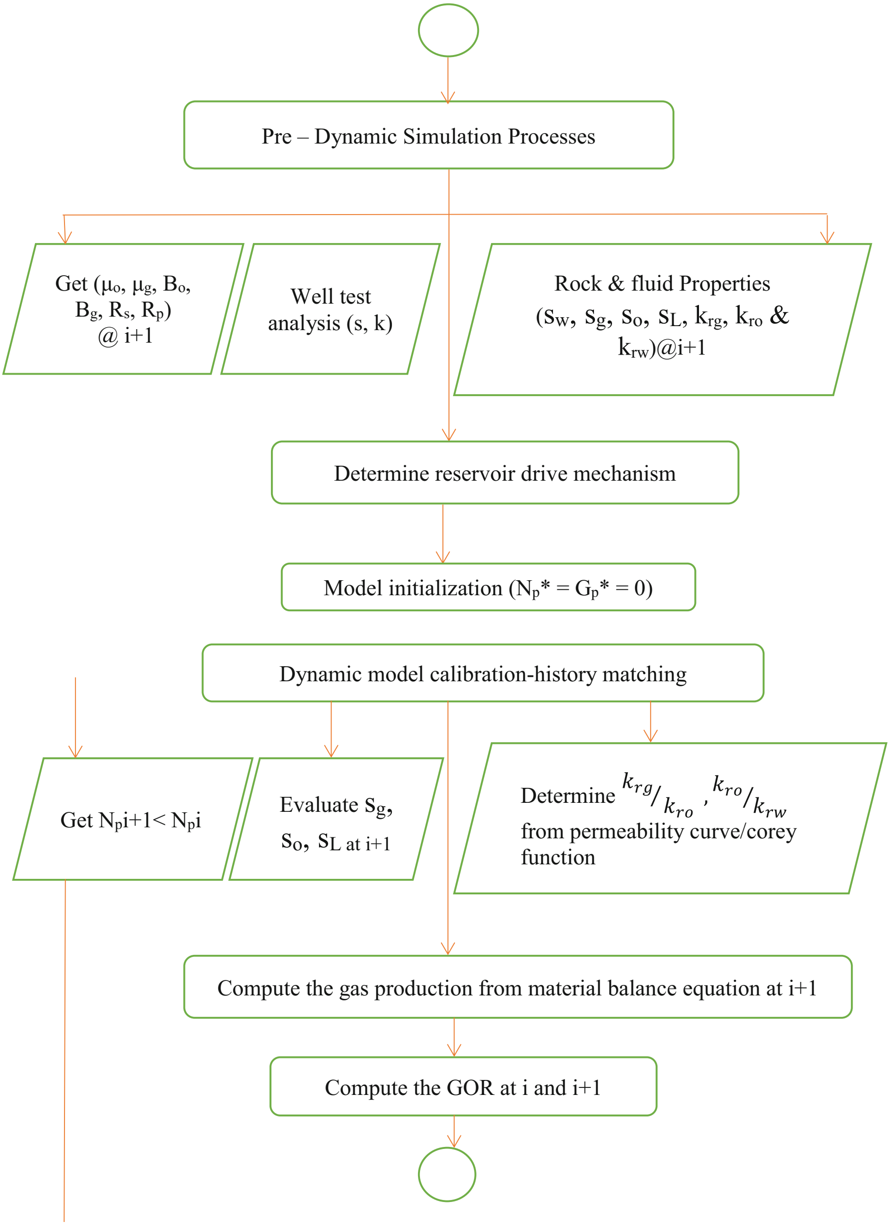
Step 7: The total gas produced $(G_p)_i$ during the first prediction period as calculated by the material balance equation $\{(G_p)_{MBE,\ i}\}$ is compared to the total gas produced as calculated by the GOR equation $\{(G_p)_{GOR,\ i}\}$. These two equations provide the two independent methods required for determining the total gas produced.

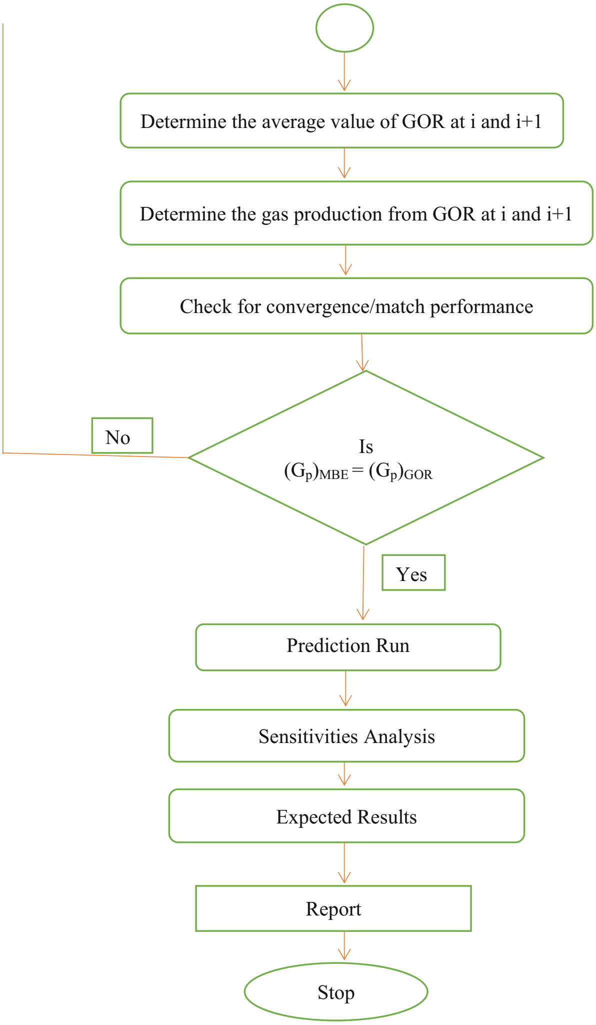
Therefore, if the cumulative gas production $\{(G_p)_{MBE,\ i}\}$ as calculated from Step 3 agrees with the value $\{(G_p)_{GOR,\ i}\}$ of Step 6, the assumed value of $(N_p)_i$ is correct and a new pressure may be selected and Steps 1 through 6 are repeated. Otherwise, assume another value of $(N_p)_i$ and repeat Steps 2 through 6.

Step 8: In order to simplify this iterative process, three values of $(N_p)_i$ can be assumed, which yield three different solutions of cumulative gas production for each of the equations (i.e., MBE and GOR equation). When the computed values of $(G_p)_i$ are plotted versus the assumed values of $(N_p)_i$, the resulting two curves (one representing results of Step 3 and the one representing Step 5) will intersect. This intersection indicates the cumulative oil and gas production that will satisfy both equations.

