I D NO.

PRESIDENCY UNIVERSITY, BENGALURU

SCHOOL OF ENGINEERING

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

Weightage: 40% | Max Marks: 80 | Max Time: 2 hrs. | 09 May, Wednesday, 2018

END TERM FINAL EXAMINATION MAY 2018 **SET B**

Even Semester 2017-18 | Course: **MEC 218 Advanced Mechanics of Solids** | VI Sem Mechanical

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Instructions:

(i) Answer all the questions.
(ii) Write neatly and legibly.

Part A

(10 M + 8 M + 8 M = 26 Marks)

1. Discuss the meaning of "solving a problem *in* polar coordinates" through an example. Your answer must state the equations and boundary conditions used, the quantities being solved for and give example solutions for these quantities.
2. Are the statements in (a) thru (d) true or false? Argue your claim in each case.

(a) The boundary conditions for a problem are derived from its loads and geometry.

(b) A particular solution satisfies only the partial differential equations of equilibrium and does not satisfy the boundary conditions for the problem in consideration.

(c) The biharmonic equation varies from one problem to another.

(d) The Airy's stress function ϕ is a solution to the boundary conditions.

3. Determine the stresses and sketch the stress distributions at Sections $B-B'$ and $C-C'$. The cross sectional areas at $B-B'$ and $C-C'$ are A and A', respectively for the "thin" flat plates in Figure 1. Assume K to be the stress concentration factor.

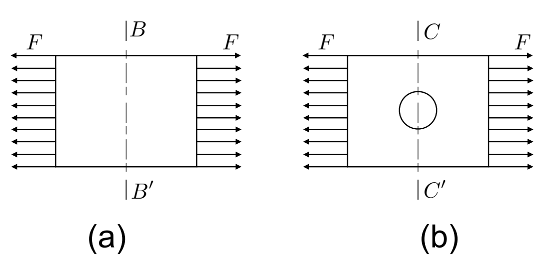

Figure 1: "Thin" Flat Plates in Tension

Part B

(12 M + 12 M = 24 Marks)

4. The following questions pertain to Point A in Figure 2.

(a) Represent A using appropriate unit vectors in Cartesian coordinates and in polar coordinates.

(b) Suppose the stresses acting at A are $\sigma_x = -90$ MPa, $\sigma_y = 95$ MPa and $\tau_{xy} = 45$ MPa. Sketch these stresses on a suitable differential element around A.

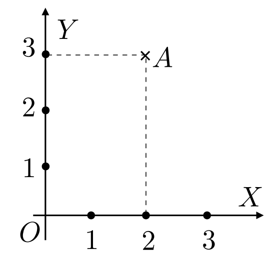

Figure 2: A Point in 2-D Space

(d) Suppose the stresses at A are $\sigma_r = -90$ MPa, $\sigma_\theta = 95$ MPa and $\tau_{r\theta} = 45$ MPa. Sketch these stresses on a suitable differential element around A.

5. This following questions pertain to the "thin" plate of unit thickness with a hole subjected to a tensile stress as shown in Figure 3.

(a) Justify the use of polar coordinates in solving this problem.

(b) Determine the boundary conditions at $r = a$ and $r = b$.

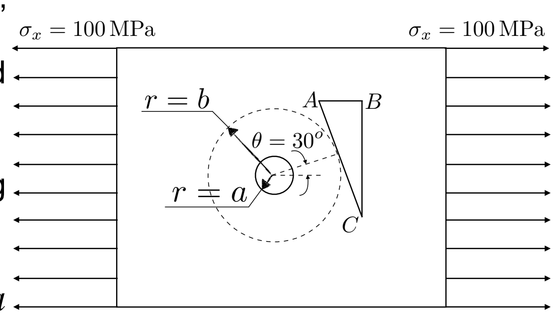

Figure 3: A "Thin" Plate with a Hole

(c) Determine the radial stresses $\sigma_r(a, 30^o)$ and $\sigma_r(b, 30^o)$?

