Presidency University, Bengaluru

School of Engineering

I Semester
2015-2016

COMPREHENSIVE EXAMINATION

Course: **MATH A 101 Engineering Mathematics I**
(Closed Book)

Max Marks:60 Max Time: 2 hours Weightage: 30 % 11th Jan' 2016 **SET A**

Instructions to Candidates
1. Write legibly.
2. Attempt all questions.
3. Assume any missing data suitably and clearly state and justify the same

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PART A (10 X 3 = 30 Marks)

1. Find $\frac{\partial^2 u}{\partial x \partial y}$ if $u(x,y)=\log\left(\frac{x^2+y^2}{xy}\right)$

2. If $u=\log[\frac{x^2+y^2}{x+y}]$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$.

3. Expand $xy^2+2x-3y$ in powers of $(x+2)$ and $(y-1)$ up to second degree terms.

4. If u + v+ w = x; v+w = xy; w = xyz then find $\frac{\partial(u,v,w)}{(x,y,z)}$.

5. Using Cayley-Hamilton theorem find the inverse of $A=\begin{bmatrix}7 & 3\\ 2 & 6\end{bmatrix}$.

6. Find the rank of the matrix $\begin{bmatrix}1 & -2 & 0 & 1\\ 2 & -1 & 1 & 0\\ 3 & -3 & 1 & 1\\ -1 & -1 & -1 & 1\end{bmatrix}$.

7. Evaluate $\int_1^2\int_1^x xy^2\,dydx$.

8. By changing into polar co-ordinates and evaluate $\iint (x^2+y^2)\,dydx$ over the circle $x^2+y^2=a^2$.

9. Find the angle between the normals to the surface $xy^3z^2 = 4$ at the points $(-1, -1, 2)$ and $(4, 1, -1)$.

10. Prove that the vector $\vec{F}$= $(Y^2 - Z^2 + 3YZ - 2X)\hat{\imath}$+ $(3xz+2xy)\hat{\jmath}$+$(3yx-2xz+2z)\hat{k}$ is both solenoidal and irrational.

PART B (6 x 5 = 30 Marks)

11. If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.

12. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

13. Using Gauss-Jordan method solve the system of linear equations

$2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16.$

14. Change the order of integration in $\int_{x=0}^{2} \int_{y=x^2/4}^{3-x} xy\, dy\, dx$ and hence evaluate it.

15. Find the area enclosed between the parabola $y = x^2$ and the straight line $y = x$.

16. State Green's theorem and evaluate $\int_c (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

Presidency University, Bengaluru

School of Engineering

I Semester
2015-2016

COMPREHENSIVE EXAMINATION

Course: **MATH A 101 Engineering Mathematics I**
(Closed Book)

Max Marks:60 Max Time: 2 hours Weightage: 30 % 11th Jan' 2016 **SET B**

Instructions to Candidates
1. Write legibly.
2. Attempt all questions.
3. Assume any missing data suitably and clearly state and justify the same

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PART A (10 X 3 = 30 Marks)

1. Find the nth derivative of $e^x sin^2 x$.

2. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

3. Expand $\frac{x}{e^x - 1}$ up to x^4.

4. If $u = 2xy, v = x^2 - y^2$ and $x = r\cos\theta, y = r\sin\theta$, evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$.

5. If $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, find A^{-1} using Cayley-Hamilton Theorem.

6. Let V denote the vector space of all real-valued functions that are integrable over the interval $[0, 1]$. Consider the transformation $T : V \to R$, where $T(f) = \int_0^1 f(x)\,dx$ for every $f \in V$. Show that T is a linear transformation.

7. Evaluate $\int_0^a \int_0^b \int_0^c \left(x^2 + y^2 + z^2\right) dx\, dy\, dz$.

8. Evaluate $\iint xy\, dx\, dy$ over the first quadrant of the circle $x^2 + y^2 = a^2$.

9. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then evaluate $div\left(\frac{\vec{r}}{r^3}\right)$.