A workflow of an expansion of Tarner's method is presented in the flow chart below




Pre – Dynamic Simulation Processes
Get (μ_o, μ_g, B_o, B_g, R_s, R_p) @ i+1
Well test analysis (s, k)
Rock & fluid Properties (S_W, S_g, S_o, S_L, k_{rg}, k_{ro} & k_{rw})@i+1
Determine reservoir drive mechanism
Model initialization ($N_p^* = G_p^* = 0$)
Dynamic model calibration-history matching
Get $N_pi+1< N_pi$
Evaluate S_g, S_o, S_L at i+1
Determine k_{rg}/k_{ro}, k_{ro}/k_{rw} from permeability curve/corey function
Compute the gas production from material balance equation at i+1
Compute the GOR at i and i+1



Example 11.2

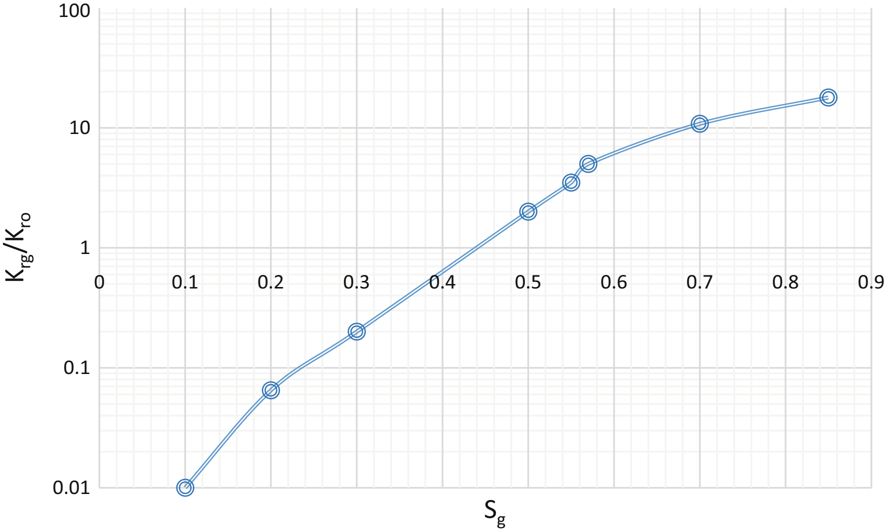
A volumetric oil reservoir presents the following characteristics:

Initial reservoir pressure, P_i	*3200 psia*
Bubble point pressure, P_b	*3200 psia*
STOIIP, N	*9,655,344 stb*
Connate water saturation, S_{wc}	*23%*
Water influx, W_e	*0*
Water injection, W_{inj}	*0*
Reservoir temperature	220^0F

The PVT data is given in the table below

Pressure (psia)	B_o (bbl/STB)	R_{so} (SCF/STB)	B_g (bbl/SCF)	Oil vis (cp)	Gas vis (cp)
3200	1.3859	1180	0.001383	0.84239	0.0238
2870	1.3784	1120	0.001618	0.89239	0.0233
2510	1.3603	1030	0.00184	0.9316	0.0231

The relative permeability curve is



Calculate the cumulative oil and gas production at 2870 psia using the Tarner's method with a convergence criteria of absolute relative error less than 5%.

Solution

Step 1: The pressure of interest = 2870 psia

Step 2: Assume the cumulative oil production $\{(N_p)_{2870}\}$ at 2870 psia = 96553.44 STB (i.e 1% of STOIIP).

Step 3: Calculate the cumulative gas production $(G_p)_{2870}$ by rearranging the MBE to give:

$$(G_p)_{MBE,\ 2870} = N\left(\{(R_{si})_{3200} - (R_s)_{2870}\} - \left\{\frac{(B_{oi})_{3200} - (B_o)_{2870}}{(B_g)_{2870}}\right\}\right) - (N_p)_{2870}\left\{\frac{B_o}{B_g} - R_s\right\}_{2870}$$

$$\left(G_p\right)_{MBE,\ 2870} = 9655344 * \left(\{1180 - 1120\} - \left\{\frac{1.3859 - 1.3784}{0.001618}\right\}\right)$$
$$- 96553.44\left\{\frac{1.3784}{0.001618} - 1120\right\}_{2870}$$
$$= 5.6045 \times 10^8\ SCF$$

Step 4: Calculate the oil and gas saturations at 2870 psia

$$(S_o)_{2870} = (1 - S_{wi})\left[1 - \frac{\left(N_p\right)_i}{N}\right]\left(\frac{(\mathrm{B_o})_i}{(\mathrm{B_{oi}})_{i-1}}\right)$$

$$(S_o)_{2870} = (1 - 0.23)\left[1 - \frac{96553.44}{9655344}\right]\left(\frac{1.3784}{1.3859}\right) = 0.7582$$

$$\left(S_g\right)_{2870} = 1 - (S_o)_i - S_w = 1 - 0.7582 - 0.23 = 0.0118$$

Step 5: Using the available relative permeability plot, the relative permeability ratio krg/kro that corresponds to the gas saturation $(S_g)_{2870}$

$$\left(\frac{\mathrm{K_{rg}}}{\mathrm{K_{ro}}}\right)_{2870} = 0.01 \quad \textit{from graph above}$$

Compute the instantaneous GOR (R_i) at 2870 psia as:

$$\mathrm{R}_{2870} = (\mathrm{R_{so}})_{2870} + \left(\frac{\mathrm{K_{rg}}}{\mathrm{K_{ro}}}\right)_{2870}\left(\frac{\mu_o \mathrm{B_o}}{\mu_g \mathrm{B_g}}\right)_{2870}$$

$$\mathrm{R}_{2870} = 1120 + 0.01\left(\frac{0.89239 * 1.3784}{0.0233 * 0.001618}\right) = 1446.284\ scf/STB$$

Step 6: Calculate again the cumulative gas production at 2870 psia given as

$$\left(G_p\right)_{GOR,\ 2870} = \left(G_p\right)_{3200} + \left[\frac{\mathrm{R}_{3200} + \mathrm{R}_{2870}}{2}\right]\left[\left(N_p\right)_{2870} - \left(N_p\right)_{3200}\right]$$

$$\left(G_p\right)_{GOR,\ 2870} = 0 + \left[\frac{1180 + 1446.284}{2}\right][96553.44 - 0] = 1.2679 \times 10^8\ SCF$$

Step 7: Since the cumulative gas production are not equal, i.e

$$\left(G_p\right)_{MBE,\ 2870} \neq \left(G_p\right)_{GOR,\ 2870}$$

We calculate the absolute relative error as

$$|Error| = \left|\frac{\left(G_p\right)_{GOR,\ 2870} - \left(G_p\right)_{MBE,\ 2870}}{\left(G_p\right)_{GOR,\ 2870}}\right| \times 100\%$$

$$|Error| = \left|\frac{1.2679 \times 10^8 - 5.6045 \times 10^8}{1.2679 \times 10^8}\right| \times 100\% = 342.03\%$$

The absolute relative error is far more than the convergence criteria. Hence, several iteration were carried out (i.e repeat step 1–6) and it converged at:

Step 1: The pressure of interest = 2870 psia

Step 2: Assume the cumulative oil production $\{(N_p)_{2870}\}$ at 2870 psia = 531043.9 STB (i.e 5.5% of STOIIP).