Part C

(15 M + 15 M = 30 Marks)

6. The question will test your knowledge of stresses in curved beams. Consider the curved beam shown in Figure 4 acted on by the moment M.

(a) State the boundary conditions that help us obtain the particular solution for the tangential stress σ_θ from the general solution.

(b) Suppose that the tangential stress experienced by the fibre gh is $\sigma_\theta = E\dfrac{\epsilon_c R + y\lambda}{R + y}$ where E is the Young's modulus of the material of the beam, ϵ_c is the strain experienced by the centroidal axis due to the action of the moment M, λ is the angular strain

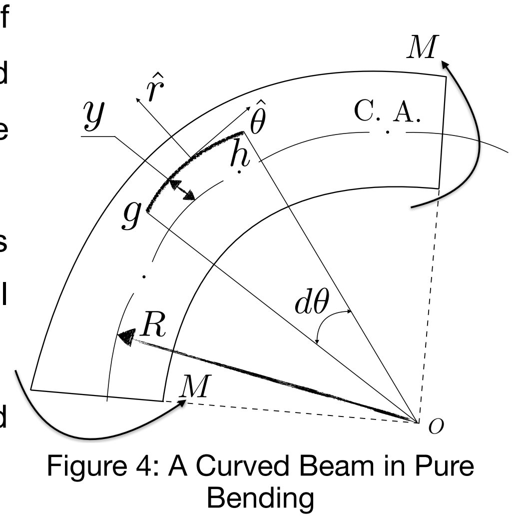

Figure 4: A Curved Beam in Pure Bending

experienced by the element gh and R and y are as shown in the figure. Use your boundary conditions from (a), $-mA = \int \frac{y}{R+y} dA$, $\int dA = A$, $\int y dA = 0$, and $\int \frac{y^2}{R+y} dA = \int (y - \frac{Ry}{R+y}) dA$ to show that $\sigma_\theta = \frac{M}{AR}(1 + \frac{y}{m(R+y)})$.

7. This question tests your knowledge of the Flamant problem. Figure 5 shows a "linear elastic wedge" of "unit thickness" acted on by the concentrated loads F_1 and F_2.

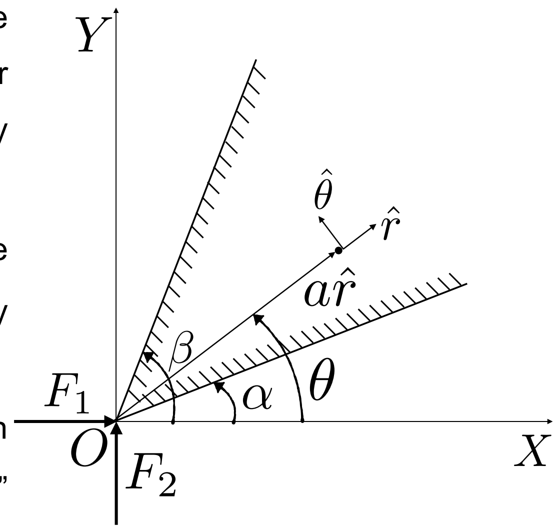

Figure 5: A Linear Elastic Wedge

(a) Use Items 7 and 9 from Table 1 to derive expressions for the shear stress $\tau_{r\theta}$ at any point in the wedge.

(b) Write down the force balance equation $\Sigma F_Y = 0$ for the part of the wedge "broken" at a radius a from the origin O.

(c) Sketch the "half plane" by modifying the wedge by taking $\alpha = -\pi$ and $\beta = 0$.

(d) Suppose we make two "legal" modifications. The first modifies the general solution $\sigma_r(r,\theta)$ to $\sigma_r(a,\theta) = c_1 \frac{2\cos\theta}{a} + c_3 \frac{2\sin\theta}{a}$. The second modifies the equilibrium condition along the X-axis for the "half plane" to $F_1 + \int_{-\pi}^{0} \sigma_r(a,\theta)\cos\theta a d\theta = 0$.

Use these two relations to generate one equation in terms of c_1 and c_3.□

Table 1 is a list of some relations and formulas that may be of use to you.