10. Given $\phi = 2x^3 y^2 z^4$, find $div\, grad\, \phi$.

PART B (6 X 5 = 30 Marks)

11. If $y = \sin\left(m \sin^{-1} x\right)$, prove that $\left(1 - x^2\right) y_2 - x y_1 + m^2 y = 0$ and hence by Leibnitz's theorem deduce that $\left(1 - x^2\right) y_{n+2} - (2n+1) x y_{n+1} + \left(m^2 - n^2\right) y_n = 0$.

12. The temperature $u(x, y, z)$ at any point in space is $u = 400\, xyz^2$. Using Lagrange's multiplier method find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = 1$.

13. Solve the system of linear equations $10x + y + z = 12,\ 2x + 10y + z = 13,\ x + y + 5z = 7$ by Gauss-Jordan method.

14. Change the order of integration in $\int_{x=0}^{a} \int_{y=x^2/a}^{2a-x} xy\, dy\, dx$ and then evaluate it.

15. Find the area enclosed by the curves $y^2 = 4ax$ and $x^2 = 4ay$.

16. State Green's theorem and hence evaluate $\int_C x^2 dx + xy\, dy$ where C is the curve in the xy plane given by $x = 0,\ y = 0,\ x = a,\ y = a\ (a > 0)$.

Presidency University, Bengaluru

School of Engineering

I Semester 2015-2016	COMPREHENSIVE EXAMINATION	Course: **MATH A 101 Engineering Mathematics I** (Open Book)
Max Marks:20	Max Time: 1 hour	Weightage: 10 % 11th Jan' 2016 **SET A**

Instructions to Candidates

1. Write legibly.
2. Attempt all questions.
3. Use of text book permitted
4. Assume any missing data suitably and clearly state and justify the same

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2 x 10 = 20 Marks

1. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

2. You might have seen dish antenna and bring the shape of the dish antenna in your answer as per the details given below. It is required to build a parabolic satellite dish, whose shape will be formed by rotating a parabolic curve $y^2 - ax^2$ about y-axis. If the dish is to have a diameter of 10 feet and a maximum depth of 2 feet, find the value of "a" and the surface area of the dish. (Hint: draw the figure as per specifications and identify the radius, value of "a" and express the given equation in terms of x, find the derivative of x with respect to y and use the maximum depth to find "a" and then apply integration technique, apply the formula, $2\pi \int x + \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$ with appropriate limits.

Presidency University, Bengaluru

School of Engineering

I Semester 2015-2016	COMPREHENSIVE EXAMINATION	Course: **MATH A 101 Engineering Mathematics I** (Open Book)

Max Marks:20 Max Time: 1 hour Weightage: 10 % 11th Jan' 2016 **SET B**

Instructions to Candidates

1. Write legibly.
2. Attempt all questions.
3. Use of text book permitted
4. Assume any missing data suitably and clearly state and justify the same

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2 x 10 = 20 Marks

1. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$.

2. a) Suppose that we have a fixed number of atoms or molecules of an ideal gas in a container that has a volume that can change with time, such as a balloon. The ideal gas law is PV = kT where k is determined by the number of atoms or molecules of the ideal gas. (Let k = 8 N.m/K). If the temperature is held constant at 320 K, what is the instantaneous rate of change of the pressure if the volume is 2 m^3 **(5)**

b) Resistors of resistances R_1, R_2, R_3....., R_n Ohms can be placed in parallel in a circuit to produce a resistor element with a new resistance R_{new} Ohms. And $R_{new} = (R_1^{-1} + R_2^{-1} + \ldots + R_n^{-1})^{-1}$. What is the instantaneous rate of change of the resistance in the new resistor element, with respect to one of the resistances R, while holding the other resistances constant? **(5)**

ID No.:										

Section No.:			Signature of Invigilator:	

Presidency University, Bengaluru

School of Engineering

I Semester 2015-2016 Quiz Course: **MATH A 101 Engineering Mathematics I**

(Closed Book)