Step 3: Calculate the cumulative gas production $(G_p)_{2870}$ by rearranging the MBE to give:

$$\begin{aligned}\left(G_p\right)_{MBE,\ 2870} &= 9655344*\left(\{1180 - 1120\} - \left\{\frac{1.3859 - 1.3784}{0.001618}\right\}\right) \\ &\quad - 531043.9\ \left\{\frac{1.3784}{0.001618} - 1120\right\}_{2870} \\ &= 6.7693 \times 10^8\ SCF\end{aligned}$$

Step 4: Calculate the oil and gas saturations at 2870 psia

$$(S_o)_{2870} = (1 - 0.23)\left[1 - \frac{531043.9}{9655344}\right]\left(\frac{1.3784}{1.3859}\right) = 0.7237$$

$$\left(S_g\right)_{2870} = 1 - \left(S_o\right)_i - S_w = 1 - 0.7582 - 0.23 = 0.0463$$

Step 5: Using the available relative permeability plot, the relative permeability ratio krg/kro that corresponds to the gas saturation $(S_g)_{2870}$

$$\left(\frac{\mathrm{K_{rg}}}{\mathrm{K_{ro}}}\right)_{2870} = 0.01 \quad \textit{from graph above}$$

Compute the instantaneous GOR (R_i) at 2870 psia as:

$$\mathrm{R}_{2870} = 1120 + 0.01\left(\frac{0.89239*1.3784}{0.0233*0.001618}\right) = 1446.284\ scf/STB$$

Step 6: Calculate again the cumulative gas production at 2870 psia given as

$$\left(G_p\right)_{GOR,\ \ 2870} = 0 + \left[\frac{1180 + 1446.284}{2}\right][531043.9 - 0] = 6.9734 \times 10^8\ SCF$$

Step 7: Since the cumulative gas production are close, the absolute relative error is calculated as

$$|Error| = \left|\frac{6.9734 \times 10^8 - 6.7693 \times 10^8}{6.9734 \times 10^8}\right| \times 100\% = 2.9268\%$$

Therefore, the cumulative oil production = 531043.9 STB and gas cumulative production = $6.9734 \times 10^8\ SCF$.

11.4 Tracy Prediction Method

Tracy (1955) developed a model for reservoir performance prediction that did not consider oil reservoirs above bubble-point pressure (undersaturated reservoir) but the computation starts at pressures below or at the bubble-point pressure. To use this method for predicting future performance, it is pertinent therefore to select future pressures at desired performance. This means that we need to select the pressure step to be used. Hence, Tracy's calculations are performed in series of pressure drops that proceed from a known reservoir condition at the previous reservoir pressure ($\boldsymbol{P_{i-1}}$) to the new assumed lower pressure ($\boldsymbol{P_i}$). The calculated results at the new reservoir pressure becomes "known" at the next assumed lower pressure. The cumulative gas, oil, and producing gas-oil ratio are calculated at each selected pressure, so the goal is to determine a table of N_p, G_p, and R_p versus future reservoir static pressure.

Tracy's Prediction Algorithm

Step 1: Select an average reservoir pressure ($\boldsymbol{P_i}$) of interest

Step 2: Calculate the values of the PVT functions $\boldsymbol{\phi_o}$, $\boldsymbol{\phi_g}$ **&** $\boldsymbol{\phi_w}$ where

$$\phi_o = \frac{B_o - R_s B_g}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]}$$

$$\phi_g = \frac{B_g}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]}$$

$$\phi_w = \frac{1}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]}$$

Step 3: Assume (estimate) the GOR ($\boldsymbol{R_i}$) at the pressure of interest

Step 4: Estimate the average instantaneous GOR ($\boldsymbol{R_{avg}}$) at the pressure of interest

The average producing gas-oil ratio for a pressure decrement from P_{i-i} *to the pressure of interest* P_i given as:

$$R_{avg} = \frac{R_{i-1} + R_i}{2}$$

Step 5: Calculate the incremental cumulative oil production $\boldsymbol{\Delta N_p}$ as:

The general material balance equation is given as

$$N = N_p\phi_o + G_p\phi_g - \left(W_e - W_p\right)\phi_w$$

For a solution gas drive reservoir (undersaturated reservoir) the equation reduces to

$$N = N_p\phi_o + G_p\phi_g$$

At pressure of interest

$$N = \left(N_p\right)_i(\phi_o)_i + \left(G_p\right)_i\left(\phi_g\right)_i$$

Note that as the pressure decreases, there is a corresponding incremental production of oil and gas designated as ΔN_p & ΔG_p. There the cumulative oil and gas production at pressure of interest are given as:

$$\left(N_p\right)_i = \left(N_p\right)_{i-1} + \Delta N_p$$

$$(G_p)_i = (G_p)_{i-1} + \Delta G_p$$

Substitute into the above equation of N at pressure of interest, we have

$$N = \left[(N_p)_{i-1} + \Delta N_p\right](\phi_o)_i + \left[(G_p)_{i-1} + \Delta G_p\right](\phi_g)_i$$

But

$$\Delta G_p = R_{avg}\Delta N_p$$

Hence

$$N = \left[(N_p)_{i-1} + \Delta N_p\right](\phi_o)_i + \left[(G_p)_{i-1} + \mathrm{R}_{avg}\Delta N_p\right](\phi_g)_i$$

$$N = (N_p)_{i-1}(\phi_o)_i + \Delta N_p(\phi_o)_i + (G_p)_{i-1}(\phi_g)_i + R_{avg}\Delta N_p(\phi_g)_i$$

$$N - (N_p)_{i-1}(\phi_o)_i - (G_p)_{i-1}(\phi_g)_i = \Delta N_p\left[(\phi_o)_i + R_{avg}(\phi_g)_i\right]$$

$$\therefore \quad \Delta N_p = \frac{N - (N_p)_{i-1}(\phi_o)_i - (G_p)_{i-1}(\phi_g)_i}{(\phi_o)_i + R_{avg}(\phi_g)_i}$$

Step 6: Calculate total or cumulative oil production from

$$(N_p)_i = (N_p)_{i-1} + \Delta N_p$$

Step 7: Calculate the oil and gas saturations at pressure P_i when the cumulative oil production $(N_p)_i$ is given as (see derivation in Chap. 5):

$$S_o = \left[1 - \frac{(N_p)_i}{N}\right]\left[\frac{(B_o)_i}{B_{oi}}\right][1 - S_{wi}]$$

$$S_g = 1 - S_o - S_{wi}$$

Step 8: Obtain the relative permeability ratio *krg*/*kro* at time i as a function of S_o *or* S_g *or* $S_L = (S_o + S_{wi})$.