Item No.	Description of the Item	Relations or Formulas
1	The partial differential equations (PDE) of equilibrium in Cartesian coordinates	$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$
2	The compatibility equation in Cartesian coordinates	$(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2})(\sigma_x + \sigma_y) = 0$
3	Relations between stresses and the Airy's stress function $\phi(x,y)$	$\sigma_x = \frac{\partial^2 \phi}{\partial y^2}, \sigma_y = \frac{\partial^2 \phi}{\partial x^2}, \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$

Item No.	Description of the Item	Relations or Formulas
4	The biharmonic equation in Cartesian coordinates	$\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}=0$
5	The PDEs of equilibrium in polar coordinates	$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{\sigma_r-\sigma_\theta}{r}=0,$ $\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{r\theta}}{\partial r}+\frac{2\tau_{r\theta}}{r}=0$
6	The compatibility equation in polar coordinates	—
7	Relations between stresses and the Airy's stress function $\phi(r,\theta)$ in polar coordinates	$\sigma_r=\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2},\sigma_\theta=\frac{\partial^2\phi}{\partial r^2},$ $\tau_{r\theta}=\frac{1}{r^2}\frac{\partial\phi}{\partial\theta}-\frac{1}{r}\frac{\partial^2\phi}{\partial r\partial\theta}$
8	The biharmonic equation in polar coordinates	$(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2})$ $\cdot(\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2})=0$
9	The Airy's stress function suited to solving the Flamant problem	$\phi=C_1 r\theta\sin\theta+C_2 r\ln r\cos\theta$ $+C_3 r\theta\cos\theta+C_4 r\ln r\sin\theta$
10	Trigonometric identities that may be useful	$\cos^2\theta=\frac{1+\cos 2\theta}{2},$ $\sin^2\theta=\frac{1-\cos 2\theta}{2},$ $\sin\theta\cos\theta=\frac{\sin 2\theta}{2}$
11	Some definite integrals that may be useful	$\int_\alpha^\beta\cos^2\theta d\theta=\frac{\beta-\alpha}{2}+\frac{1}{4}(\sin 2\beta-\sin 2\alpha),$ $\int_\alpha^\beta\sin\theta\cos\theta d\theta=\frac{1}{2}(\cos 2\alpha-\cos 2\beta)$

Table 1: Some Relations and Formulas That May Be Useful

The End

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SCHOOL OF ENGINEERING

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Weightage: 40% Max Marks: 80 Max Time: 2 hrs. 09 May, Wednesday, 2018

SOLUTIONS TO END TERM FINAL EXAMINATION MAY 2018 **SET B**

Even Semester 2017-18 Course: **MEC 218 Advanced Mechanics of Solids** VI Sem Mechanical

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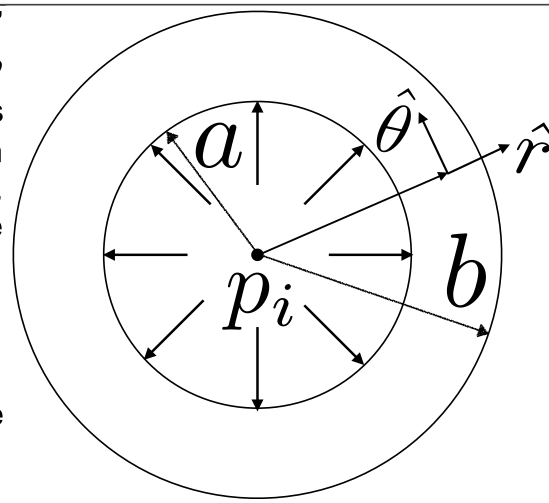
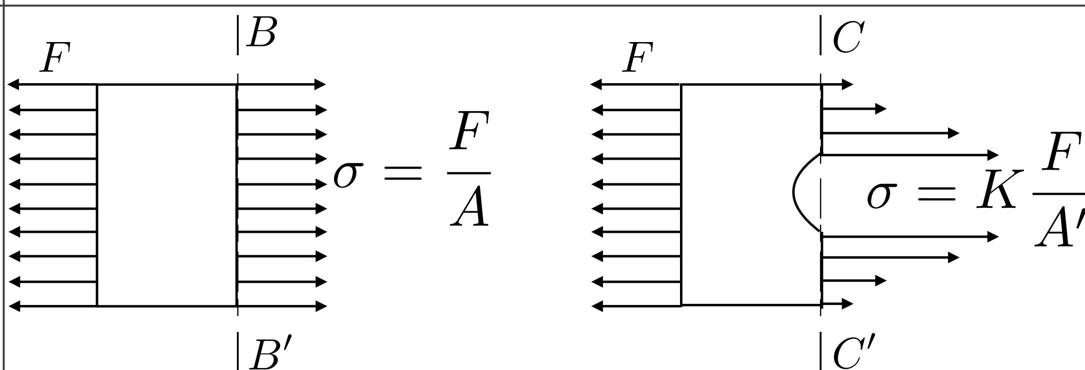
Part A