Max Marks: 20 Max Time: 30 Min Weightage: 10% 17th Dec' 2015 **Set A**

Instructions to Candidates

1. Write legibly using pen only.
2. Do not overwrite.
3. Answer in the question paper itself, there will be no separate answer book provided.
4. Enter your ID No. and Section No. in the designated place

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20 x 1 = 20 Marks

1. State true or false: 4^{th} derivative of $(2x+1)^3$ is 0

2. Definition of a homogeneous function of two variables is____

3. When we differentiate an expression with respect to one of a number of independent variables, we are engaged in:

a) Total derivation b) Partial derivation c) Finding definite integrals d) Integration

4. The nth derivative of $\dfrac{1}{(ax+b)}$ is ________________

5. If $x = r\cos\theta,\ y = r\sin\theta$, then $\dfrac{\partial\theta}{\partial x}$ equals

(a) $\dfrac{\sin\theta}{r}$ (b) $\dfrac{\cos\theta}{r}$ (c) $\dfrac{-\sin\theta}{r}$ (d) $\dfrac{\tan\theta}{r}$

6. If $u(x,y)$ is a homogeneous function of degree n, then the value of the expression $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$ is

(a) $n(n+1)u$ (b) $n(n-1)u$ (c) nu (d) $(n-1)u$

7. A function U(x,y) satisfies the Laplace equation, then U is said to be________________ function.

8. Lagrange's multiplier method is used to solve (a) Unconstrained optima (b) Constrained optima

9. At a minimum turning point:

a) first derivative = 0 and second derivative is +ve

b) first derivative = -ve and second derivative is +ve

c) first derivative = 0 and second derivative is -ve

d) first derivative = +ve and second derivative is +ve

10. State True or False: If $f_1(u,v,w,x,y,z)=0,\ f_2(u,v,w,x,y,z)=0,\ f_3(u,v,w,x,y,z)=0$ are three implicit relations and if $u,v,\ w$ are implicit functions of $x,y,\ z$, then $\frac{\partial(u,v,w)}{\partial(x,y,z)}=(-1)^3\frac{\frac{\partial(f_1,f_2,f_3)}{\partial(u,\ v,\ w)}}{\frac{\partial(f_1,f_2,f_3)}{\partial(x,\ y,\ z)}}$.

11. If $A^T = -A$, A is called as........

12. Define a Linear Transformation

13. Characteristic equation of $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is ....

14. A system of homogeneous equations always has a solution: True or False?

15. Cayley Hamilton theorem states that ______________________________-
____

16. State True or False: Every square real matrix can be written as the sum of symmetric and skew-symmetric matrices

17. State True or False:

The product of two eigen values of the matrix A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is 3 and third eigen value of A is 1.

18. System of simultaneous linear equation is non-homogenous if the solution vector b is __________

19. If the matrix A is Hermitian, then the matrix $i\,A$ is

(a) Hermitian (b) Unitary (c) Orthogonal (d) Skew-Hermitian

20. State True or False: A^{-1} does not exist if 0 is an eigen value of the matrix A.

For official use (students shall not write beyond this line)

Marks scored out of 20 [ ]

Name and Signature of Examiner with Date

ID No.:										

Section No.:			Signature of Invigilator:	

Presidency University, Bengaluru

School of Engineering

I Semester 2015-2016 Quiz Course: **MATH A 101 Engineering Mathematics I**

(Closed Book)

Max Marks: 20 Max Time: 30 Min Weightage: 10% 17th Dec' 2015 **Set B**

Instructions to Candidates

1. Write legibly using pen only.
2. Do not overwrite.
3. Answer in the question paper itself, there will be no separate answer book provided.
4. Enter your ID No. and Section No. in the designated place

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20 x 1 = 20 Marks

1. State true or false: For u = u(x, y, z), $\frac{\partial^3 u}{\partial x \partial y \partial z} = \frac{\partial^3 u}{\partial z \partial y \partial x}$