Step 9: Make a plot of $\frac{krg}{kro}$ *Versus* S_o *or* S_L on a semi log graph

Step 10: Calculate the new instantaneous GOR at time, i given as

$$R_{avg}{}^{New} = \left[\frac{B_o k_{rg} \mu_o}{B_g k_{ro} \mu_g}\right]_i + (R_s)_i$$

Step 11: Compare the assumed or estimated GOR in Step 3 with the calculated GOR in Step 10. If the values are within acceptable tolerance, the incremental cumulative oil produced is correct (step 5), then proceed to the next step. If not within the tolerance, set the assumed GOR equal to the calculated new GOR and repeat the calculations from Step 3.

Step 12: Calculate the cumulative gas production.

$$(G_p)_i = (G_p)_{i-1} + \Delta G_p = (G_p)_{i-1} + R_{avg} \Delta N_p$$

Step 13: Make a final check on the accuracy of the prediction which should be made on the MBE as:

$$N_p \phi_o + G_p \phi_g = N \pm \boldsymbol{Tolerance}$$

If the STOIIP is based on 1 STB in step 5, the final check equation reduces to

$$N_p \phi_o + G_p \phi_g = 1 \pm \boldsymbol{Tolerance}$$

Step 14**:** Repeat from Step 1 for a new (lower) pressure value.

As the calculation progresses, a plot of GOR versus pressure can be maintained and extrapolated as an aid in estimating GOR at each new pressure.

Example 11.3

Apply the data in Example 11.1 to calculate the cumulative oil and gas production at 2870 psia using Tracy's method.

Tracy Method

Step 1: The average reservoir pressure of interest = 2870 psia
Step 2: Calculate the values of the PVT functions ϕ_o, ϕ_g & ϕ_w where

The is no gas cap, hence m = 0

$$\phi_o = \frac{B_o - R_s B_g}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]}$$

$$\phi_o = \frac{1.3784 - \{1120*0.001618\}}{(1.3784 - 1.3859) + \{(1180 - 1120)*0.001618\}} = -4.84215$$

$$\phi_g = \frac{B_g}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi}\left[\frac{B_g}{B_{gi}} - 1\right]}$$

$$\phi_g = \frac{0.001772}{(1.3859 - 1.3784) + \{(1180 - 1120)*0.001618\}} = 0.018062$$

Step 3: Assume $R_{assume} = 1447$ *SCF/STB* at 2870 psia

Step 4: Estimate the average instantaneous GOR (R_{avg}) at 2870 psia

$$R_{avg} = \frac{R_{i-1} + R_i}{2} = \frac{1180 + 1447}{2} = 1313.5$$

Step 5: Calculate the incremental cumulative oil production ΔN_p as:

$$\Delta N_p = \frac{N - (N_p)_{i-1}(\phi_\mathrm{o})_i - (G_\mathrm{p})_{i-1}(\phi_g)_i}{(\phi_\mathrm{o})_i + R_{avg}(\phi_g)_i}$$

Note

$$(N_p)_{i-1} = (G_p)_{i-1} = 0$$

$$\Delta N_p = \frac{9655344 - (0* - 4.84215) - (0*0.018062)}{-4.84215 + (0.018062*1313.5)} = 511343.99\ STB$$

Step 6: Calculate total or cumulative oil production from

$$(N_p)_{2870} = (N_p)_{i-1} + \Delta N_p = 0 + 511343.99 = 511343.99\ \ STB$$

Step 7: Calculate the oil and gas saturations at 2870 psia

$$S_o = \left[1 - \frac{\left(N_p\right)_i}{N}\right]\left[\frac{(B_o)_i}{B_{oi}}\right][1 - S_{wi}]$$

$$S_o = \left[1 - \frac{511343.99}{9655344}\right]\left[\frac{1.3784}{1.3859}\right][1 - 0.23] = 0.7253$$

$$S_g = 1 - 0.7253 - 0.23 = 0.0447$$

Step 8: Obtain the relative permeability ratio *krg/kro* at 2870 psia.

From the relative permeability curve given in Example 11.2,

$$\frac{k_{rg}}{k_{ro}} = 0.010$$

Step 9: Calculate the new instantaneous GOR at time, i given as

$$\mathrm{R}_{2870} = (\mathrm{R}_{\mathrm{so}})_{2870} + \left(\frac{\mathrm{K}_{\mathrm{rg}}}{\mathrm{K}_{\mathrm{ro}}}\right)_{2870}\left(\frac{\mu_{\mathrm{o}}\mathrm{B}_{\mathrm{o}}}{\mu_{\mathrm{g}}\mathrm{B}_{\mathrm{g}}}\right)_{2870}$$

$$\mathrm{R}_{2870} = 1120 + 0.01\left(\frac{0.89239 * 1.3784}{0.0233 * 0.001618}\right) = 1446.284\ scf/STB$$

Step 10: Compare the assumed GOR in Step 3 with the calculated GOR in Step 9.

$$R_{assume} = 1447\ SCF/STB$$

$$\mathrm{R}_{2870} = 1446.28$$

Since these values are closed, thus the cumulative oil production is:

$$\left(N_p\right)_{3335} = 511343.99\ \ STB$$

Step 12: Calculate the cumulative gas production.

$$\left(G_p\right)_{2870} = \left(G_p\right)_{3200} + \Delta G_p = \left(G_p\right)_{3200} + \left(R_{avg}\Delta N_p\right)_{2870}$$

$$\left(G_p\right)_{2870} = 0 + 511343.99 * 1313.5 = 671650330.9\ SCF$$

Comparing Results of Tarner and Tracy Method

Parameter	Tarner method	Tracy method
Cum oil production (STB)	531043.9	511343.99
Cum gas production (SCF)	6.9734×10^8	6.7165×10^8