(10 M + 8 M + 8 M = 26 Marks)

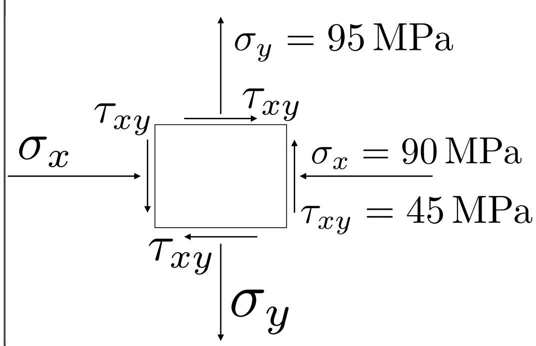
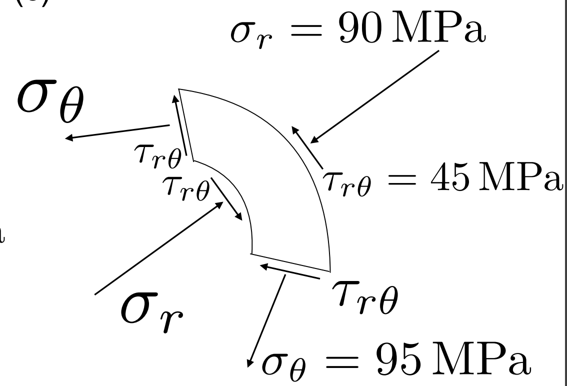
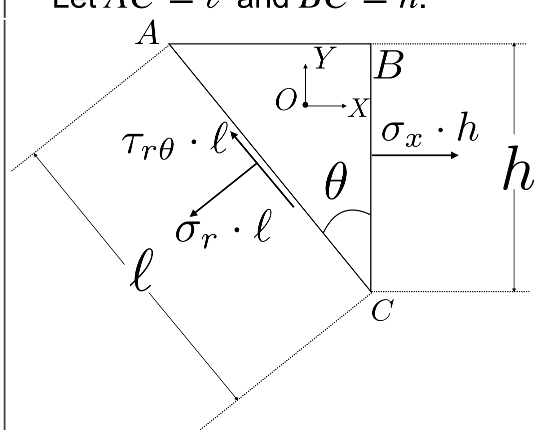
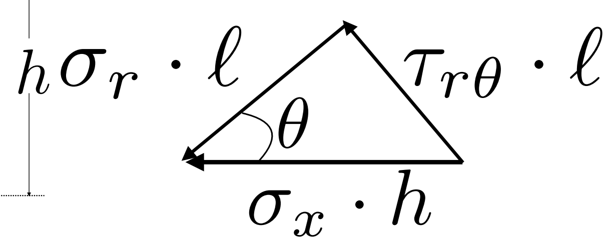
Solution to Q. 1	Figure 1 shows a “plane stress pipe” with inner radius a and outer radius b subjected to internal pressure p_i. This is an example of a problem that can be solved by using polar coordinates. To solve the problem we use the partial differential equations $\dfrac{\partial \sigma_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial \tau_{r\theta}}{\partial \theta} + \dfrac{\sigma_r - \sigma_\theta}{r} = 0,$ $\dfrac{1}{r}\dfrac{\partial \sigma_\theta}{\partial \theta} + \dfrac{\partial \tau_{r\theta}}{\partial r} + \dfrac{2\tau_{r\theta}}{r} = 0,$ and the boundary conditions $\sigma_r(a,\theta) = p_i$ and $\sigma_r(b,\theta) = 0$. The quantities being solved for are $\sigma_r(r,\theta)$, $\sigma_\theta(r,\theta)$ and $\tau_{r\theta}(r,\theta)$. Any differentiable functions of r and θ for these quantities that satisfy the PDEs and the boundary conditions are a solution to the problem selected as an example.  Figure 1: A “Plane Stress Pipe” Subjected to Internal Pressure p_i
Scheme of Marking	<u>NOTE</u>: Students could write correct examples other than the one suggested. ✦ Selecting a suitable example - 3 M ✦ The PDEs of equilibrium - 2 M ✦ Boundary conditions matching the example chosen - 2 M ✦ Identifying the unknown quantities - 2 M ✦ Example solutions - 1 M
L. O. No.	(v)
Max Time	15 minutes