2. If $\phi = y/x, x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} = ?$

3. If $y = 3w^4 - w^2x + x^2z - z^3$ then the partial derivative of y with respect to w is:
a) $-w^2 + 2xz$ b) $12w^3 - w^2x + 2xz$ c) $3w^3 - wx$ d) $12w^3 - 2wx$

4. If $y = \frac{1}{ax+b}$, then y_n equals

(a) $\frac{(-1)^n (n!) a^n}{(ax+b)^{n+1}}$ (b) $\frac{(-1)^{n-1} (n!)\, a^n}{(ax+b)^n}$ (c) $\frac{(-1)^n (n-1)!\, a^n}{(ax+b)^n}$ (d) $\frac{(-1)^{n-1} (n-1)!\, a^n}{(ax+b)^{n+1}}$

5. If $u = \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}$, then $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y}$ equals (a) $2u$ (b) $\frac{1}{2}u$ (c) $3u$ (d) $\frac{3}{2}u$

6. The asymptote of the curve $x^2y^2 - xy^2 + x + y + 1 = 0$ parallel to the x-axis is

(a) $y = 1$ (b) $y = 0$ (c) $y = -1$ (d) $x = y$

7. Expansion of cos(x) in Mclaurin's series up to 3rd degree terms is:………………….

8. When $rt - s^2 > 0, r < 0, the\ function\ is$ ....

9. State True or False: The expansion of $\sin x$ in powers of x is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+-----$

10. The Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for the functions $u=e^x \sin y$ and $v=x+\log\sin y$ is

(a) 1 (b) 0 (c) $\frac{e^x}{x}$ (d) $e^x \sin x \cos y$

11. Eigen values of $\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$ are……….

12. When do we say a system of equations inconsistent?

13. Say true or false: Eigen vector of a matrix can be a zero vector

14. If $A=\begin{bmatrix}1&0\\0&3\end{bmatrix}, A^{-1}=?$

15. Trace of the matrix is ____________________

16. State True or False: $\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$ is an orthogonal matrix

17. Let u and v are functions of two variables and their Jacobian is zero the u and v are said to be _____

18. Given that AX= b represents a system of simultaneous linear equations and the determinant of the matrix A is zero then the given system has _______ number of solution(s).

19. The rank of the matrix $\begin{bmatrix}1&1&1\\0&1&2\\0&-1&-2\end{bmatrix}$ is (a) 3 (b) 2 (c) 1 (d) 0

20. The nullity of the matrix $\begin{bmatrix}1&0&1&0&1\\0&1&-2&0&3\\0&0&0&1&-5\\0&0&0&0&0\end{bmatrix}$ is (a) 4 (b) 3 (c) 2 (d) 1

For official use (students shall not write beyond this line)

Marks scored out of 20

Name and Signature of Examiner with Date

ID No.:										

Section No.:			Signature of Invigilator:	

Presidency University, Bengaluru

School of Engineering

I Semester 2015-2016 Quiz Course: **MATH A 101 Engineering Mathematics I**

(Closed Book)

Max Marks: 20 Max Time: 30 Min Weightage: 10% 17th Dec' 2015 **Set C**

Instructions to Candidates

1. Write legibly using pen only.
2. Do not overwrite.
3. Answer in the question paper itself, there will be no separate answer book provided.
4. Enter your ID No. and Section No. in the designated place

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20 x 1 = 20 Marks

1. Fill in blanks: nth derivative of 1/(x+1) is...............
2. Differentiation refers to the process whereby we:

    a) Calculate the intercept of a curve with the vertical axis

    b) Calculate the area underneath a curve

    c) Calculate the gradient to a curve at any point on the curve

    d) Calculate the intercept of a curve with the horizontal axis

3. The nth derivative of sin(ax+b) = ____________________

4. State true or false: Laplace equation in two dimensions is $u_{xx} + u_{yy} = 0$

5. State true or false: The function $u = e^{x/y}\sin(x/y)$ is not a homogeneous function.

6. If $f(x,y) = c$ is an implicit function, then $\frac{dy}{dx}$ equals (a) $\frac{f_x}{f_y}$ (b) $\frac{-f_y}{f_x}$ (c) $\frac{-f_x}{f_y}$ (d) $\frac{f_y}{f_x}$

7. *If* $u = x + y, v = xy$, *what is* $\frac{\partial(u,v)}{\partial(x,y)}$?

