PRESIDENCY UNIVERSITY
BENGALURU

SET A

SCHOOL OF ENGINEERING
END TERM EXAMINATION - JAN 2024

Semester : Semester III - 2022
Course Code : MAT2004
Course Name : Discrete Mathematical Structures
Program : B.Tech.

Date : 10-JAN-2024
Time : 9:30AM - 12:30 PM
**Max Marks :** 100
Weightage : 50%

Instructions:
(i) Read all questions carefully and answer accordingly.
(ii) Question paper consists of 3 parts.
(iii) Scientific and non-programmable calculator are permitted.
(iv) Do not write any information on the question paper other than Roll Number.

PART A

ANSWER ALL THE QUESTIONS   **5 X 4M = 20M**

1. Obtain the Disjunctive normal form of $\neg(p \to (q \wedge r))$.

    (CO1) [Knowledge]

2. If $f : R \to R$ is defined by $f(x) = ax + b,$ for $a, b \in R$ and $a \neq 0$. Show that f is invertible and find the inverse of f.

    (CO2) [Knowledge]

3. Write the relation of the given poset and draw the directed graph of ($\{1, 2, 3, 4\}$, <).

    (CO2) [Knowledge]

4. Find the minimal and maximal elements of the given figure and greatest lower bound and the least upper bound of $\{4,5\}$
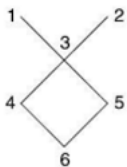
    (CO3) [Knowledge]

5. How many ways are there to place 10 indistinguishable balls into 8 distinguishable bins?

    (CO4) [Knowledge]

PART B

ANSWER ALL THE QUESTIONS   **5 X 10M = 50M**

6. Verify that $R \to S$ can be derived from the given premises $P \to (Q \to S), \neg R \lor P$ and Q.

   (CO1) [Comprehension]

7. If X = {1, 2, 3, ..., 7} and R ={(x, y) | x − y is divisible by 3}. Show that R is an equivalence relation.

   (CO2) [Comprehension]

8. Prove that Cancellation laws hold in Boolean Algebra.
   i . e for any three elements a, b, c in a Boolean algebra such that, a ∧ b = a ∧ c, a ∨ b = a ∨ c ⇒ $b = c$.

   (CO3) [Comprehension]

9. Determine whether the given posets $(\{1, 2, 3, 4, 5\}, |)$ and $(\{1, 2, 4, 8, 16\}, |)$ are lattices. ('|' represents divisibility relation).

   (CO3) [Comprehension]

10. (i) How many ways are there to distribute hands of 5 cards to each of 6 players from the standard deck of 52 cards?
    (ii) How many ways are there to pack five copies of the same book into four identical boxes, where a box can contain as many as five books?

    (CO4) [Comprehension]

PART C

ANSWER ALL THE QUESTIONS   **2 X 15M = 30M**

11. a) Obtain the Principal disjunctive normal form of $p \land \neg(q \land r) \lor (p \to q)$.
    b) Prove that $\forall x(P(x) \to Q(x)), \forall x(R(x) \to \neg Q(x)) \implies \forall x(R(x) \to \neg P(x))$.

    (CO1) [Application]

12. Prove that $(D_{30}, |)$ is a distributive lattice where D_{30} is the set of all positive divisors of 30.

    (CO3) [Application]