Roll No												

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

PRESIDENCY UNIVERSITY
BENGALURU

Department of Research & Development

Mid - Term Examinations - AUGUST 2024

Odd Semester: Ph.D. Course Work
Course Code: MAT844
Course Name: Numerical Linear Algebra
Department: Mathematics

Date: 13-08-2024
Time: 09.30am to 11.00am
Max Marks: 50
Weightage: 25%

Instructions:

(i) Read the all questions carefully and answer accordingly.
(ii) Do not write any matter on the question paper other than roll number.

PART A (THOUGHT PROVOKING)

Answer all the Questions. Each question carries 5 marks. (4Q x 5M = 20M)

1. Let V be the set of all pairs (x, y) of real numbers. Let F be the field or real numbers. Define

$$(x, y) + (x_1, y_1) = (3y + 3y_1, -x - x_1)$$

$$c\,(x, y) = (3cy, -cx).$$

Verify that V with this operation, is not a vector space over the field of real numbers.

2. Define linearly independent and linearly dependent. Check whether or not the following vectors are linearly independent of R^3:

   (a) $(1, -2,1), (2,1, -1), (7, -4,1)$

   (b) $(2,3,5), (4,9,25)$

3. Determine whether or not the following vectors (1,2,1), (2,1,0), (1, -1,2), forms a basis of R^3.

4. If T is a linear transformation on a finite dimensional vector space V such that range $(T)$is a proper subset of V, show that there exists a non-zero element α in V with $T(\alpha) = 0$.

S

PART B (PROBLEM SOLVING)

Answer all the Questions. Each question carries 10 marks. **(3Q x 10M = 30M)**

5. Define the following matrices
   (i) Jordan matrix.
   (ii) Defective matrix.
   (iii) Non defective matrix.
   (iv) Positive definite matrix.

6. State and prove Cauchy schwarz inequality?

7. (i) Prove that every finite dimensional inner product space has an orthonormal basis.

   (ii) Apply the Gram Schmidt process to the vectors $\beta_1 = (1,0,1)$, $\beta_2 = (1,0,-1)$, $\beta_3 = (0,3,4)$, to obtain an orthonormal basis for $V_3\ (R)$ with the standard inner product space.