8. At a maximum turning point:

a) first derivative = 0 and second derivative is +ve

b) first derivative = -ve and second derivative is +ve

c) first derivative = 0 and second derivative is -ve

d) first derivative = +ve and second derivative is +ve

9. State true or false: The expansion of $\log(1+x)$ in powers of x is $1-x+\frac{x^2}{2}-\frac{x^3}{3}+\frac{x^4}{4}+-----$

10. If $f(x,y)=1+x^2y^2$, then the stationary point of the function $f(x,y)$ is

(a) (0, 0) (b) (1, 0) (c) (0, 1) (d) (1, 1)

11. State Cayley-Hamilton's theorem

12. Rank of $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix}$ is.........

13. Say true or false: Set of all polynomials of degree 4 is a vector space

14. Relationship between the product of all eigen values and the determinant of the matrix ___________

15. Write a formula for the characteristic polynomial of a 2X2 matrix is ____________________

16. State True or False: If λ is an eigen value of a matrix, then $\frac{1}{\lambda}$ is an eigen value of the matrix A^{-1}

17. If A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ then inverse of A is _____________

18. State true or false: Any complex square matrix can be expressed as the sum of a Hermitian matrix and a skew-Hermitian matrix

19. The eigen values of the matrix $\begin{bmatrix} 2 & 5 & 13 \\ 0 & 3 & 9 \\ 0 & 0 & 4 \end{bmatrix}$ are

20. State true or false: The mapping $T:R \to R,\ T(x)=x^3$ is a linear transformation

Presidency University, Bengaluru
School of Engineering

I Semester 2015-2016 Test 1 Course: **MATH A 101 Engineering Mathematics I**
(Closed Book)

Max Marks: 30 Max Time: 50 Min Weightage: 15 % 19 October 2015 **Set A**

Instructions to Candidates
1. Write legibly
2. Attempt all questions serially, in order of question paper
3. Assume suitable data wherever necessary and justify the same.

Part A (5 X 1 = 5 Marks)

1. Find the nth derivative of $(a\,x + b)^n$
2. Find the derivative of $tan(5 - sinx)$
3. Choose the correct option: According to Euler's theorem,
$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} =$
(a) nu (b) $(n-1)u$ (c) $n(n-1)u$ (d) $n(n+1)u$
4. State TRUE or FALSE: z = f(x,y) represents a curve in 2D space
5. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, find u_x .

Part B (5 X 2 = 10 Marks)

6. Evaluate $\frac{d}{dx}(\frac{x+y}{x^2+y^2})$
7. If $x = r\,cos\theta, y = r\,sin\theta$, find $\frac{\partial r}{\partial x}$ and $\frac{\partial x}{\partial r}$.
8. If $u = f(x+ct) - g(x-ct)$ then prove that $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$
9. If $y = x \sin x$,find y_n
10. If $u = \sin^{-1}(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}})$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Part C (3 x 5 = 15 Marks)

11. If $u = e^{xyz}$, find $\frac{\partial^3 u}{\partial x\, \partial y\, \partial z}$.
12. State and prove extension of Euler's theorem
13. If $cos^{-1}(\frac{y}{b}) = nlog(\frac{x}{n})$ then prove that $x^2y_{n+2} + (2n+1)\,xy_{n+1} + n^2y_n = 0$

Presidency University, Bengaluru
School of Engineering

I Semester 2015-2016 Test 1 Course: **MATH A 101 Engineering Mathematics I**
(Closed Book)

Max Marks: 30 Max Time: 50 Min Weightage: 15 % 19 October 2015 **Set B**

Instructions to Candidates
1. Write legibly
2. Attempt all questions serially, in order of question paper
3. Assume suitable data wherever necessary and justify the same.