Solution to Q. 2	(a) <u>True</u>! One acceptable argument is to present a boundary condition, like the ones in the solution to Q. 1. (b) <u>False</u>! A general solution satisfies only the PDEs of equilibrium. The particular solution satisfies both, the PDEs and the boundary conditions. (c) <u>True</u>! The biharmonic equation changes from one coordinate system to another. Since coordinate systems may be different for different problems the biharmonic equation does change from problem to problem. However, since all problems can be solved with one coordinate system it is okay to say that the statement is <u>false</u> too. (d) <u>False</u>! The Airy's stress function ϕ is a solution to the biharmonic equation.
Scheme of Marking	✦ The right claim - 1 M ✦ Arguing the claim correctly - 1 M
L. O. No.	(v)
Max Time	10 minutes
Solution to Q. 3	 Figure 2: Stress Distributions and Stresses in "Thin" Flat Plates With and Without Discontinuities in Their Cross-Sectional Areas
Scheme of Marking	✦ Each stress distribution drawn right - 2 M ✦ Each stress expressed correctly - 2 M
L. O. No.	(viii)
Max Time	5 minutes

Part B

(12 M + 12 M = 24 Marks)

Solution to Q. 4	(a) In Cartesian coordinates we write the point A as $2\hat{i} + 3\hat{j}$. The radial distance of the point from the origin is $\sqrt{(2)^2 + (3)^2} = \sqrt{13}$. The vector starting at O and finishing at A makes the angle $\tan^{-1} = \frac{3}{2}$ $= 56.3^o$ with the $X-$axis. This vector is represented as $\sqrt{13}\hat{r}$.

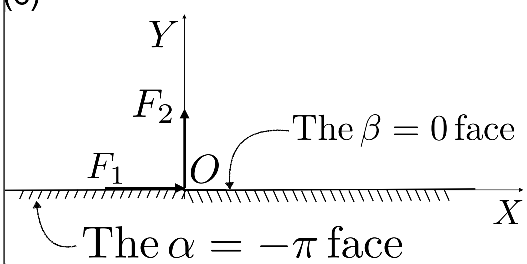
Solution to Q. 4 Continues	(b)  Figure 3: Stresses Acting at A in Cartesian Coordinates (c)  Figure 4: Stresses at A in Polar Coordinates
Scheme of Marking	✦ Representation of the point in Cartesian coordinates - 2 M ✦ Representation of the point in polar coordinates - 2 M ✦ Shape of each stress element - 1 M ✦ Each stress shown right - 1 M
L. O. No.	(v)
Max Time	10 minutes
Solution to Q. 5	(a) The use of polar coordinates to solve this problem is justified as follows. We know from the concept of stress concentration around discontinuities in cross sectional areas that the stress σ_x will be maximum at $r = a$ and will gradually settle to a state of near uniformity as r increases. This means that there exists a radius b beyond which we can assume that the stress σ_x is constant and equals the applied stress. Our objective is now reduced to finding the stresses in the annulus with inner radius a and outer radius b. It is now natural to use polar coordinates to solve this reduced problem of stress determination in this annulus. (b) The boundary condition at $r = a$ is $\sigma_r(a, \theta) = 0$. Let $AC = \ell$ and $BC = h$.  Figure 5: The Geometry and Stresses on the Differential Element $ABCA$ at $r = b$  Figure 6: Resolution of the Applied Force on $ABCA$ Along the Radial and Tangential Directions

Solution to Q. 5 Continues	Then we see from Figure 5 that $\cos\theta = \frac{h}{l}$. We observe the following from Figure 6: $(\sigma_x \cdot h)\cos\theta = \sigma_r \cdot \ell \implies \sigma_r = \sigma_x \cos^2\theta$, and $(\sigma_x \cdot h)$ $\sin\theta = -\tau_{r\theta} \cdot \ell \implies \tau_{r\theta} = -\sigma_x \sin\theta\cos\theta$. Therefore, the boundary conditions at $r = b$ are $\sigma_r(b,\theta) = \sigma_x \cos^2\theta$ and $\tau_{r\theta}(b,\theta) = -\sigma_x \sin\theta \cdot \cos\theta$. (c) From the boundary condition $\sigma_r(a,\theta) = 0$ we see that $\sigma_r(a,30^o) = 0$. From the boundary condition $\sigma_r(b,\theta) = \sigma_x \cos^2\theta$ we can see that $\sigma_r(b,30^o) = 100\cos 30^o = 100 \cdot \frac{\sqrt{3}}{2} = 50\sqrt{3}$.
Scheme of Marking	✦ Justifying the use of polar coordinates to solve this problem - 4 M ✦ The boundary condition at $r = a$ - 2 M ✦ The boundary conditions at $r = b$ - 4 M ✦ Determination of $\sigma_r(a,30^o)$ and $\sigma_r(b,30^o)$ - 2 M
L. O. No.	(viii)
Max Time	20 minutes

Part C

(15 M + 15 M = 30 Marks)

Solution to Q. 6	(a) Two boundary conditions that will help us determine σ_θ are $\int \sigma_\theta dA = 0$ and $\int \sigma_\theta y dA = M$. (b) The tangential stress experienced by the fibre gh is $\sigma_\theta = E\frac{\epsilon_c R + y\lambda}{R + y}$. This can be rewritten as $\sigma_\theta = E\frac{\epsilon_c R + y\lambda + \epsilon_c y - \epsilon_c y}{R + y} = E[\epsilon_c + (\lambda - \epsilon_c)\frac{y}{R + y}]$. Substituting this into the two boundary conditions in (a) and using the integral relations mentioned in the question gives us $\epsilon_c = \frac{M}{EAR}$ and $\lambda - \epsilon_c = \frac{M}{mEAR}$. By substituting for ϵ_c and $(\lambda - \epsilon_c)$ in the expression for σ_θ we get $\sigma_\theta = \frac{M}{AR}(1 + \frac{y}{m(R + y)})$.
Scheme of Marking	✦ The two boundary conditions - 6 M ✦ Rewriting σ_θ and substituting it into the two boundary conditions - 5 M ✦ Expressions for ϵ_c and $(\lambda - \epsilon_c)$ - 2 M ✦ Final expression for σ_θ - 2 M