11.4.1 Schilthuis Prediction Method

Schilthuis develop a method of reservoir performance prediction using the total produced or instantaneous gas-oil ratio which was defined mathematical as:

$$R = \mathrm{R_{so}} + \frac{\mathrm{K_{rg}}}{\mathrm{K_{ro}}} \frac{\mu_o \mathrm{B_o}}{\mu_g \mathrm{B_g}}$$

Schilthuis method requires a trail-and-error approach to achieve an appropriate result of incremental oil recovery. Schilthuis rearranged the material balance equation to initiate a convergence criteria with minimum tolerance of error. This equation is expressed as:

$$\frac{\frac{N_p}{N}\left[B_o + \left(\mathrm{R_p} - \mathrm{R_s}\right)\mathrm{B_g}\right]}{\left(B_o - B_{oi}\right) + \left(\mathrm{R_{si}} - \mathrm{R_s}\right)\mathrm{B_g}} - 1 \pm \boldsymbol{Tolerance}$$

In a scenario where this criteria is not achieved, a new increment oil recovery should be guessed and the procedure repeated until the criteria is satisfied. To help reduce the number of interaction process, it is advisable to make two initial guesses of the incremental oil recovery for the first pressure drop. Therefore, to determine the next guess value for the incremental oil recovery, secant method is employed which is given as:

$$x_{i+1} = x_i - \left[\frac{f(x_i)\{x_i - x_{i-1}\}}{f(x_i) - f(x_{i-1})}\right]$$

Where

$$x_{i+1} = New\ guess$$

$$x_i \quad \& \quad x_{i-1} = the\ initial\ guesses$$

$$f(x_i)\ \ \&\ \ f(x_{i-1}) = \textit{functions of the initial guesses}$$

To employ the secant method into the Schilthuis method, we rearrange the convergence criteria to the equation; the function of the initial guesses as a function of incremental oil recovery. Thus,

$$f(x_i) = \frac{\left(\frac{N_p}{N}\right)_{x_i}\left[B_o + \left(\mathrm{R_p} - \mathrm{R_s}\right)\mathrm{B_g}\right]}{(B_o - B_{oi}) + (\mathrm{R_{si}} - \mathrm{R_s})\mathrm{B_g}} - 1 = \textit{Tolerance}$$

$$f(x_{i-1}) = \frac{\left(\frac{N_p}{N}\right)_{x_{i-1}}\left[B_o + \left(\mathrm{R_p} - \mathrm{R_s}\right)\mathrm{B_g}\right]}{(B_o - B_{oi}) + (\mathrm{R_{si}} - \mathrm{R_s})\mathrm{B_g}} - 1 = \textit{Tolerance}$$

Where

$$x_i = \left(\frac{N_p}{N}\right)_{x_i} \quad \& \quad x_{i-1} = \left(\frac{N_p}{N}\right)_{x_{i-1}}$$

That is

$$\left(\frac{N_p}{N}\right)_{x_{i+1}} = \left(\frac{N_p}{N}\right)_{x_i} - \left[\frac{f(x_i)\left\{\left(\frac{N_p}{N}\right)_{x_i} - \left(\frac{N_p}{N}\right)_{x_{i-1}}\right\}}{f(x_i) - f(x_{i-1})}\right]$$

Schilthuis's Prediction Algorithm

Step 1: Assume value for the incremental oil recovery at the current pressure of interest given as:

$$\left(\frac{\Delta N_p}{N}\right)_1 \quad \& \quad \left(\frac{\Delta N_p}{N}\right)_2$$

Step 2: Determine the cumulative oil produced to the current pressure of interest by adding all the previous incremental oil produced.

$$\left(\frac{N_p}{N}\right)_1 = \sum\left(\frac{\Delta N_p}{N}\right)_1 \quad \& \quad \left(\frac{N_p}{N}\right)_2 = \sum\left(\frac{\Delta N_p}{N}\right)_2$$

Step 3: Determine the oil saturation from material balance equation given as:

$$S_o = (1 - S_{wi})\left[1 - \frac{N_p}{N}\right]\left(\frac{B_o}{B_{oi}}\right)$$

The total fluid saturation can be calculated as:

$$S_L = S_o + S_w$$

The gas saturation is calculated as:

$$S_g = 1 - S_o - S_{wi}$$

Step 4: Determine the relative permeability ratio

$$\frac{k_{rg}}{k_{ro}} \; or \; \frac{k_{rw}}{k_{ro}}$$

Step 5: Calculate the instantaneous gas-oil ratio at the current pressure of interest

$$R = \mathrm{R_{so}} + \frac{\mathrm{K_{rg}}}{\mathrm{K_{ro}}} \frac{\mu_o \mathrm{B_o}}{\mu_g \mathrm{B_g}}$$

Step 6: Calculate the average gas-oil ratio over the current pressure drop

$$R_{avg} = \frac{R_{i-1} + R_i}{2}$$

Step 7: Calculate the incremental gas production

$$\left(\frac{\Delta G_p}{N}\right)_1 = \left(\frac{\Delta N_p}{N}\right)_1 * R_{avg} \quad \& \quad \left(\frac{\Delta G_p}{N}\right)_2 = \left(\frac{\Delta N_p}{N}\right)_2 * R_{avg}$$

Step 8: Determine the cumulative gas produced to the current pressure of interest by adding all the previous incremental gas produced.

$$\left(\frac{G_p}{N}\right)_1 = \sum \left(\frac{\Delta G_p}{N}\right)_1 \quad \& \quad \left(\frac{G_p}{N}\right)_2 = \sum \left(\frac{\Delta G_p}{N}\right)_2$$

Step 9: Determine the cumulative produced gas-oil ratio given as:

$$(R_p)_1 = \frac{\left(\frac{G_p}{N}\right)_1}{\left(\frac{\Delta N_p}{N}\right)_1} \qquad \& \qquad (R_p)_2 = \frac{\left(\frac{G_p}{N}\right)_2}{\left(\frac{\Delta N_p}{N}\right)_2}$$

Step 10: Check for convergence

$$f(x_i) = \frac{\left(\frac{N_p}{N}\right)_{x_i}\left[B_o + \left((R_p)_1 - \mathrm{R_s}\right)\mathrm{B_g}\right]}{(B_o - B_{oi}) + (\mathrm{R_{si}} - \mathrm{R_s})\mathrm{B_g}} - 1 = Tolerance$$

$$f(x_{i-1}) = \frac{\left(\frac{N_p}{N}\right)_{x_{i-1}}\left[B_o + \left((R_p)_2 - \mathrm{R_s}\right)\mathrm{B_g}\right]}{(B_o - B_{oi}) + (\mathrm{R_{si}} - \mathrm{R_s})\mathrm{B_g}} - 1 = Tolerance$$

Step 11: If convergence is satisfied, then stop the iteration process, else calculate the new incremental oil recovery using the equation below and repeat the entire process.

$$\left(\frac{N_p}{N}\right)_{x_{i+1}} = \left(\frac{N_p}{N}\right)_{x_i} - \left[\frac{f(x_i)\left\{\left(\frac{N_p}{N}\right)_{x_i} - \left(\frac{N_p}{N}\right)_{x_{i-1}}\right\}}{f(x_i) - f(x_{i-1})}\right]$$

Example 11.4

Apply the data in Example 11.2 to calculate the cumulative oil and gas production at 2870 psia using Schilthuis's method with a convergence criteria of absolute relative error less than 2%.