Part A (5 X 1 = 5 Marks)

1. State TRUE or FALSE: nth derivative of $(x+1)^{n-1}$ is 0

2. Choose the correct option: Euler's theorem is applicable only for
(a) Non-homogeneous functions (b) Homogeneous functions (c) both (d) neither

3. State Leibnitz's theorem.

4. If $u = f\left(\frac{x}{y}\right)$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

5. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^y$

Part B (5 X 2 = 10 Marks)

6. Verify $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ for $z = \sqrt{x+y^2}$

7. If $u = (1-2xy+y^2)^{-\frac{1}{2}}$ prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$

8. Find the total derivative of u with respect to t , where $u = y^2 - 4ax$ and $x = at^2$; $y = 2at$

9. If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}$.

10. Find the nth derivative of $\cos x\ \cos 2x$.

Part C (3 x 5 = 15 Marks)

11. State and prove extension Euler's theorem

12. If $\frac{1}{u^2} = x^2 + y^2 + z^2$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

13. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)\,y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.

Presidency University, Bengaluru
School of Engineering

I Semester 2015-2016 | Test 1 Makeup | Course: **MATH A 101 Engineering Mathematics I** (Closed Book)

Max Marks: 30 | Max Time: 50 Min | Weightage: 15 % | 27th Nov' 2015

Instructions to Candidates
1. Write legibly
2. Attempt all questions serially, in order of question paper
3. Assume suitable data wherever necessary and justify the same.

Part A (5 X 1 = 5 Marks)

1. Write the Laplace equation.
2. Define harmonic function.
3. Find the derivative of sec(5-4x)
4. Find the derivative of cosec(3-kx)
5. Find the nth derivative of $(ax + b)^{-1}$

Part B (5 X 2 = 10 Marks)

6. Verify Laplace equation for the function $z= - x^2 + y^2$
7. Verify Euler's theorem for the function $u = (8x^3 + y^2) (\log x - \log y)$
8. Find the nth derivative of $1/(x^2 - a^2)$
9. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$, where $u = x^2y + y^2x$
10. Compute $\frac{\partial^2 u}{\partial \theta^2}$ for u(x,y) where $x = r\cos\theta$; $y = r\sin\theta$

Part C (3 x 5 = 15 Marks)

11. State and Prove Euler's theorem for homogenous functions.
12. If $y = \sin(m \sin^{-1}x)$ then prove that $(1-x^2)y_{x+2} - (2n+1)y_{n+1} + (m^2 + n^2)y_n = 0$
13. If $u = \frac{x+y+z}{\sqrt{x}+\sqrt{y}+\sqrt{z}}$ then compute $x\frac{\delta u}{\delta x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$

Presidency University, Bengaluru
School of Engineering

I Semester 2015-2016 Test 2 Course: **MATH A 101 Engineering Mathematics I**
(Closed Book)

Max Marks: 30 Max Time: 50 Min Weightage: 15 % 14th Dec' 2015 **Set C**

Instructions to Candidates
1. Write legibly
2. Assume suitable data wherever necessary and justify the same.

PART A (5 X 1 = 5 Marks)

1. Say true or false: $(AB)^T = A^T B^T$.

2. Define orthogonal matrix.

3. State the condition (using rank) to be satisfied when the non-homogeneous system $AX = B$ has a unique solution.

4. The trace of a square matrix A is equal to the: (a) Sum of eigen values (b) Spectral radius (c) Product of eigen values (d) Mean of eigen values

5. In case a function $f(x,y)$ is found to have $f_x = 0$ *and* $f_y = 0$ at a point and $rt - s^2$ is found to be less than zero at that point (where $r = f_{xx}, s = f_{xy}$ *and* $t = f_{yy}$), then the point is __________.