L. O. No.	(viii)
Max Time	25 minutes
Solution to Q. 7	(a) From Items 7 and 9 in Table 1 we can use the Airy's stress function $\phi = C_1 r\theta \sin\theta + C_2 r \ln r \cos\theta + C_3 r\theta \cos\theta + C_4 r \ln r \sin\theta$ and the relations $\tau_{r\theta} = \frac{1}{r^2}\frac{\partial\phi}{\partial\theta} - \frac{1}{r}\frac{\partial^2\phi}{\partial r\partial\theta}$ to find the shear stress $\tau_{r\theta}$. We get $\tau_{r\theta} = c_2(\frac{\sin\theta}{r}) - c_4(\frac{\cos\theta}{r})$. (b) $\Sigma F_Y = 0 : F_2 + \int_\alpha^\beta [\sigma_r(a,\theta)\sin\theta + \tau_{r\theta}(a,\theta)\cos\theta] a\, d\theta = 0.$ (c)  Figure 7: The "Half Plane" (d) From the two given relations we get $F_1 + 2\int_{-\pi}^{0} c_1 \cos^2\theta d\theta + 2\int_{-\pi}^{0} c_3 \sin\theta\cos\theta d\theta = 0.$ From Item 11 we get $F_1 + c_1\pi = 0.$
Scheme of Marking	✦ The expression for shear stress - 3 M ✦ Each term in the left hand side of the equation - 2 M ✦ Sketch of the "half plane" - 3 M ✦ Substituting for $\sigma_r(a,\theta)$ in the given equilibrium condition - 1 M ✦ Using Item 11 - 1 M ✦ Obtaining $F_1 + c_1\pi = 0$ - 1 M
L. O. No.	(ix)
Max Time	25 minutes

The End

ID NO:	

PRESIDENCY UNIVERSITY, BENGALURU

SCHOOL OF ENGINEERING

Weightage: 20% Max Marks: 40 Max Time: 1 Hour Tuesday, March 27, 2018

TEST – 2 **SET A**

Even Semester 2017-18 Course: **MEC 218 Advanced Mechanics of Solids** VI Sem Mechanical

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Instructions

(i) Answer all the questions!
(ii) It is your duty to make your work understood to the evaluator of your test. Write neatly!
(iii) Freehand drawings are encouraged to save you time.

Part A

(6 M + 2 M + 2 M = 10 Marks)

1. Sketch only the shapes of the 2-D differential stress elements in Cartesian and polar coordinates. Next, show the stresses acting *only* on the element in polar coordinates.
2. Identify suitable stress elements for the analysis of the following engineering structures.
   (a) Beams (b) Cylinders (c) Rotating disks (d) A Plate with a Hole
3. Choose two suitable unit vectors to help locate a point in 2-D space in the polar coordinate system. Can these unit vectors move or they fixed? Explain your answer.

Part B

(6 M + 5 M + 5 M = 16 Marks)

4. Check if the stresses as defined by the Airy's stress function $\phi(x, y)$ satisfy $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$ and $\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0.$
5. Write down the partial differential equations of equilibrium and the compatibility equation for a 2-D differential stress element in Cartesian coordinates. Explain briefly the concept of solution to these equations.
6. Write down the biharmonic equation in Cartesian coordinates. Explain how the biharmonic equation in polar coordinates can be derived from the biharmonic equation in Cartesian coordinates. Do not perform the actual derivation, just explain the steps involved in the process.

Part C

(8 M + 6 M = 14 Marks)

7. Write down the boundary conditions for the beam and thick cylinder with loads as described in (a) and (b). Choose numbers or suitable letters of the English and Greek alphabet to denote any geometric parameters, loads and stresses.
   (a) A beam in pure bending,
   (b) A "plane stress pipe" subjected only to internal pressure and no external pressure.
8. Check to see if the Airy's stress function $\phi(x, y) = a_1 x + b_1 y$, where a_1 and b_1 are constants, satisfies the biharmonic equation in Cartesian coordinates and the boundary condition $\int_{-h}^{h} 4\tau_{xy} dy = 2000.$

The End

ID NO:

PRESIDENCY UNIVERSITY, BENGALURU
SCHOOL OF ENGINEERING

Weightage: 20% Max Marks: 40 Max Time: 1 Hour Thursday, Feb 22, 2018

TEST – 1

Even Semester 2017-18 Course: **MEC 218 Advanced Mechanics of Solids** VI Sem Mechanical

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Instructions

(i) Answer all the questions!
(ii) You have sixty minutes to answer the test. Plan your test!
(iii) It is your duty to make your work understood to the evaluator of your test. Write neatly!
(iv) Freehand drawings are encouraged to save you time.
(v) You will need to use non-programmable calculators to answer this test.

Part A

(2 Q x 5 M = 10 Marks)

1. Draw a neat sketch of a 2-D differential stress element subjected to the state of plane stress. Is the element on which the stresses act a 3-D element or a 2-D element?
2. Explain (in a sentence or two!) principal stresses for the plane stress situation for a 2-D differential stress element with a sketch. What is the magnitude and direction of the shear stress on each of the principal planes?

Part B

(2 Q x 8 M = 16 Marks)

3. The following questions pertain to the uniaxial and the generalised Hooke's law.
   (i) State the mathematical relations for the uniaxial and the generalised Hooke's law. Define all terms in both the mathematical relations. Do not write any explanations!
   (ii) Determine the change in height of the element of a machine member subjected to the forces shown in Figure 1. Assume the Young's modulus $E = 200$ GPa and the Poisson's ratio $\nu = 0.3$.

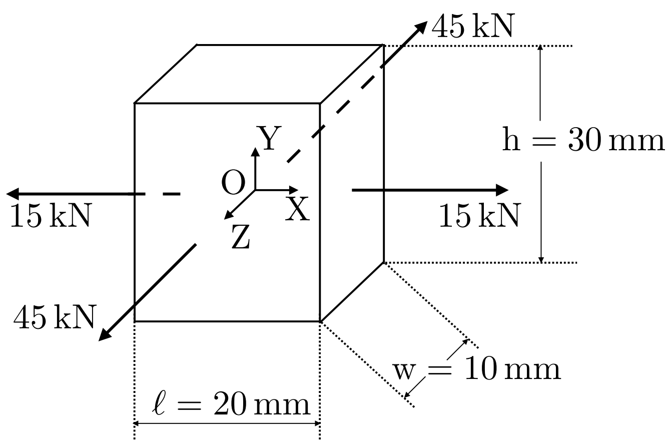

Figure 1: An Element in a Machine Member

4. The following questions test your knowledge on concepts related to the hydrostatic state of stress and strain energy.
   (i) Sketch the hydrostatic state of stress using a 3-D differential stress element.
   (ii) We have seen in class that the strain energy stored in an elastic body per unit volume $\mathscr{U}$ is given by $\mathscr{U} = \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$ where σ_1, σ_2 and σ_3 are the principal stresses. Starting from this relation derive an expression for the hydrostatic strain energy in an elastic body per unit volume in terms of the principal stresses σ_1, σ_2 and σ_3.

Part C

(2 Q x 7 M = 14 Marks)

5. We now test your knowledge on the theories of failure in the following questions.
   (i) Suppose a machine element is made of cast iron. Name a failure theory that is suitable to check if the machine element fails or is safe to the subjected stresses.
   (ii) Figure 2 shows an element of a machine member made of steel subjected to stresses. Using the distortion energy theory of failure for ductile materials check if the element fails or is safe when subjected to the stresses shown in the figure. Assume the yield strength S_y of the material of the machine member is 245 MPa.

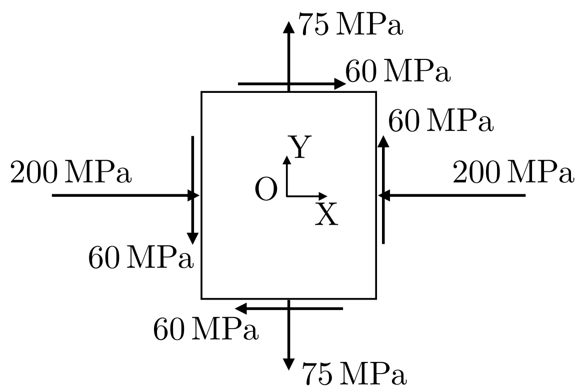

Figure 2: An Element in a Machine Member

6. The partial differential equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + F_x = 0 \text{ and}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + F_y = 0$$

are the equations of equilibrium for a 2-D differential stress element. Starting with the right differential stress element show the appropriate stresses and body forces acting on it and *derive any one* of the two partial differential equations.

The End