Solution

Step 1: Assume value for the incremental oil recovery at the current pressure of interest given as:

$$\left(\frac{\Delta N_p}{N}\right)_1 = 0.05 \qquad \& \qquad \left(\frac{\Delta N_p}{N}\right)_2 = 0.055$$

Step 2: The cumulative oil produced to the current pressure of interest

$$\left(\frac{N_p}{N}\right)_1 = \sum\left(\frac{\Delta N_p}{N}\right)_1 = 0.05 \qquad \& \qquad \left(\frac{N_p}{N}\right)_2 = \sum\left(\frac{\Delta N_p}{N}\right)_2 = 0.055$$

Step 3: The oil saturation from material balance equation given as:

$$S_o = (1 - S_{wi})\left[1 - \frac{N_p}{N}\right]\left(\frac{B_o}{B_{oi}}\right)$$

$$S_{o1} = (1 - 0.23)[1 - 0.05]\left(\frac{1.3784}{1.3859}\right) = 0.7275$$

$$S_{o2} = (1 - 0.23)[1 - 0.055]\left(\frac{1.3784}{1.3859}\right) = 0.7237$$

The gas saturation is calculated as:

$$S_g = 1 - S_o - S_{wi}$$

$$S_{g1} = 1 - 0.7275 - 0.23 = 0.0425$$

$$S_{g2} = 1 - 0.7237 - 0.23 = 0.0463$$

Step 4: The relative permeability ratio

$$\left(\frac{k_{rg}}{k_{ro}}\right)_1 @S_{g1} = \left(\frac{k_{rg}}{k_{ro}}\right)_2 @S_{g2} = 0.01$$

Step 5: The instantaneous gas-oil ratio at the current pressure of interest

$$R = \mathrm{R_{so}} + \frac{\mathrm{K_{rg}}}{\mathrm{K_{ro}}}\frac{\mu_\mathrm{o}\mathrm{B_o}}{\mu_\mathrm{g}\mathrm{B_g}}$$

$$R_1 = R_2 = 1120 + 0.01\left(\frac{0.89239 * 1.3784}{0.0233 * 0.001618}\right) = 1446.284\ scf/STB$$

Step 6: The average gas-oil ratio over the current pressure drop

$$R_{avg1} = R_{avg2} = \frac{R_{i-1} + R_i}{2} = \frac{1180 + 1446.284}{2} = 1313.142\ scf/STB$$

Step 7: The incremental gas production

$$\left(\frac{\Delta G_p}{N}\right)_1 = \left(\frac{\Delta N_p}{N}\right)_1 * R_{avg} = 0.05 * 1313.142 = 65.6571$$

$$\left(\frac{\Delta G_p}{N}\right)_2 = \left(\frac{\Delta N_p}{N}\right)_2 * R_{avg} = 0.055 * 1313.142 = 72.2228$$

Step 8: The cumulative gas produced to the current pressure of interest

$$\left(\frac{G_p}{N}\right)_1 = \sum \left(\frac{\Delta G_p}{N}\right)_1 = 65.6571 \quad \& \qquad \left(\frac{G_p}{N}\right)_2 = \sum \left(\frac{\Delta G_p}{N}\right)_2 = 72.2228$$

Step 9: The cumulative produced gas-oil ratio given as:

$$(R_p)_1 = \frac{\left(\frac{G_p}{N}\right)_1}{\left(\frac{\Delta N_p}{N}\right)_1} = \frac{65.6571}{0.05} = 1313.142 \ \& \ (R_p)_2 = \frac{\left(\frac{G_p}{N}\right)_2}{\left(\frac{\Delta N_p}{N}\right)_2} = \frac{72.2228}{0.055} = 1313.142$$

Step 10: Check for convergence

$$Tolerance = \pm 2\%$$

$$f(x_i) = \frac{\left(\frac{N_p}{N}\right)_{x_i}\left[B_o + \left((R_p)_1 - \mathrm{R_s}\right)\mathrm{B_g}\right]}{(B_o - B_{oi}) + (\mathrm{R_{si}} - \mathrm{R_s})\mathrm{B_g}} - 1 = Tolerance$$

$$f(x_i) = \frac{0.05[1.3784 + (1313.142 - 1120)0.001618]}{(1.3784 - 1.3859) + (1180 - 1120)0.001618} - 1 = -0.0562$$

$$f(x_{i-1}) = \frac{\left(\frac{N_p}{N}\right)_{x_{i-1}}\left[B_o + \left((R_p)_2 - \mathrm{R_s}\right)\mathrm{B_g}\right]}{(B_o - B_{oi}) + (\mathrm{R_{si}} - \mathrm{R_s})\mathrm{B_g}} - 1 = Tolerance$$

$$f(x_{i-1}) = \frac{0.05[1.3784 + (1313.142 - 1120)0.001618]}{(1.3784 - 1.3859) + (1180 - 1120)0.001618} - 1 = 0.0382$$

Step 11: The convergence criteria is not satisfied because neither assumed values are within the chosen tolerance value of ±2%. Else the new incremental oil recovery is calculated using the equation below and the entire process is repeated.

$$\left(\frac{N_p}{N}\right)_{x_{i+1}} = \left(\frac{N_p}{N}\right)_{x_i} - \left[\frac{f(x_i)\left\{\left(\frac{N_p}{N}\right)_{x_i} - \left(\frac{N_p}{N}\right)_{x_{i-1}}\right\}}{f(x_i) - f(x_{i-1})}\right]$$

$$\left(\frac{N_p}{N}\right)_{x_{i+1}} = 0.055 - \left[\frac{0.0382\{0.055 - 0.05\}}{0.0382 - (-0.0562)}\right] = 0.0529$$

Step 1: New guess

$$\frac{\Delta N_p}{N} = 0.0529$$

Step 2: The cumulative oil produced to the current pressure

$$\frac{N_p}{N} = \sum \frac{\Delta N_p}{N} = 0.0529$$

Step 3: Oil saturation from material balance equation given as:

$$S_o = (1 - 0.23)[1 - 0.0529]\left(\frac{1.3784}{1.3859}\right) = 0.7253$$

The gas saturation is calculated as:

$$S_g = 1 - 0.7253 - 0.23 = 0.0447$$

Step 4: The relative permeability ratio

$$\frac{k_{rg}}{k_{ro}} @S_g = 0.01$$

Step 5: Instantaneous gas-oil ratio at the current pressure of interest

$$R = 1120 + 0.01\left(\frac{0.89239*1.3784}{0.0233*0.001618}\right) = 1446.284 \; scf/STB$$

Step 6: The average gas-oil ratio over the current pressure drop

$$R_{avg} = \frac{R_{i-1} + R_i}{2} = \frac{1180 + 1446.284}{2} = 1313.142\ scf/STB$$

Step 7: Calculate the incremental gas production

$$\frac{\Delta G_p}{N} = \frac{\Delta N_p}{N} * R_{avg} = 0.0529 * 1313.142 = 69.4652$$

Step 8: The cumulative gas produced to the current pressure of interest

$$\frac{G_p}{N} = \sum \frac{\Delta G_p}{N} = 69.4652$$

Step 9: The cumulative produced gas-oil ratio given as:

$$R_p = \frac{\frac{G_p}{N}}{\frac{\Delta N_p}{N}} = \frac{69.4652}{0.0529} = 1313.142$$

Step 10: Check for convergence

$$Tolerance = \pm 2\%$$

$$f\left(\frac{N_p}{N}\right) = \frac{\frac{N_p}{N}\left[B_o + \left(\left(R_p\right)_1 - \mathrm{R_s}\right)\mathrm{B_g}\right]}{\left(B_o - B_{oi}\right) + \left(\mathrm{R_{si}} - \mathrm{R_s}\right)\mathrm{B_g}} - 1 = Tolerance$$

$$f\left(\frac{N_p}{N}\right) = \frac{0.0529[1.3784 + (1313.142 - 1120)0.001618]}{(1.3784 - 1.3859) + (1180 - 1120)0.001618} - 1 = -0.00146 < \pm 2\%$$

Step 11: If convergence is satisfied, thus the iteration process is stopped.

Therefore, the incremental oil recovery at 2870 psia is 0.0529. Given the STOIIP = 9,655,344. It implies that the cumulative oil produced at 2870 psia is:

$$N_p = 0.0529 * N = 0.0529 * 9655344 = 510767.69\ \mathrm{STB}$$

The cumulative gas produced at 2870 psia:

$$\frac{G_p}{N} = 69.4652$$

$$G_p = 69.4652 * N = 69.4652 * 9655344 = 670710402\ \text{SCF} = 6.7071 \times 10^8\ SCF$$

Comparing Results with Tarner and Tracy Method

Parameter	Tarner method	Tracy method	Schilthuis method
Cum oil production (STB)	531043.9	511343.99	510767.69
Cum gas production (SCF)	$6.9734 * 10^8$	$6.7165 * 10^8$	$6.7071 * 10^8$

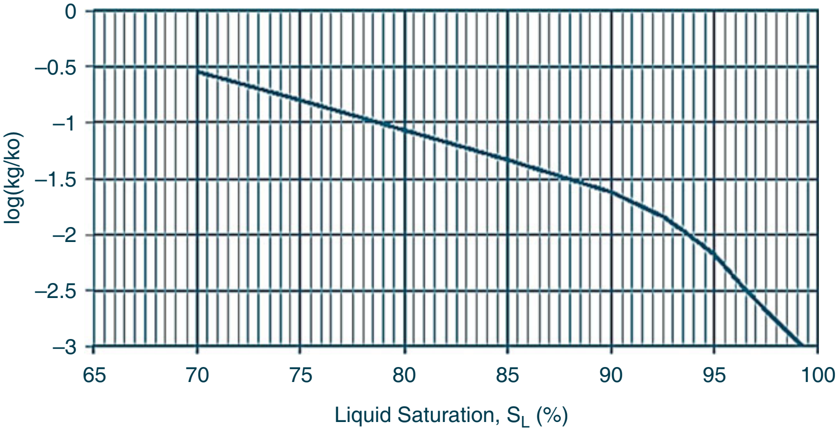
Exercises

Ex 11.1 Given the data below of a volumetric oil reservoir

Bubble point pressure, P_b	*1700 psia*
STOIIP, N	*77.89 MMstb*
Connate water saturation, S_{wc}	*25%*
Water influx, W_e	*0*
Water injection, W_{inj}	*0*
Reservoir temperature	200^0F

Fluid properties

P (psi)	B_o (bb//STB)	R_s (scf/STB)	B_g (cuft/SCF)	μ_o (cp)	μ_g (cp)
1700	1.265	962	0.00741	1.19	0.0294
1500	1.241	873	0.00842	1.22	0.0270
1300	1.214	784	0.00983	1.25	0.0251
1099	1.191	689	0.01179	1.3	0.0235
900	1.161	595	0.01471	1.35	0.0232
700	1.147	495	0.011931	1.5	0.0230
501	1.117	392	0.02779	1.8	0.0226
300	1.093	282	0.04828	2.28	0.0223
100	1.058	150	0.15272	3.22	0.0209



kg/ko Curve versus liquid saturation

Using the Tarner method and adopting the following criteria for the maximum allowable error:

$$|Error| = \left| \frac{(G_p)_{GOR,\ \ 2870} - (G_p)_{MBE,\ \ 2870}}{(G_p)_{GOR,\ \ 2870}} \right| \times 100\% \leq 1\%$$

Calculate the following:

- The oil cumulative production for (P = 1500, 1300, 1099)
- The instantaneous gas-oil production ratio
- The gas cumulative production

Ex 11.2 Repeat Ex 11.1 using Muskat method

Ex 11.3 Given the following data of Level GT oil reservoir in Ugbomro:

Bubble point pressure, P_b	*2650 psia*
STOIIP, N	*12.89 MMstb*
Connate water saturation, S_{wc}	*23%*
Water influx, W_e	*0*
Water injection, W_{inj}	*0*
Reservoir temperature	200^0F

Pressure (psia)	Bo (rb/STB	Bg (rb/STB)	Rs (scf/STB)	Uo (cp)	Uo (cp
2650	1.3814	0.000895	680	0.956	0.018
2180	1.3791	0.000947	574	1.236	0.0165
1825	1.3572	0.000988	528	1.492	0.0152

The relative permeability ratio is calculated as

$$\frac{k_{rg}}{k_{ro}} = 0.000128e^{17.257S_g}$$

Predict the performance (oil and gas production) of the reservoir at 2180 psia and 1825 psia

Ex 11.4 The following data are obtained from a depletion drive reservoir:

P.psia	2600	2400	**2100**	1800	1500	1200	1000	700	400
Rsi, SCF/STB	1340	1340	**1340**	1280	1150	985	860	662	465
B_O, bbl/STB	1.45	1.46	**1.480**	1.468	1.440	1.339	1.360	1.287	1.202
B_g, B/SCF $\times 10^{-3}$	…	…	**1.283**	1.518	1.853	2.365	2.885	4.250	7.680
μ_O/μ_g	…	…	**34.1**	38.3	42.4	48.8	53.6	62.5	79.0

Additional Data:

Initial reservoir pressure, P_i	*2925 psia*
Bubble point pressure, P_b	*2100 psia*
STOIIP, N	*100 MMstb*
Connate water saturation, S_{wc}	*15%*
Initial oil formation volume factor, β_{oi}	*1.429 bbl/stb*

Kg/Ko	26	12.5	3.3	0.8	0.19	0.022	0.01
So,%	30	40	50	60	70	80	84

Predict the reservoir performance, using Tarner method, effective from the time when the pressure is 2400 psia up to the time when the pressure becomes 400 psia. The productivity index was determined as 0.5 bbl/day/psi when the reservoir pressure was 2400 psia. Assume $P_{wf} = 200$ psia and $J_2 = J_1 (\beta_{O1}/ \beta_{O2})$ to plot P, Np, Gp, Rp & qo Vs. time.

Ex 11.5 Given the following data for a depletion drive reservoir, calculate the cumulative oil and gas production and the average GOR when the pressure reaches 700 psi using Tarner method.

Oil viscosity, μ_o	*1.987 cp*
Gas viscosity, μ_g	*0.01426 cp*
STOIIP, N	*90.45 MMstb*
Connate water saturation, S_{wc}	*20.5%*

P, psi	$B_{O,}$ bbl/STB	R_s SCF/STB	B_g, bbl/SCF	N_p, MMSTB	G_p, MMSCF	R_i. SCF/STB
1125	1.1236	230	--------	0.0	0.0	------
900	-------	--------	--------	6.76	--------	-------
800	1.0965	150	--------	9.41	4708	850
700	1.0925	132	0.003748	?	?	?

Kg/Ko	0.018	0.02	0.025	0.028	0.033	0.038	0.044	0.050
S_g,%	10	10.5	11	11.5	12	12.5	13	13.5

Ex 11.6 The following data are obtained from a gas cap drive reservoir:

P.psia	1710	1400	1200	1000	800	600	400	200
Rsi, SCF/STB	462	399	359	316	272	225	176	122
B_O, bbl/STB	1.205	1.18	1.164	1.148	1.131	1.115	1.097	1.075
B_g, bbl/SCF	0.00129	0.00164	0.00197	0.00245	0.00316	0.00436	0.0068	0.0143
μ_O/μ_g	. . .	113.5	122	137.5	163	197	239	284

Additional Data:

Initial reservoir pressure, P_i	*1710 psia*
Current point pressure, P	*1400 psia*
STOIIP, N	*40 MMstb*
Gas initially in place, G	*790*N*
Cumulative oil produced, N_p @1400 psia	*0.176*N stb*
Solution GOR, R_s	*8490 scf/stb*
Gas cap size, m	*4.0*
Connate water saturation, S_{wc}	*15%*
Reservoir, β_{oi}	*1.429 bbl/stb*

Kg/Ko	0.9	0.4	0.18	0.075	0.034	0.02	0.01	0.0028
S_L,%	70	75	80	85	90	92.5	95	97.5

(a) Predict the reservoir performance, using Tarner method, effective from the time when the pressure is 1400 psia up to the time when the pressure becomes 200 psia.

(b) Plot the predicted reservoir performance (Np Vs. P. & GOR)

Ex 11.7 Given the following data for a saturated depletion drive reservoir. Calculate the cumulative oil and gas production and the average GOR, when the pressure reaches 2100 psi using Schilthuis method. μ_O / μ_g = 41.645 at 2100 psi, Initial reservoir pressure = 2500 psi, and connate water saturation = 0.20.

P, psi	$B_{O,}$ bbl/STB	R_s SCF/STB	$B_g \times 10^{-3}$, bbl/SCF	N_p/N	G_p/N	R_i. SCF/STB
2500	1.498	721	1.048	0.0	0.0	721
2300	1.463	669	1.155	0.0168	11.67	669
2100	1.429	617	1.280	?	?	?

Kg/Ko	27.0	7.5	0.3	0.55	0.2	0.05	0.01	0.001
S_L,%	30	40	50	60	70	80	90	93

Ex 11.8 Given the following data for a depletion drive reservoir, calculate the cumulative oil and gas production and the average GOR when the pressure reaches 1200 psi using Schilthuis method. N = 10.025 MM STB, Sw = 0.22. μ_o / μ_g = 108.96 at 1200 psi. Pi = 3013 psi, Pb =2496 psi.

P, psi	$B_{O,}$ bbl/STB	R_s SCF/STB	B_g bbl/SCF	N_p/N	G_p/N	R_i. SCF/STB
3013	1.315	650	--------	0.0	0.0	650
2496	1.325	650	-------	-------	--------	650
1302	1.233	450	---------	1.179	1.123	2080
1200	1.224	431	0.001807	?	?	?

Kg/Ko	0.71	0.255	0.095	0.03	0.01
S_L,%	70	75	80	85	89

Ex 11.9 Given the following data for a saturated depletion drive reservoir. Calculate the cumulative oil and gas production and the average GOR, when the pressure reaches 1900 psi using Schilthuis method. μ_O / μ_g = 41. 645 at 1900 psi, Initial reservoir pressure = 2500 psi, and connate water saturation = 0.20.

P, psi	$B_{O,}$ bbl/STB	R_s SCF/STB	$B_g \times 10^{-3}$, bbl/SCF	N_p/N	G_p/N	R_i. SCF/STB
2500	1.498	721	1.048	0.0	0.0	721
2300	1.463	669	1.155	0.0168	11.67	669
2100	1.429	617	1.280	0.0427	28.87	658
1900	1.395	565	1.440	?	?	?

Kg/Ko	0.012	0.018	0.02	0.025	0.033	0.044	0.057	0.074
S_g,%	9	10	10.5	11	12	13	14	15

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