PART B (5 X 2 = 10 Marks)

6. What is the maximum area of a rectangle whose perimeter is 20 cm?

7. Find the rank and nullity of $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$.

8. Write the series of $\log(1 + x)$

9. Find the Jacobian of $u = a\cosh x \cos y$ *and* $v = a \sinh x \sin y$

10. Find the inverse of $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$.

PART C (3 X 5 = 15 Marks)

11. Test the consistency and solve the system of linear equations

$$x_1 + 2x_2 - 3x_3 = 1, 2x_1 + x_2 + 4x_3 = 2 \text{ and } 3x_1 + 3x_2 + 4x_3 = 1$$

12. Find the minimum value of the function $x^2 + y^2 + xy + ax + by$.

13. A manufacturer can produce three different products in quantities p,q and r respectively and thereby derive a profit f(p,q,r) = 2p+8q+24r. Find the value of p,q and r that maximize the profit of production subject to the constraint $p^2 + 2q^2 + 4r^2 = 4,500,000,000$.

Presidency University, Bengaluru
School of Engineering

I Semester 2015-2016 Test 2 Course: **MATH A 101 Engineering Mathematics I**
(Closed Book)

Max Marks: 30 Max Time: 50 Min Weightage: 15 % 14th Dec' 2015 **Set A**

Instructions to Candidates
1. Write legibly
2. Assume suitable data wherever necessary and justify the same.

PART A (5 X 1 = 5 Marks)

1. Say true or false: Taylor's series is used to approximate functions.

2. What are the Eigen values of a diagonal matrix?

3. Define the rank of a matrix.

4. Define unitary matrix.

5. Write down Taylor series expansion for two variables.

PART B (5 X 2 = 10 Marks)

6. Expand $\tan^{-1} x$ in powers of (x-1).

7. Find Eigen values of $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$.

8. If λ is an Eigen value of a square matrix A then prove that $1/\lambda$ is an Eigen value of A^{-1}.

9. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

10. Find the inverse of $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$.

PART C (3 X 5 = 15 Marks)

11. Find the extreme value of the function $x^m y^n z^p$ when $x + y + z = a$

12. Find the minimum value of the function $x^2 + y^2 + xy + ax + by$.

13. Solve the equations $x + y + z = 9$, $2x + 5y + 7z = 52$ and $2x + y - z = 0$ using Gauss Jordan method.

Presidency University, Bengaluru
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I Semester 2015-2016 Test 2 Course: **MATH A 101 Engineering Mathematics I**
(Closed Book)

Max Marks: 30 Max Time: 50 Min Weightage: 15 % 14th Dec' 2015 **Set B**

Instructions to Candidates
1. Write legibly
2. Assume suitable data wherever necessary and justify the same.

PART A (5 X 1 = 5 Marks)

1. Say true or false: A matrix in invertible if its determinant= 0.

2. When two functions are said to be functionally dependent?

3. Write down the expansion of $\frac{1}{1-x}$ in powers of x.

4. Define saddle point.

5. Determine $\rho(A)$ if A is an $m \times n$ matrix.

PART B (5 X 2 = 10 Marks)

6. Write the Maclaurin's series for sinh(x).

7. Find $\frac{\partial(u,v)}{\partial(x,y)}$ if $u = x^2 + y^2$ and $v = xy$.

8. Find the Taylor's series expansion of $x^3 + xy^2$ about the point $(2,1)$ up to first degree.

9. Find $\frac{\partial(u,v)}{\partial(x,y)}$ if $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$.

10. Check and comment on the consistency of $x_1 + x_2 = 1, 2x_1 + 2x_2 = 2$

PART C (3 X 5 = 15 Marks)

11. Find the maximum and minimum of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$

12. Test for consistency and solve

$$\begin{aligned} x+2y-z&=3 \\ 3x-y-2z&=1 \\ 2x-2y+3z&=4 \\ x-y+z&=1 \end{aligned}$$

13. A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction.